Effect of Frequency Offset and Multiple Interferences on the Performance of Random Spreading downlink MC-CDMA

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*Abstract*__ **The performance of multi-carrier code division multiple access (MC-CDMA) systems that employ random spreading sequence in downlink frequency selective fading channels is analyzed. An expression of the probability density function (pdf) of multiple access interference (MAI), inter-carrier-interference (ICI) and noise is derived for MC-CDMA systems in terms of the number of users, the spreading factor, the number of sub-carriers, mobile station's velocity, and frequency offset. The results show that the effects of frequency offset become crucial as system loading increases. In this paper the synchronous downlink of MC-CDMA in a time variant channel is considered where, a Gaussian approximation of the MAI and ICI is derived to reduce the computational complexity in calculating the system performance.**

Index Terms — **MC-CDMA, Frequency Offset, Inter-Carrier-Interference (ICI), OFDM, Bit Error Rate (BER), Multiple Access Interference (MAI).**

I. INTRODUCTION

Future generations of broadband wireless mobile systems will aim to support a wide range of services and bit rates by employing a variety of techniques capable of achieving the highest possible spectrum efficiency [1]. Multi-carrier Code Division Multiple Access (MC-CDMA) has gained increasing popularity for providing high data rate services in recent years. This is mainly due to its capabilities to support high data rate transmission in a mobile environment characterized by being a highly hostile radio channel [2]. Like in Orthogonal Frequency Division Multiplexing (OFDM) systems, Frequency Offset (FO) between the transmitter and receiver may cause severe performance degradation in the MC-CDMA system [3]. However, ICI due to carrier FO in MC-CDMA can degrade the system performance significantly. The source of such FO can be due to the frequency mismatch between the transmitter and receiver oscillators, or a Doppler shift due to vehicle motion. The main problem with the FO is that it introduces interference from other carriers since they are no longer orthogonal [4]. In [5], Kim analytically evaluated MC-CDMA in multipath channels in terms of Multiple Access Interference (MAI), ICI, and noise. The effect of FO was analyzed using Hadamard-Walsh code in [3], [4], [5]. CDMA systems with random spreading sequences have received special interest in [7], [8], since their spectrum affords a convenient way of analysis due to its statistical symmetry and can provide an upper bound on the MAI performance. In practice, long codes can be considered as random spread spectrum, which has been employed in some CDMA systems such as IS-95 and CDMA 2000.

Random spreading sequences are used in our system, where our contribution is the BER calculation expressed explicitly in terms of the number of users, spreading length, frequency offset, fading parameters, user signal power and channel noise. Another crucial contribution of our work is the closed form expression for the pdf of the MAI, ICI and noise. In this paper, a closed form pdf of the MAI in synchronous downlink CDMA systems is obtained. Then the analysis is extended to MC-CDMA systems with random spreading sequences in frequency selective fading channels. To reduce the computational complexity in calculating BER using the obtained pdf of the MAI and ICI, a Gaussian approximation that provides an accurate estimation of the BER is derived. Our observations are consistent with those reported in [6] & [4]. The paper is organized as follows, in section II, system model is proposed, and then the MAI and ICI are analyzed in section III. The performance analysis is presented in section IV, then the numerical results are in section V, and section VI concludes the paper.

II. SYSTEM MODEL

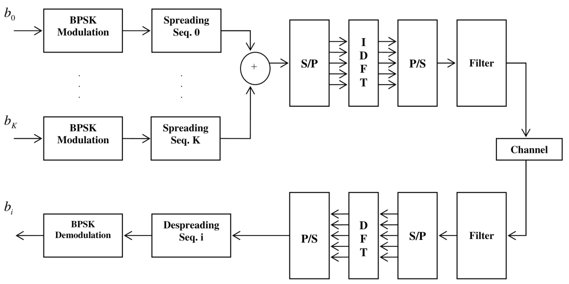


Fig. 1 MC-CDMA transmitter and receiver scheme

The block diagram of MC-CDMA is shown in **F**ig. 1. The K+1 symbol, obtained by BPSK modulation, are spread and added together. Random spreading sequences have been used under perfect synchronization assumption. A cyclic prefix is added at the inverse discrete Fourier transform (IDFT) output to protect the MC-CDMA from inter-symbol-interference (ISI). At the receiver side, the discrete Fourier transform (DFT) is taken to the M samples of each multi-carrier symbol.

