On the Mathematical Modeling of Arcs in AC Circuit Breakers

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ABSTRACT

This paper is aimed at modeling arcs in an ac circuit breaker protecting a short transmission line. The model is based on combining Cassie and Mayr differential equations. The characteristics of the arc struck between the breaker contacts are investigated. These characteristics include the arc current, voltage and conductance. The dependency of the characteristics on the transmission line inductance, the source frequency and the arc time constant is discussed. The calculated temporal variation of the arc current agreed reasonably with oscillograms recorded experimentally for arcs ignited in a horn gap simulating circuit breaker arcs.

1. Introduction

The growth of short-circuit currents of power systems has brought into prominence the short-line fault limitation of circuit breakers, especially those of air-blast or SF_6 type [1]. These faults cause high initial rates of rise of the breaker terminal voltage at current zero that the breaker arc may be reignited long before fault system recovery voltage has time to appear. Physically, such reignition results from an early restoration of the energy balance required for maintenance of the conducting arc column. Calculation of the energy relations involved requires knowledge not only of the transient circuit properties, but also the dynamic arc characteristics. Thus, the breaker arc phenomena are extremely complex in detail so that complete mathematical models of the arc comparable in simplicity and accuracy with engineering models of the connected system are not possible.

As the Alternating current does not need an artificial interruption of the arc, therefore small circuit breakers are capable of interrupting very considerable resistive currents. However, the performance becomes less favorable if inductive circuits are to be interrupted such as short-line faults. The current will have a substantial phase displacement with respect to the voltage. Thus, the zero passage of the current no longer occurs at low instantaneous voltage.

This paper is aimed at modeling reignited arcs in ac circuit breakers protecting a short transmission line to obtain the temporal variations of the arc voltage and current over the successive half cycles of the applied voltage. The effects of the arc time constant, supply frequency and line inductance on the arc current, voltage and conductance are discussed.

2. Method of Analysis

Figure 1 shows a schematic representation of a power system which consists of a short-line connected to a high-voltage source through a step-up transformer and a circuit breaker. The objective is to determine the arc voltage v_a, the arc current i_a and the arc conductance g_a.

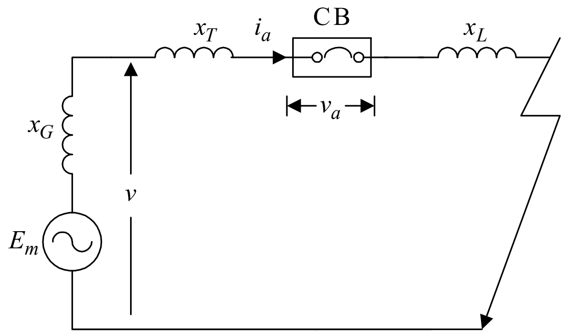

Fig. 1 A power circuit with a fault at the end of a short line protected by a circuit breaker.

The proposed arc model is based on combining Cassie [2] and Mayr [3] differential equations being derived on principal power loss and energy storage mechanisms in the arc column [4]. The model employs the nonlinear equations; Cassie equation before current zero until the arc conductance g_a reached the linearized solution value $g_o = \omega\theta I/E_o$; then the Mayr equation until g_a reached a minimum value g_{min} before starting to increase, then the Cassie equation again but with a new steady-state arc voltage $E_o'(= E_o\sqrt{g_o/g_{min}})$ corresponding to the computed conductance g_{min}. The angular frequency of the supply voltage is ω, I is the rms value of interrupted current in amperes, E_o is the near-constant arc voltage just before its sudden drop as the falling current approaches zero and θ is arc time constant.

