A Novel Adaptive EEVSLMS Algorithm

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***Abstract* __ In this paper, we propose a new adaptive error estimation Variable Step-Size Least Mean Square (VSLMS) algorithm in which, we use the error for weight adjustment based on previous error estimations. Also stability of this adaptive algorithm is investigated. Simulation results show that it has a faster convergence rate compared with the VSLMS algorithm.**

***Index Terms* __ adaptive filter, error estimation, VSLMS, LMS.**

I. INTRODUCTION

The LMS algorithm is the simplest and the most universally applicable adaptive algorithm to be used which uses a gradient-based method of steepest decent. This algorithm is used in different applications, due to its simplicity. Note that, the convergence rate of the LMS algorithm is proportional to the step-size parameter [1]-[3]. Small value of this parameter results in a slow convergence rate. Hence, the VSLMS algorithm proposed in [1],[4],[6],[7] has a faster convergence rate than the LMS algorithm. Also, a new method based on error estimation presented in [5] that is not appropriate for online applications. In this paper, Error Estimation VSLMS (EEVSLMS) adaptive algorithm is proposed based on error estimation by means of previous errors. In this approach, coefficient adjustment is not computed by the last error value but with several previous values.

This paper is organized as follows: In Section II, the VSLMS algorithm expressed. The proposed algorithm is described in Sections III and IV. Finally, simulation results and conclusions are presented in Sections V and VI, respectively.

II. VSLMS ADAPTIVE FILTER

The process of adaptive adjustment is consisting of error computation and update of filter weights vector. The structure of an adaptive filter is shown in Fig. 1, where $X(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T$, $y(n)$, $d(n)$, $W(n) = [w_1(n), w_2(n), \ldots, w_N(n)]$ and N respectively denote, the input vector of the system at time n, corresponding output, is the desired signal, filter weights vector and the filter length. In the VSLMS algorithm unlike the LMS, the amount of the step- size parameter is variable at each iteration. The VSLMS algorithm is [1]:

A. Filtering

$$y(n) = \mathbf{W}^T(n)\,\mathbf{X}(n) \quad (1)$$

B. Error estimation

$$e(n) = d(n) - y(n) \quad (2)$$

C. Coefficient and step-size parameter update

$$\begin{aligned}
&(i = 0,1,\ldots,N-1)\\
&g_i(n) = e(n)x(n-i)\\
&\mu_i(n) = \mu_i(n-1) + \rho\,\mathrm{sign}[g_i(n)]\,\mathrm{sign}[g_i(n-1)]\\
&\text{if } \mu_i(n) > \mu_{max},\ \mu_i(n) = \mu_{max}\\
&\text{if } \mu_i(n) < \mu_{min},\ \mu_i(n) = \mu_{min}\\
&w_i(n+1) = w_i(n) + 2\mu_i(n)g_i(n)
\end{aligned} \quad (3)$$

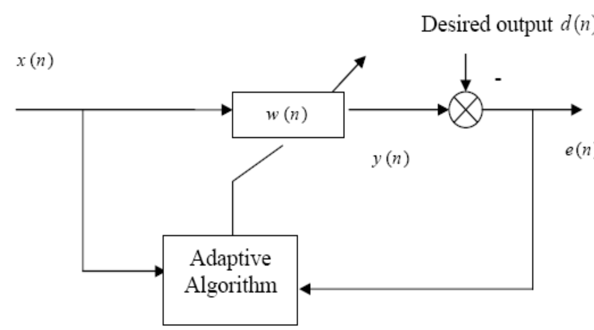

Fig. 1. Adaptive filter structure

III. PROPOSED EEVSLMS ALGORITHM

Here, instead of using the n^{th} instantaneous error, we make use of the errors obtained from p stages earlier. Each error weight determines the related amount of trace error in filter weight vector, whose values are obtained via using different experiments. The proposed algorithm is shown in Fig. 2.