Let v_m be the summation of K+1 users' m^{th} chip during the i^{th} bit interval:

$$v_m = \sum_{k=0}^{K} b_i^k \; c_{i,m}^k \qquad , m = 0,1,\ldots, M-1 \tag{1}$$

where $c_{i,m}^k$ is the k^{th} user's m^{th} chip in the i^{th} bit interval. The spreading factor (n) is assumed to be equal to the number of sub-carriers (M) although the case can be generalized. The h^{th} IDFT of v_m corresponding to a block of M symbols is

$$w_h = \frac{1}{M}\sum_{m=0}^{M-1} v_m \exp(j2\pi m\frac{h}{M}) \qquad , h = 0,.......M-1 \tag{2}$$

Let the normalized FO, ε, be $f_o / \Delta f$, where f_o is the frequency offset and $\Delta f = 1/MT_c$. At the receiver input the noiseless component of the received signal during the bit interval, impaired by the frequency offset, is

$$\hat{r}(t) = \frac{1}{M}\sum_{h=0}^{M-1}\sum_{m=0}^{M-1} v_m \, q(t-hT_c)\exp[j2\pi(m+\varepsilon)\frac{h}{M}] \tag{3}$$

where the first summation is due to collecting M branches of the IDFT, $q(t)$ is the unit rectangular pulse over a chip interval.

$$y_h = \frac{1}{M}\sum_{m=0}^{M-1} v_m \exp\,[\,j2\pi\, h\frac{m+\varepsilon}{M}] \tag{4}$$

$$\begin{aligned} Z_g &= \sum_{h=0}^{M-1} y_h \exp[-j2\pi h\frac{g}{M}] + \eta_g \\ &= \sum_{h=0}^{M-1}\sum_{m=0}^{M-1} v_m \frac{1}{M}\exp[j\frac{2\pi}{M}(m+\varepsilon-g)h] + \eta_g \end{aligned} \tag{5}$$

$$\begin{aligned} \sum_{g=0}^{M-1} c_{i,g}^d Z_g &= b_i^d \sum_{h=0}^{M-1}\frac{1}{M}\exp[j\frac{2\pi h\varepsilon}{M}] \\ &+ b_i^d \sum_{g=0}^{M-1}\sum_{h=0}^{M-1}\sum_{\substack{m=0\\ m\neq g}}^{M-1} c_{i,g}^d c_{i,m}^d \frac{1}{M}\exp\left[j\frac{2\pi}{M}(m+\varepsilon-g)h\right] \\ &+ \sum_{g=0}^{M-1}\sum_{h=0}^{M-1}\sum_{\substack{k=0\\ k\neq d}}^{K} b_i^k \, c_{i,g}^d \, c_{i,m}^k \frac{1}{M}\exp\left[j\frac{2\pi\varepsilon h}{M}\right] \\ &+ \sum_{g=0}^{M-1}\sum_{h=0}^{M-1}\sum_{m=0}^{M-1}\sum_{\substack{k=0\\ k\neq d}}^{K} b_i^k c_{i,g}^d c_{i,m}^k \frac{1}{M}\exp\left[j\frac{2\pi}{M}(m+\varepsilon-g)h\right] + \eta_g \end{aligned} \tag{6}$$

The frequency offset included in (3) produces ICI. The h^{th} noiseless DFT input is shown in (4). The M values of y_h corrupted by AWGN samples are fed to the DFT. The g^{th} DFT output is shown in (5), where η_g is the noise variable that is still Gaussian, since a phase rotation due to FO doesn't change the statistics of a complex Gaussian process [4]. The output of the DFT is then fed into the despreading block after the P/S conversion as shown in Fig. 1. Thus, the output of the despreader matched to the desired user's spreading sequence during the i^{th} bit interval is shown in (6), where b_i^d and $c_{i,m}^d$ are the desired user's bit and chip respectively, during the i^{th} bit and m^{th} chip interval. The first, second, third and fourth term in (6), at the right hand side of the equal represent the desired information, self- interference, the MAI and the ICI, respectively. The estimated bit for the desired user can be obtained after a hard limiter that follows the BPSK demodulation.