2.1 Describing equations of the arc

The arc voltage v_a, the arc current i_a and hence the arc conductance g_a as functions of the time are obtained through the solution of Cassie and Mayr differential equations. Cassie equation states [1]:

$$(1/g_a)dg_a/dt = 1/\theta\left((v_a/E_o)^2 - 1.0\right) \quad (1)$$

Mayr equation states that:

$$(1/g_a)dg_a/dt = 1/\theta\left(v_a i_a/N_o - 1.0\right) \quad (2)$$

Where N_o is the Mayr equation constant. The rms value of the interrupted short-circuit current I_{sc} in the circuit of Fig. 1 is considered the initial value of the arc current. The value of N_o is expressed as:

$$N_o = I_{sc}\,\omega\,\theta\,E_o \qquad (3)$$

The value g_o is the arc conductance at time at which the current equals zero. It is expressed as:

$$g_o = I_{sc}\,\omega\theta / E_o \qquad (4)$$

2.2 Evaluation of initial conditions

The solution of the differential equations of Cassie (or Mayr) calls for initial conditions of the circuit shown in Fig. 1, where the circuit resistance is neglected. x_L is the reactance of the line connecting the horn gap to the source, x_G is the internal impedance of the source, E_m is the maximum source voltage

As the current is considered linear prior current zero [2], then the arc current in Cassie mode can be expressed as:

$$i_a = \sqrt{2}\,I_{sc}\,\omega\,t$$

As the calculation for Cassie mode starts at time τ before the current zero, so the initial conditions are expressed as:

$$i_a = \sqrt{2}\,I_{sc}\,\omega\,\tau \qquad (5)$$

$$v_a = E_o \qquad (6)$$

$$g_a = i_a / v_a \qquad (7)$$

2.3 Calculation of E_o

As the circuit of Fig. 1 is considered an inductive circuit, the source voltage is at the peak value near the current zero. The loop equation for the circuit of is written as:

$$E_m = \big((x_L + x_T + x_G)/\omega\big)\,di/dt - E_o$$

$$\therefore\ di/dt = (E_m + E_o)\,\omega/(x_L + x_T + x_G) \qquad (8)$$

The terminal voltage of the source v is expressed as:

$$v = ((x_L + x_T)/\omega)di/dt - E_o \qquad (9)$$

Eliminating di/dt from equations (8) and (9), one obtains

$$v = E_m(1-F) - FE_o \qquad (10)$$

where $F = x_G/(x_L + x_G)$

Assume $v = 0.5\,E_m$ [5] when the circuit breaker starts to open, then the initial arc voltage E_o can be obtained from eqn. (9) as

$$E_o = \big(E_m(1-F) - 0.5E_m\big)/F \qquad (11)$$

2.4 Calculation of the initial short-circuit current I_{sc} and circuit voltage V

The short circuit current I_{sc} is determined by the loop impedance of the circuit of Fig. 1 and is expressed as

$$I_{sc} = E_m/(x_G + x_T + x_L) \qquad (12)$$

The terminal voltage v before opening the circuit breaker (at current zero) can be expressed as:

$$v = E_m - I_{sc}x_G \qquad (13)$$

2.5 Cassie-Mayr-Cassie model

25.1 First Cassie mode

Cassie equation is applied before current zero until the current becomes equal to zero. With reference to Cassie equation (1), the change of the arc conductance dg_a in a time interval dt is determined as:

$$dg_a = \big(g_a/\theta\big)\big((v_a/E_o)^2 - 1.0\big)\,dt$$

so the updated value of the arc conductance is:

$$g_{a\,new} = g_{a\,old} + dg_a \qquad (14)$$

The variation of the arc current di_a over the time interval dt is evaluated as:

$$di_a = \sqrt{2}\,I_{sc}\,\omega\ dt$$

so the updated value of the arc current is:

$$i_{a\,new} = i_{a\,old} + di_a \qquad (15)$$

With the knowledge of how the current and the conductance of the arc are changing with time, the updated arc voltage is calculated as

$$v_{a\,new} = i_{a\,new}/\,g_{a\,new} \qquad (16)$$

As the circuit equation of Fig. 1 is written as:

$$(x_G/\omega)\,di/dt = E_m - v \qquad (17)$$

Then, the circuit voltage $v(t)$ is expressed as

$$v_{new} = E_m - (x_G/\omega)(di_a/dt) \qquad (18)$$

The Cassie mode is determined when the arc current drops to the zero value and the arc voltage also reaches zero. The arc conductance reaches the value g_o.