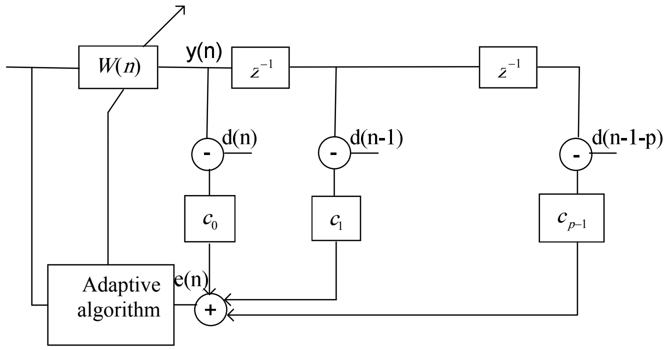

Fig. 2. Structure of adaptive EEVSLMS algorithm

The P differences between the filter output and the desired signal are multiplied by c_i (i=0,…,p) coefficients, then they are summed up to form the error signal, e(n). This error is utilized to compute filter weights.

Assume that $\boldsymbol{E}_p(n)$ and $\boldsymbol{C}_P$ are the subsets consisting of the error vector and the corresponding error weights at time n as

$$\boldsymbol{E}_p(n) = [e(n), e(n-1), \dots, e(n-p+1)]^T \tag{4}$$

$$\boldsymbol{C}_P = [c_0, c_1, \dots, c_{p-1}]^T \tag{5}$$

where $e(n\text{-}i)$ is the filter error at time i(i=0,…,p-1), c_i is the i^{th} weight error filter, p is the number of previously used errors, and T is matrix transposition. Thus, the error values can be formulated as

$$e(n) = \boldsymbol{C}_p^{\,T} \boldsymbol{E}_p(n) \tag{6}$$

Then, the weight vector recursion is written as

$$\boldsymbol{W}(n) = \boldsymbol{W}(n-1) + 2\mu \boldsymbol{X}(n) \boldsymbol{C}_p^T \boldsymbol{E}_p(n) \tag{7}$$

As a result, the EEVSLMS algorithm can be classified into a tri-state algorithm as

A. Filtering

$$\hat{y}(n) = \boldsymbol{W}^T(n)\boldsymbol{X}(n) \tag{8}$$

B. Weight vector update equation

$$e(n) = d(n) - \hat{y}(n) \tag{9}$$

C. Error Estimation

$$\hat{e}(n) = \boldsymbol{C}_p^{\,T} \boldsymbol{E}_p(n) \tag{10}$$

where $\boldsymbol{E}_p(n) = [e(n), e(n-2), \dots, e(n-p-1)]^T$.

D. Coefficient and step size Parameters update

$$\begin{aligned}
&(i = 0,1,\dots,N-1)\\
&g_i(n) = \hat{e}(n)x(n-i)\\
&\mu_i(n) = \mu_i(n-1) + \rho\, sign[g_i(n)]\, sign[g_i(n-1)]\\
&if\ \mu_i(n) > \mu_{max},\ \mu_i(n) = \mu_{max}\\
&if\ \mu_i(n) < \mu_{min},\ \mu_i(n) = \mu_{min}\\
&w_i(n+1) = w_i(n) + 2\mu_i(n) g_i(n)
\end{aligned} \tag{11}$$

IV. STABILTY ANLYSIS

Suppose that the input and output of the adaptive filter are WSS processes, and $\boldsymbol{W}_0$ and e_0 indicate the optimum values of weights and error, respectively. We consider update equations as

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + 2\boldsymbol{\mu}(n)\boldsymbol{C}_P \boldsymbol{X}(n)e(n) \tag{12}$$

where

$$\boldsymbol{\mu}(n) = [\mu_0(n) \quad \mu_1(n) \dots \mu_{p-1}(n)]^T \tag{13}$$

is the step-size vector. By defining

$$\boldsymbol{V}(n) = \boldsymbol{W}(n) - \boldsymbol{W}_0 \ , \tag{14}$$

we can rewrite (12) as

$$\boldsymbol{V}(n+1) = \boldsymbol{V}(n) + 2\boldsymbol{\mu}(n)\boldsymbol{C}_P \boldsymbol{X}(n)e(n) \tag{15}$$

where we have

$$\begin{aligned}
e(n) &= d(n) - \boldsymbol{W}^T(n)\boldsymbol{X}(n)\\
&= d(n) - \boldsymbol{W}_o^T \boldsymbol{X}(n) - (\boldsymbol{W}_o^T(n) - \boldsymbol{W}_o)\boldsymbol{X}(n)\\
&= e_o(n) - \boldsymbol{V}^T(n)\boldsymbol{X}(n)
\end{aligned} \tag{16}$$

