# Viterbi Detection of OFDM-CPM Signals

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Abstract— Recently, a new class of Orthogonal Frequency Division Multiplexing - Continuous Phase Modulation (OFDM-CPM) signals that out performs conventional OFDM-PSK signals was introduced. In these signals binary data sequence is mapped to complex symbols using the concept of correlated phase states of a CPM signal. While optimum and suboptimum minimum error probability algorithms have been employed to arrive at this superiority, in this paper we propose Viterbi algorithm for detection of OFDM-CPM signals and present their performance over multipath fading channels. It is shown that Viterbi receivers offer performance that is nearly the same as that of the minimum BER receiver and are easily implemented.

## I. Introduction

OFDM is a good candidate for wireless multimedia communication by virtue of its excellent properties in frequency-selective fading environment [1], [2]. In OFDM, data is transmitted over several parallel low data rate channels. This provides data integrity due to fading, relative to modulation methods that employ single channel for high data rate transmission. Among other benefits of OFDM is that it fully exploits the advantages of digital signal processing concepts [3].

A typical OFDM transmitter works as follows. Serial encoded data is sent to a mapper that outputs a complex number. The mapper could be PSK, QAM, DPSK or DAPSK. A serial-to-parallel converter serially takes in the complex numbers and forms a parallel stream by increasing their time period. This stream has as many complex numbers as the number of subchannels. Inverse fast Fourier transform (IFFT) is then applied to the parallel stream that results in orthogonal signals on the subchannels. The orthogonal signals are then converted back into a serial stream and after up converting the signal to desired carrier frequency the signal is transmitted.

At the receiver, the process described above is reversed. It starts with down converting the received signal. The resulting signal is then passed through a serial-to-parallel converter, a fast Fourier transform (FFT) block, parallel-to-serial converter and finally a de-mapper to eventually obtain the transmitted data sequence. In the absence of noise and fading, transmitted data is recovered without errors. In order to mitigate the effects of intersymbol interference (ISI) a guard interval is inserted at the transmitter and later removed at the receiver.

While in the literature OFDM-PSK, -QAM, -DPSK and -DAPSK have been considered [4]–[7], OFDM-CPM signals that use the concept of correlated phase states of a CPM signal have recently been introduced [8], [9]. In [10], [11], multiple-symbol-observation receivers have been proposed for the detection of OFDM-CPM signals. One of these is the minimum BER receiver that compares $n$ samples of the received signal with all the possible transmitted signals and arrives at an optimum decision on the data transmitted during the first bit interval. This would require the received samples to be compared with $2^n$ possible signals where $n$ is the observation interval. Thus, for longer symbol observations the receiver becomes uneconomical and too complex from the viewpoint of implementation. Therefore, we propose and investigate detection of OFDM-CPM signals using VA which performs maximum likelihood sequence estimation (MLSE) and is computationally less expensive than the minimum BER receiver.

The paper is organized as follows. In Section II we describe OFDM-CPM signaling scheme. We present the Viterbi detection algorithms in Section III. Numerical results are presented in Section IV and the paper is concluded in Section V.

## II. OFDM-CPM Signaling Scheme

As shown in Fig. 1, serial bit stream $b_i$, $i = 0, 1, 2, \ldots$, with bit duration of $T_b$ seconds is converted into blocks of $N$ bits represented by $a_{k,p}$,

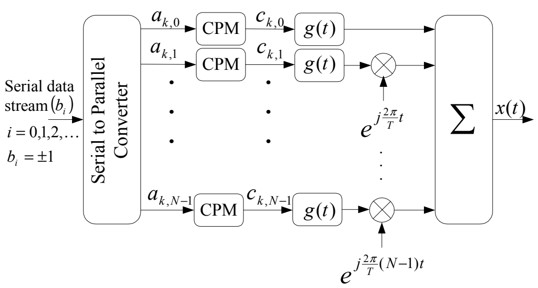

Fig. 1. OFDM-CPM Transmitter

$k = 0, 1, 2, \ldots,$ and $p = 0, 1, 2, \ldots, N-1$, where $N$ denotes the number of carriers and $a_{k,p} = \pm 1$. For example, $a_{0,p}$ would denote the first block of $N$ bits and $a_{1,p}$ the second block of $N$ bits and so on. The CPM mappers transform the incoming $a_{k,p}$ into appropriate complex numbers $c_{k,p}$ given by