2.5.2 Mayr Mode

This mode starts following Cassie mode and the last values obtained from Cassie mode are considered the initial conditions of Mayr's mode, i.e: $i_a = 0$, $v_a = 0$, $g_a = g_o$, $v = v_{new}$ of eqn. (18)

With reference to Mayr equation (2), the change of the arc conductance dg_a in the time interval dt is determined as

$$dg_a = \big(g_a/\theta\big)\big((v_a\,i_a/N_o) - 1.0\big)\,dt$$

so the updated arc conductance is:

$$g_{a\,new} = g_{a\,old} + dg_a \qquad (19)$$

From the circuit equation (16), the change of arc current di_a in the time interval dt is

$$di_a = (E_m - v)/(x_G\,dt/\omega)$$

so the updated value of the arc current is:

$$i_{a\,new} = i_{a\,old} + di_a \qquad (20)$$

The arc voltage is determined as

$$v_{a\,new} = i_{a\,new}/\,g_{a\,new} \qquad (21)$$

The updated circuit voltage of Fig. 1 is also written as

$$(x_L / \omega)\, di / dt = v - v_a \quad (22)$$

Combining eqns. (17) and (22) to obtain the updated circuit voltage:

$$v_{new} = \frac{E_m + A v_{a\,new}}{A+1} \quad (23)$$

where $A = x_G / x_L$

The Mayr mode terminates when the arc conductance reaches its minimum value $g_{\min}$.

2.5.3 Second Cassie mode

This mode is based on Cassie equation when the arc conductance starts to increase. The Cassie assumption of constant arc voltage E_o is change to $E_o^{'}$ as

$$E_o^{'} = E_o\,(g_o / g_{\min}) \quad (24)$$

The last values obtained from Mayr mode are considered the initial values of the second Cassie mode.

From Eqn. (1); one can write:

$$dg_a = \big(g_a/\theta\big)\big((v_a / E_o^{'})^2 - 1.0\big)\, dt$$

so the updated arc conductance is:

$$g_{a\,new} = g_{a\,old} + dg_a \quad (14)$$

As

$$g_a = i_a / v_a \quad (25)$$

$$dg_a / dt = d / dt\,(i_a / v_a) = \big(i_a\, dv_a / dt - v_a\, di_a / dt\big) / v_a^2$$

From eqns. (1), (25) and (26), one can obtain

$$dv_a = \Big((1/\theta)\big((v_a / E_o^{'})^2 - 1.0\big) + (1/i_a)\, di_a / dt\Big) v_a\, dt \quad (26)$$

so the updated arc voltage is:

$$v_{a\,new} = v_{a\,old} + dv_a \quad (27)$$

The updated arc current is determined as

$$i_{a\,new} = v_{a\,new}\, g_{a\,new} \quad (28)$$

The circuit voltage is also determined from eqn. (22).

3. Results and Discussion

The investigated system described by the circuit shown in Fig. 1 has the following data [6] for the base case:

The generator has the following parameters:

Substation reactance $x_G = 0.26\ pu$

Rated $MVA = 420\, MVA$

Rated voltage $= 18\, kV$

Frequency $= 50\, Hz$

The step-up transformer has the following parameters:

Reactance $x_T = 0.15\, pu$

Rated $MVA = 420\, MVA$

Voltage ratio $= 18/220\, kV$

The gaseous transmission line has the following parameters:

Reactance $x_L = 0.13357\, pu$

The calculated temporal variation of the arc conductance is shown in Fig. 2, where the calculation is started at $2\mu s$ before current zero. When the current equals zero, the arc conductance g_a equals $0.28x10^{-4}\, S$ for the base case. After the current zero, the arc conductance continuous to decrease until it reaches its minimum value $g_{\min}$ at $0.27x10^{-4}\, S$. Following the minimum value, the arc conductance starts to increase again with time indicating unsuccessful interruption of the arc. The arc conductance depends on the line reactance being represented by the factor F of eqn. (10) as shown in Fig. 2. The conductance depends also on the arc time constant, Fig. 3, and the source frequency, Fig. 4.