By substitution of (16) in (15), we obtain

$$\begin{aligned}
\boldsymbol{V}(n+1) &= \boldsymbol{V}(n) + 2\boldsymbol{\mu}(n)\boldsymbol{C}_P \boldsymbol{X}(n)[e_o(n)\\
&\quad - \boldsymbol{V}^T(n)\boldsymbol{X}(n)]\\
&= \boldsymbol{V}(n)(I - 2\boldsymbol{C}_P \boldsymbol{X}(n)\boldsymbol{X}^T(n)\boldsymbol{\mu}(n))\\
&\quad + 2\boldsymbol{\mu}(n)\boldsymbol{C}_P \boldsymbol{X}(n)e_o(n)
\end{aligned} \tag{17}$$

The expectation of both sides of (17) leads to

$$E(\boldsymbol{V}(n+1)) = E[(I - 2\boldsymbol{C}_P \boldsymbol{X}(n)\boldsymbol{X}^T(n)\boldsymbol{\mu}(n))$$
$$\boldsymbol{V}(n)] + 2\boldsymbol{C}_P E[e_o(n)\boldsymbol{\mu}(n)\boldsymbol{X}(n)] \quad (18)$$
$$= E[I - 2\boldsymbol{C}_P \boldsymbol{X}(n)\boldsymbol{X}^T(n)\boldsymbol{\mu}(n)]E(\boldsymbol{V}(n))$$

where we used the orthogonality principle of the error and input signals. Considering the independency of the input signal, weight and step-size vectors, we get

$$E[\boldsymbol{V}(n+1)] = (I - 2\boldsymbol{C}_P \boldsymbol{R}\boldsymbol{\mu}(n))E[\boldsymbol{V}(n)] \quad (19)$$

where $R = E[X(n)X^T(n)]$ is the input covariance matrix. Now, the system stability condition can be written as

$$tr\left|\boldsymbol{C}_P \boldsymbol{R}\boldsymbol{\mu}(n)\right| < 1/3 \quad (20)$$

In practice, we can use (21) as the worst case similar to what reported for the VSLMS algorithm

$$\mu_i(n) < \frac{1}{3tr\left|\boldsymbol{C}_P \boldsymbol{R}\right|} \qquad i = 1,\ldots,N \quad (21)$$

V. SIMULTION RESULTS

In this section, we show the simulation results for the EEVSLMS and VSLMS algorithms. The input signal is defined as

$$x(n) = 5cos(n)(2.3)^{cos(n)} \quad (22)$$

Also, the noise is zero-mean white Gaussian with variance 0.5. The filter length is 2. Based on different experiments, we have selected the weight filter error estimation as

$$p = 3, \quad C_p = [0.251 - 0.354 - 0.214]^T \quad (23)$$

Fig. 1 shows the learning curve of the EEVSLMS algorithm based on the average of 10000 independent trials of the experiment. The proposed algorithm converges at the 38^{th} iteration while the VSLMS algorithm converges at the 73^{th} iteration. It is seen that the convergence rate of the proposed algorithm compared to that of the VSLMS algorithm is significantly faster.

VI. CONCLUSION

In this paper, we introduced to the EEVSLMS algorithm based on error estimation. In this algorithm, unlike the VSLMS algorithm, we estimated the error of n'th time by last p-stages. The proposed algorithm achieves a faster convergence rate compared to the VSLMS algorithm.

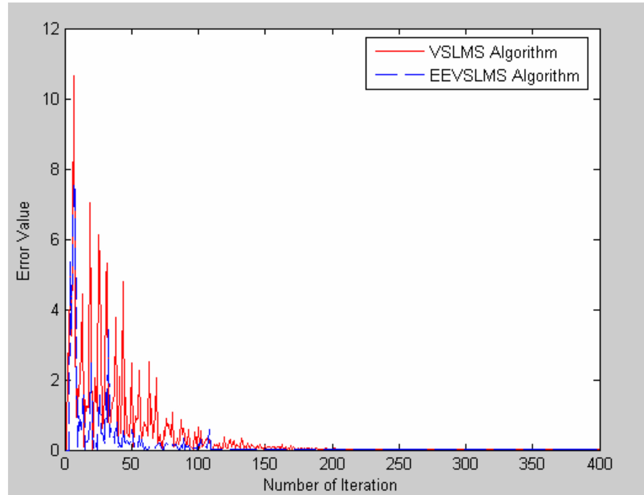

Fig. 3. Comparing learning curve of two algorithms VSLMS, EEVSLMS

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