$$c_{k,p} = \cos\left(\theta_{k,p}\right) + j\sin\left(\theta_{k,p}\right), \tag{1}$$

with

$$\theta_{k,p} = a_{k,p}\pi h + \pi h \sum_{q=0}^{k-1} a_{q,p} + \phi; \tag{2}$$

where parameter $h$ defines the CPM mapper and $\phi$ represents the initial mapping point that is assumed zero without loss of generality. The angles $\theta_{k,p}$ depend not only on the current data but also on the past data. Fig. 2 shows values of $\theta_{k,p}$ as a function of time when $h = \frac{1}{2}$. Current value of $\theta$ is determined by adding $+\pi h$ (if data bit is a $+1$) or $-\pi h$ (if data bit is a $-1$) to the previous value of $\theta$. The corresponding complex numbers lie on a circle.

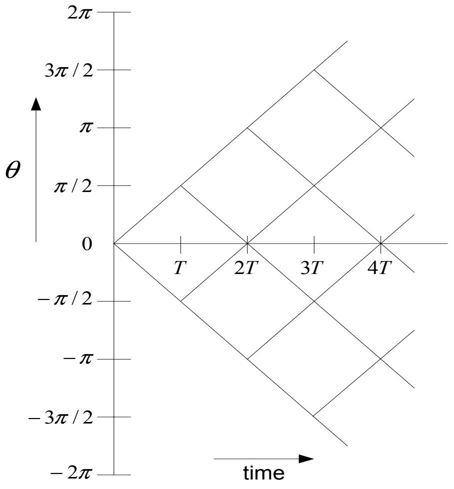

Fig. 2. Phase trellis for OFDM-CPM signaling

The complex numbers from the output of CPM mappers are passed through pulse shaping filters $g(t)$, then modulated by orthogonal carriers and finally summed to give the transmitted OFDM symbol which is mathematically represented as

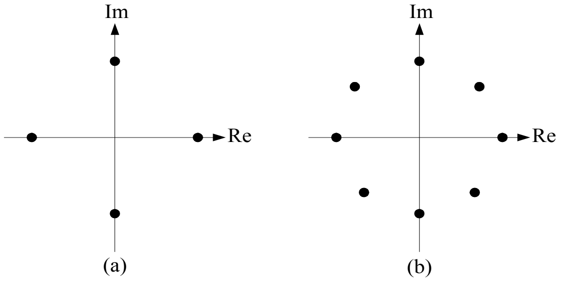

Fig. 3. Constellation diagram of CPM mapper for (a) $h = 0.5$ and (b) $h = 0.25$

$$x(t) = \sum_k \sum_p c_{k,p} g(t - kT') e^{j\frac{2\pi}{T}pt}, 0 \le t < \infty \tag{3}$$

where

$$g(t) = \begin{cases} \frac{1}{\sqrt{T}} & -T_g \le t \le LT \\ 0 & t < -T_g, t > LT. \end{cases} \tag{4}$$

In Eq. (3), $T (= NT_b)$ is the OFDM symbol duration, $T_g$ is guard interval (assumed to be sufficient to remove ISI completely), $T' = T_g + LT$ is the observation period and the value of $L$ is chosen depending upon whether the scheme is full response ($L = 1$) or partial response ($L > 1$). In full response signaling, the incoming bit during current symbol interval influences the phase only in the current symbol interval unlike in partial response signaling where this bit would influence phases in the subsequent $L$ symbols as well.

The parameters $h$ and $L$ can be chosen to obtain several subclasses of OFDM-CPM signals. Some of these are described next.

### A. OFDM-CPFSK Signals

In this case, the value of $h$ remains constant for all OFDM symbols. By choosing $h$ to be rational and $0 < h < 1$ it is possible to have finite number of points in the CPM constellation. In Fig. 3 is shown the constellation diagram of CPM mapper for $h = \frac{1}{2}$ and $h = \frac{1}{4}$.

### B. OFDM-multi-$h$ CPFSK Signals

The value of $h$ is cyclically chosen from a set of $K$ values. That is, the value of $h$ employed during the $i$th symbol is given by $h_l$, $l = i$ modulo $K$ [12].