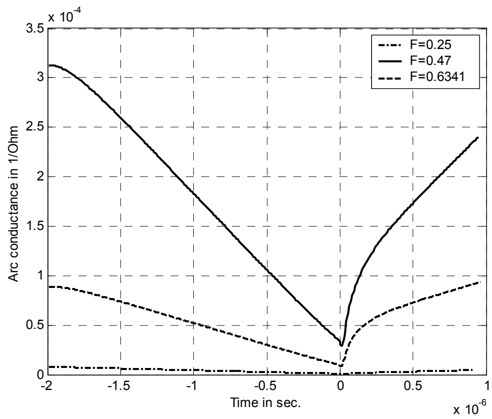

Fig.2 Temporal variation of arc conductance as influenced by line inductance.

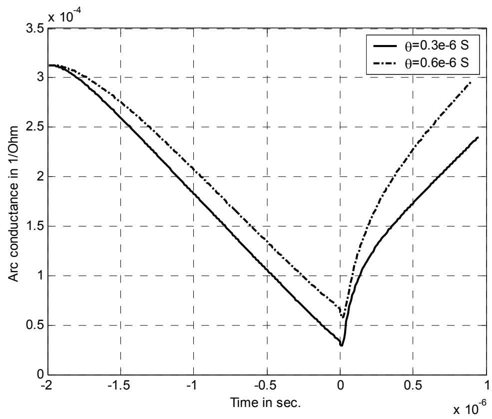

Fig. 3 Temporal variation of arc conductance as influenced by arc time constant.

The calculated temporal variation of the arc voltage is shown in Fig. 5, where the calculation is started at $2\mu s$ before current zero. When the current equals zero, the arc voltage is also zero. The rate of rise of the arc recovery voltage before current zero is 46 $kV/\mu s$ for the base case. The arc voltage depends on the line reactance being represented by the factor F as shown in Fig. 5. The conductance depends also on the arc time constant, Fig. 6, and the source frequency, Fig. 7.

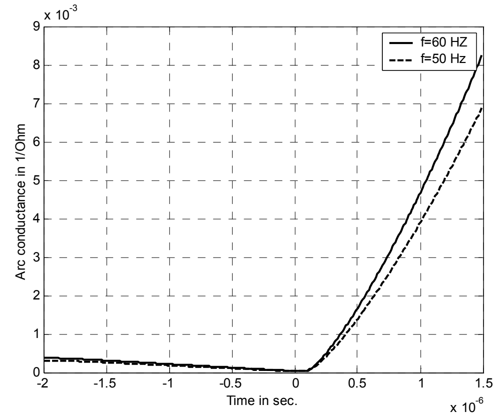

Fig. 4 Temporal variation of arc conductance as influenced source frequency.

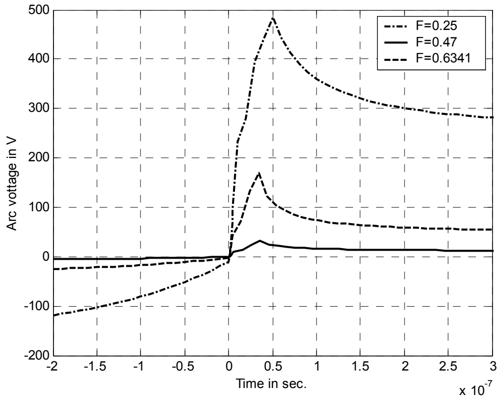

Fig. 5 Temporal variation of arc voltage as influenced by line inductance.