III. MULTIPLE ACCESS AND INTER-CARRIER INTERFERENCE

Random variables characterizing various aspects of the MAI are described using random spreading sequences, data bits and correlation functions of the spreading sequences. The conditional characteristic function of the MAI is the basis of our derivation. Beginning with the MAI from two users that are arbitrarily chosen, assuming real spreading sequences, the cross-correlation can be written as [4]

$$R_i^{k,j} = \sum_{m=0}^{n-1} c_{i,m}^k \; c_{i,m}^j \tag{7}$$

Where $c_{i,m}^k$ are independent identically distributed (i.i.d) with $p_r(c_{i,m}^k = 1/\sqrt{n}) = p_r(c_{i,m}^k = -1/\sqrt{n}) = 1/2, \forall k,i,m$. Here, we assume that the random spreading is dynamically changing every bit interval, although the results can be applied to static random sequence. Then the cross-correlation will be $R_i^{k,j} = (n-2l)/n,\ l = 0,1,\ldots n$ where l denotes the number of chip positions that differ. The MAI conditioned on specific spreading sequence is characterized by a binomial distribution. Conditioning the decision statistic on a specific spreading sequence such as the first user's spreading sequence is necessary for conditional independence of pair-wise interference. Assuming a two user system with spreading factor n and equal power A^2, the discrete characteristic function of the MAI taking the interference amplitude values x_l is

$$\Phi_1(w) = \sum_{l=0}^{n}\binom{n}{l} p^l \; q^{n-l}\exp\{\,jwx_l\} \tag{8}$$

For K interferers, the characteristic function of MAI and noise is shown in (9). The corresponding pdf can be obtained by its inverse Fourier transform as shown in (10). We can see that the pdf is the sum of nK +1 Gaussian pdfs with the mean equal to A(K-2l/n). The variance of the MAI and noise is given in (11).

$$\Phi(w) = \sum_{l=0}^{nK}\binom{nK}{l} p^l \; q^{nK-l}\exp\{jwA(K-2l/n)\}.$$
$$\exp\{-\sigma^2 w^2/2\} \tag{9}$$

$$f_{CDMA}(x)=\frac{1}{2^{nK}}\sum_{l=0}^{nK}\binom{nK}{l}\frac{1}{\sqrt{2\pi\sigma^2}}.$$
$$.\exp\{-\left(x-A(K-\frac{2l}{n})\right)^2/2\sigma^2\} \quad (10)$$

$$\sigma_{CDMA}^2=\sigma^2+KA^2/n \quad (11)$$

The variable of the MAI, ICI and noise for the desired user, *d,* during the i^{th} bit and m^{th} chip interval, f_i^d, can be written as

$$f_i^d=\sum_{m=0}^{M-1}u_m.s_m=u_0.s_0+\sum_{m=1}^{M-1}u_m.s_m \quad (12)$$

$$s_m=\frac{1}{M}\sum_{h=0}^{M-1}\exp\{j\frac{2\pi}{M}(m+\varepsilon)h\} \quad (13)$$

$$u_m=\sum_{l=0}^{M-1}c_{i,l}^d\mu_m+n_m \quad ,\mu_m=\sum_{\substack{k=0\\k\neq d}}^{K}b_i^k c_{i,m}^k \quad (14)$$

where μ_m denotes the MAI in the m^{th} sub-carrier [9]. In (12) the real part of S_m has been taken since real signature waveforms and real binary data are assumed. Without loss of generality, we consider the MAI, ICI and noise at zeroth sub-carrier and the source of the interference is *K* chips from the *K* interferers. From (12), the pdf of f_i^d is shown in (15) where *f*(.) is the pdf defined in (10) and * indicates the convolution operator. The fading vector is given as $\vec{\alpha}=\{\alpha_0,....\alpha_{M-1}\}$, where α_i is the fading parameter at the i^{th} sub-carrier. In frequency selective fading channels the amplitude *A* is replaced with $\alpha_i A$ at the i^{th} sub-carriers. The conditional pdf of the MAI, ICI, and noise can be written as shown in (16). The variance of the MAI and ICI in slowly fading channel can be obtained as shown in (17) [4]

$$f_{MC-CDMA}(x)=\frac{1}{|S_0|}f\left(\frac{x}{S_0}\right)*\frac{1}{|S_1|}f\left(\frac{x}{S_1}\right)*,......$$
$$....,*\frac{1}{|S_{M-1}|}f\left(\frac{x}{S_{M-1}}\right) \quad (15)$$

$$f_{MC-CDMA}(x\,|\,\vec{\alpha})=\frac{1}{2^{nKM}}\sum_{l_0=0}^{nK},....,\sum_{l_{M-1}=0}^{nK}\binom{nK}{l_0},......$$
$$.....,\binom{nK}{l_{M-1}}\frac{1}{\sqrt{2\pi\sigma^2\sum_{k=0}^{M-1}S_k^2}}*$$
$$\exp\{-(x-A\left[K\sum_{k=0}^{M-1}\alpha_k S_k-2(\sum_{k=0}^{M-1}l_k\alpha_k S_k)/n\right])^2/(2\sigma^2\sum_{k=0}^{M-1}s_k^2)\} \quad (16)$$

$$\sigma_{MC-CDMA}^2=\sigma^2\sum_{k=0}^{M-1}S_k^2+A^2\frac{K}{n}\sum_{k=0}^{M-1}\alpha_k^2S_k^2 \quad (17)$$