For example, the complex numbers of a 4-carrier OFDM-CPM signal with $H_2 = \{\frac{1}{2}, \frac{1}{4}\}$ for first two

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blocks of data sequences are shown below (assuming initial mapping points to be $1 + j0$):

$$[a_{k,p}, a_{k+1,p}] \implies [c_{k,p}, c_{k+1,p}]\,,$$

$$\begin{bmatrix} +1 & -1 \\ +1 & +1 \\ -1 & +1 \\ +1 & -1 \end{bmatrix} \implies \begin{bmatrix} +j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ +j & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ -j & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \\ +j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \end{bmatrix}.$$

### C. OFDM-multi-$h$ CPFSK with Asymmetric Parameters

While in OFDM-multi-$h$ CPFSK signals $h$ values are independent of data bits $a_{k,p}(= \pm 1)$, in asymmetric multi-$h$ signals $h$ is made a function of $a_{k,p}$. That is, the value of $h$ during the $i$th symbol interval is chosen $h_{+i}$ or $h_{-i}$ accordingly as data is a $+1$ or $-1$ respectively. This gives additional flexibility to the designers to optimize system performance.

### D. Partial Response OFDM-CPFSK Signals

In Eq. (4), by making $L > 1$ the pulse duration can be extended to more than one OFDM symbol. Using a value of $L = 2, 3, \ldots$ systematic correlation can be furthered amongst OFDM symbols which in turn can be exploited for improvement in system performance.

## III. Viterbi Detection

In 1967, A.J. Viterbi [13] proposed Viterbi Algorithm (VA) as a method for decoding convolutional codes. VA can be regarded as a recursive optimal solution to the problem of estimating the state sequence of a discrete-time finite-state Markov process observed in memoryless noise.

In [10], [11], multiple-symbol-observation receivers have been proposed for the detection of OFDM-CPM signals. One of these is the minimum BER receiver that compares $n$ samples of the received signal with all the possible transmitted signals and arrives at an optimum decision on the data transmitted during the first bit interval. This would require the received samples to be compared with $2^n$ possible signals where $n$ is the observation interval. Thus, for longer symbol observations the receiver becomes uneconomical and too complex from the viewpoint of implementation. Therefore, we propose and investigate detection of OFDM-CPM signals using VA which performs maximum likelihood sequence estimation (MLSE) and is computationally less expensive than the minimum BER receiver.

### A. VA for Detection of OFDM-CPFSK Signals

VA can be used for the detection of OFDM-CPFSK signals when the parameter $h$ is chosen as $p/q$ ($p, q$ integers). Then the number of states at each symbol interval are $q$ if $p$ is even and $2q$ if $p$ is odd [14]. Figure 4 shows an example of possible states (i.e., $\theta$ and corresponding complex numbers) and paths through the trellis traversed by data sequences with $h = 2/3$ for an arbitrary subcarrier. Starting from state $\theta = 0$ ($c = 1 + j0$), one can trace a particular path based on the data sequence using (2). For illustration, a data sequence 1101 is also shown using thick lines.

The algorithm involves calculating a measure of similarity or distance, between the received signal at an instant $i$ and all the trellis paths entering each state at instant $i$. The Viterbi algorithm removes from consideration those trellis paths that could not possibly be candidates for the maximum likelihood choice. When two paths enter the same state, the one having the best metric is chosen and is called the surviving path. Selection of surviving paths is performed for all the states. The algorithm continues in this way to advance deeper into the trellis, making decisions by eliminating the least likely paths. The decisions made in this way are not truly maximum likelihood but they can be made almost as good provided that the decision depth is long enough.

The metrics or distances between the received sequence and the likelihood sequence are computed as follows. Let $\hat{c}_{k,p} = \hat{u}_{k,p} + j\hat{v}_{k,p}$ represent the received sequence of complex numbers at the output of FFT for $p$th subcarrier with $k = \{1, 2, \ldots, m\}$ and $m$ being decision depth. Also, let $\theta_{k,p}$ represent the phases (or states) of possible transmitted sequences that correspond to complex numbers given by $c_{k,p} = u_{k,p} + jv_{k,p}$. Then the metric $d_{k,p}$ is the squared distance between the two sequences of complex numbers and is given by,

$$d_{k,p} = (u_{k,p} - \hat{u}_{k,p})^2 + (v_{k,p} - \hat{v}_{k,p})^2$$

These metrics are successively updated at each symbol interval and all possible state transitions are extended. All paths are deleted at the next symbol interval except the path with the highest likelihood. Once the whole signal sequence is traced, all candidate paths terminate in the same common node at the far end of the trellis and the most likely of these is the desired sequence.