The first peak of the arc voltage for the base case reaches 33 kV at the same instant the arc conductance is minimum as shown in Fig. 5. Following the peak, the arc voltage decreases to an almost constant value indicating that the arc is struck between the breaker contacts, i.e. the arc is not successfully interrupted. This is confirmed in Figs. 5-7, where the arc voltage tends to approach a constant value corresponding to what was named before [2] the "burning voltage" of the arc. The positive sign of the burning voltage holds for positive current and the negative sign holds for negative current.

The calculated temporal variation of the arc current is shown in Fig. 8 for the base case. Before the current zero by $2\mu s$, the value of the arc current declines approaching zero, and increases after current zero indicating unsuccessful interruption of the arc between the circuit breaker contacts. The rate of change of the arc current is 0.85 $A/\mu s$ before current zero for the base case. The arc current depends on the line reactance being represented by the factor F as shown in Fig. 8. The arc current depends also on the arc time constant, Fig. 9, and the source frequency, Fig. 10.

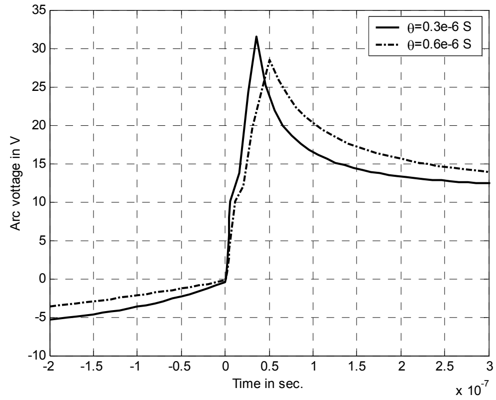

Fig. 6 Temporal variation of arc voltage as influenced by arc time constant.

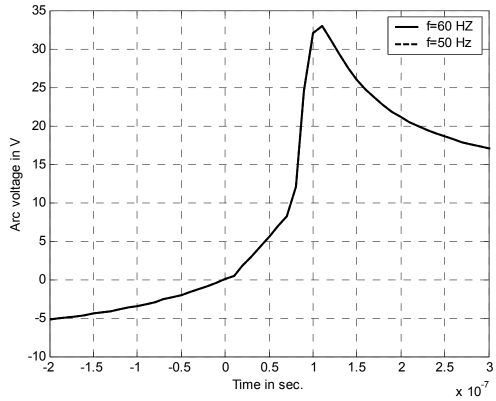

Fig. 7 Temporal variation of arc voltage as influenced by arc source frequency.

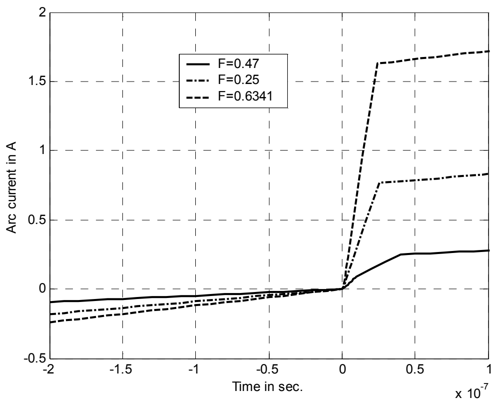

Fig. 8 Temporal variation of arc voltage as influenced by line inductance.

It has been reported before [2] that the arc voltage at current zero will be equal to or greater than the instantaneous value of the system voltage and the arc cannot reignite immediately after its extinction. There develops an interval through which the arc current stays at zero until the system voltage again has reached the arc voltage. During this interval the voltage in the external circuit also disappears, resulting in a very distorted curve shape for this voltage. This is not in agreement with the present calculated temporal variations of the arc current, Figs. 8-10 and the recorded oscillograms [7] which did not show any marked interruption (i.e. zero value) of the arc current. Therefore, the arc current cannot be considered as composed of two components [5], the first of which represents the steady-state sinusoidal inductive current developing with entirely closed circuit-breaker and lagging by 90^0 behind the voltage. The second component represents a current which changes linearly with time.