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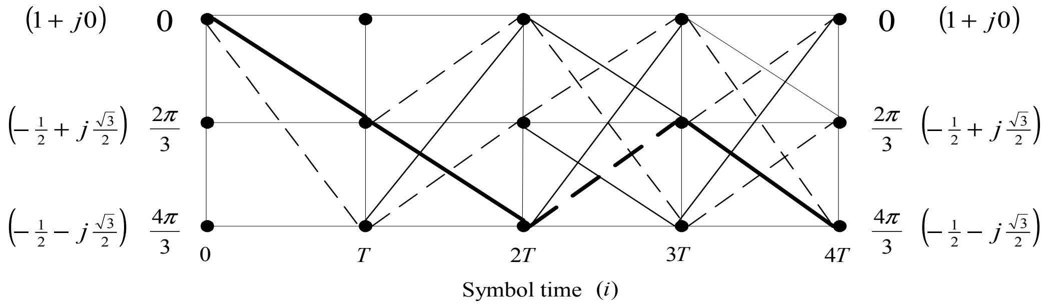

Fig. 4. Possible paths through trellis for $h = 2/3$. A data sequence 1101 is also shown using thick lines. (Legend: solid line - data 1, broken line - data 0)

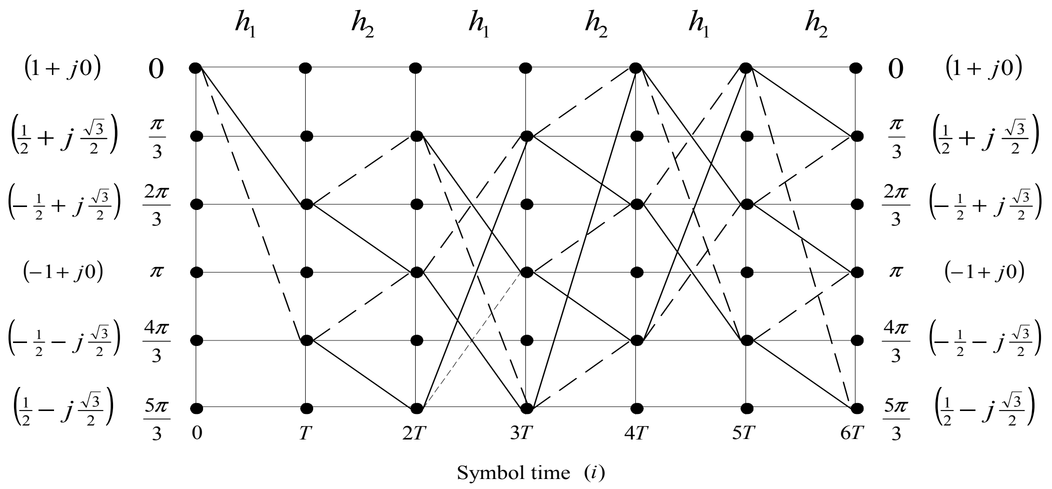

Fig. 5. Possible paths through trellis for $H_2 = \{2/3, 1/3\}$.(Legend: solid line - data 1, broken line - data 0)

### B. VA for Detection of OFDM-multi-$h$ CPFSK Signals

Figure 5 shows an example of possible states and paths through the trellis for an OFDM-multi-$h$ CPFSK scheme with $H_2 = \{2/3, 1/3\}$. Starting from state $\theta = 0$ ($c = 1 + j0$), one can trace a particular path based on the data sequence using (2). In this scheme, the value of $h$ is not constant for each interval and is chosen from the set $H_2$. For this particular case, the parameter $h$ alternates between 2/3 and 1/3.

There will be $2q$ distinct states when the $h$ values are chosen such that they have a common denomenator, for example, for a 2-$h$ scheme, $H_2 = \{p_1/q, p_2/q\}$. However, the Viterbi algorithm has to keep track of only $q$ states at any symbol interval as is shown in Fig. 5. The desired sequence is decided much the same way as is done for OFDM-CPFSK signals by successively updating metrics at each symbol interval.

## IV. Numerical Results

The performance of proposed decoders was evaluated using a two ray multipath channel and was compared with the minimum BER receiver proposed in [10], [11]. Impulse response of the channel is given by [15],

$$h(t) = \alpha_1\delta(t) + \alpha_2\delta(t-\tau), \tag{5}$$

where $\alpha_1$ and $\alpha_2$ are complex Gaussian random variables and $\tau$ is the delay between the direct and indirect ray. A 32-carrier OFDM-CPM system was simulated having a guard interval of 8 $\mu$s, $\mathrm{E}\left\{|\alpha_1|^2\right\}/\mathrm{E}\left\{|\alpha_2|^2\right\} = 10$ dB and $\tau = 6$ $\mu$s.

Figure 6 shows a plot of BER as a function of SNR for two OFDM-CPFSK schemes with $h = 4/5$ and $h = 4/7$. Solid and broken lines indicate the performances obtained by using minimum BER receivers that arrive at an optimum decision based on $n$ samples while stars and squares show the performances of Viterbi decoders. The decision depth of Viterbi decoder is 7 for both the signaling schemes. The decoder has to keep track of 5 states for $h = 4/5$ and 7 states for $h = 4/7$. It is observed that the performances of Viterbi decoders and minimum BER receivers are nearly the same.

Figure 7 shows a plot of BER as a function of SNR for an OFDM-multi-$h$ CPFSK scheme with $H_2 =$

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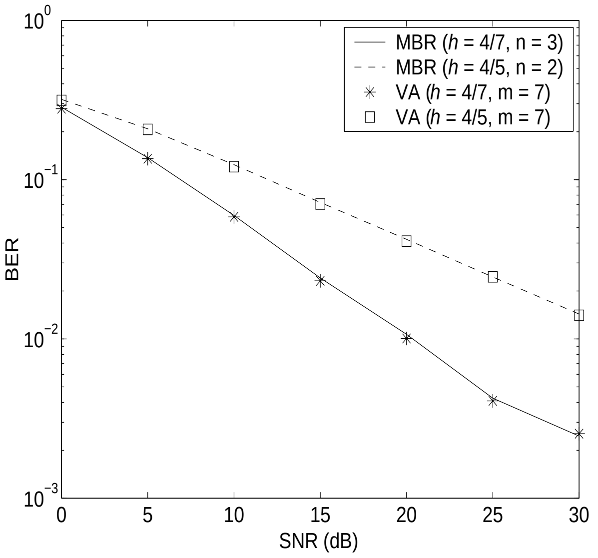

Fig. 6. Bit error rate versus SNR for two OFDM-CPFSK schemes with $h = 4/5, 4/7$.

$\{8/16, 9/16\}$. The decision depth in the Viterbi decoder is 4. It is noted that the performances of Viterbi decoders and minimum BER receivers are nearly the same. It is also noted that OFDM-multi-$h$ CPFSK scheme achieves the same error performance as that of OFDM-CPFSK ($h = 4/7$) with a decision depth of only 4.

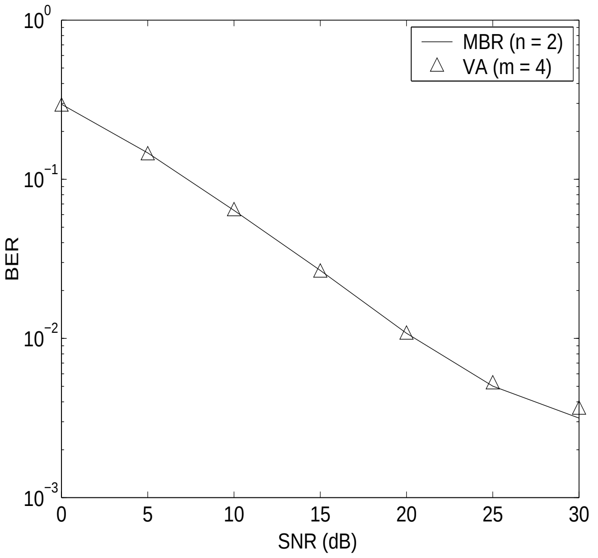

Fig. 7. Bit error rate versus SNR for an OFDM-multi-$h$ CPFSK scheme with $H_2 = \{8/16, 9/16\}$

## V. Conclusions

Viterbi algorithm is an attractive alternative for detection of OFDM-CPM signals. The only restriction while using VA is that the parameter $h$ is required to be rational. In the paper, we have presented the Viterbi algorithm for OFDM-CPFSK and OFDM-multi-$h$ CPFSK signals. However, the technique can be easily extended to other subclasses of OFDM-CPM signals.

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