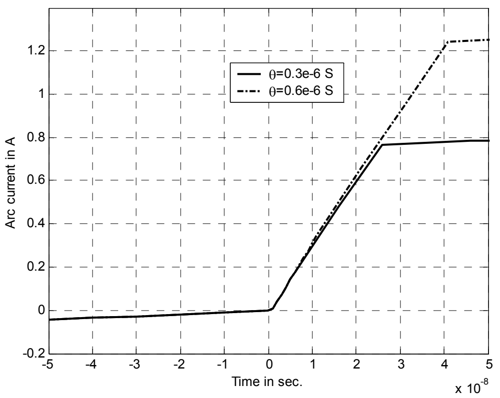

Fig. 9 Temporal variation of arc voltage as influenced by arc time constant.

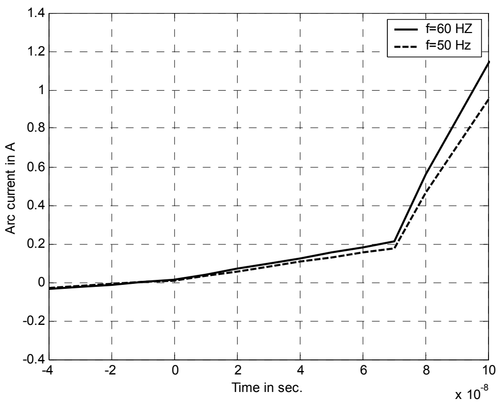

Fig.10 Temporal variation of arc voltage as influenced by source frequency.

To simulate the reignited arc in a circuit breaker in the laboratory, the circuit breaker in Fig. 1 is replaced by a horn gap and the arc current is recorded on an oscilloscope. On the base of $30\,kVA$, the reactance $x_G\ (=.04\,pu)$ is that of the source voltage in the laboratory. The reactance $x_T\ (=.0112\,pu)$ is that of the transformer stepping-up $(0.22/150\,kV)$ the source voltage to trigger the arc in the horn gap. The reactance x_L $(3.9x10^{-7}\ pu)$ is that of the circuit connecting the gap to the high voltage source. The capacitance at generator terminals is ignored. Fig. 11 shows the calculated arc-current waveform against that recorded in the laboratory. The calculated temporal variation of the arc current agreed reasonably with oscillograms recorded experimentally for arcs reignited in a horn gap representing circuit breaker arcs in ac circuits.

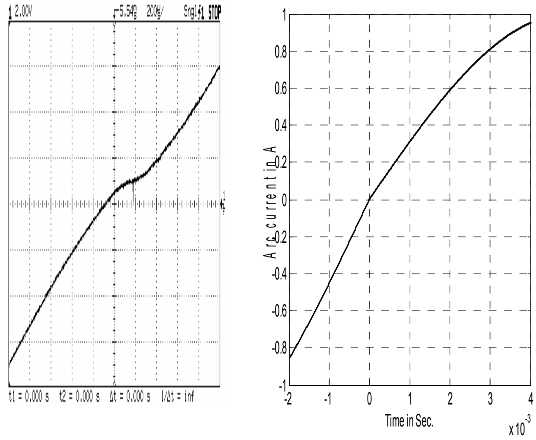

Fig. 11 Pu recorded and calculated arc current waveform (plotted to the same scale).

4. Conclusions

On the basis of the present analysis the following conclusions may be drawn:

1) A model is developed for arcs struck between the contacts of a circuit-breaker protecting a short transmission line. The model is based on combining Cassie and Mayr non-linear differential equations of the arc.
2) The temporal variations of the arc current, voltage and conductance are determined. The dependency of these variations on the transmission line inductance, the source frequency and the arc time constant is discussed.
3) The calculated temporal variation of the arc current agreed reasonably with oscillograms recorded experimentally for arcs ignited in a horn gap simulating circuit breaker arcs in laboratory.

References

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