We can find the pdf and the variance of flat fading channels and AWGN channels from (15) and (16) by simply replacing $\alpha_k=\alpha, \forall k$ for flat fading or $\alpha_k=1, \forall k$ for AWGN.

The pdf of the MAI, ICI and noise for the MC-CDMA is the weighted sum of *nKM* Gaussian pdfs (16). Heuristically, the effect of frequency offset is to increase the number of Gaussian pdfs with different means. With zero frequency offset, the number of Gaussian pdfs is reduced to *nK* in (10).

IV. PERFORMANCE ANALYSIS

The Gaussian approximation of the MAI and ICI in frequency selective fading channel, Eq. (16) and (17), can be written as follows: The conditional BER is given as $P_b(E/\vec{\gamma})=Q(\sqrt{2\gamma_t})$ [11], where γ_t is the total conditional SNR per symbol, $\vec{\gamma}=[\gamma_1,...\gamma_{M-1}]$, $\gamma_i=\alpha_i^2$ and $f_{\vec{\gamma}}(\vec{\gamma})$ is the joint pdf of $\gamma_1,....,\gamma_{M-1}$

$$P_b(E/\vec{\gamma})=Q\left(\sqrt{\sum_{i=0}^{M-1}\frac{S_0^2\gamma_i A^2/n}{\sigma^2\sum_{k=0}^{M-1}S_k^2+\frac{K}{n}A^2(\sum_{j=0}^{M-1}\gamma_j S_j^2)}}\right) \quad (18)$$

The Gaussian approximation of the average BER is given as (19), where E_c/N_0 is the chip energy-to-noise ratio. The phase shift can be estimated from the received signal without error because the channel fading is assumed to be slow. For flat fading $\gamma_i=\gamma, \forall i$ Eq. (19) is reduced to Eq. (20) [11]. Eq (20) is further reduced to AWGN channels for $\gamma_i=1, \forall i$ as Eq (21) which reduces to CDMA without ICI. This can be evaluated numerically as illustrated in the appendix.

$$P_b=\int_0^\infty Q\left(\sqrt{\sum_{i=0}^{M-1}\frac{2S_0^2\gamma_i(E_c/N_0)}{\sum_{k=0}^{M-1}S_k^2+2\frac{K}{n}(E_b/N_0)(\sum_{j=0}^{M-1}\gamma_j S_j^2)}}\right)f_{\vec{\gamma}}(\vec{\gamma})\,d\vec{\gamma} \quad (19)$$

$$P_b=\int_0^\infty Q\left(\sqrt{\frac{2S_0^2\gamma(E_b/N_0)}{\left(1+2\frac{K}{n}\gamma(E_b/N_0)\right)\left(\sum_{j=0}^{M-1}S_j^2\right)}}\right)e^{-\gamma}\,d\gamma \quad (20)$$

$$P_b=Q\left(\sqrt{\frac{S_0^2(2E_b/N_0)}{\left(1+\frac{K}{n}\frac{2E_b}{N_0}\right)\left(\sum_{j=0}^{M-1}S_j^2\right)}}\right) \quad (21)$$

V. NUMERICAL RESULTS

a) Flat Fading Channel: we use random spreading sequences in the following. Fig. 2 shows the BER in flat Rayleigh fading channels for one and five interferers, the result was also observed in [4]. In flat fading channels, the BER begins to saturate at low SNR for a large number of users due to large MAI. For a small number of users the BER is still linear at high SNR regardless of frequency offsets. We have also observed that the BER saturation point decreases as frequency offset increases, This is also shown for different number of interferers and SNR=20dB in Fig.3, this is because a large frequency offset increases the ICI and subsequently the aggregated interference. As the number of sub-carriers increases the effect of FO on the performance will appear significantly as shown in Fig.4 , for large number of sub-carriers the frequency separation between sub-carriers is small, by the additional FO, the orthogonality between subcarriers will be lost and degrades the performance.

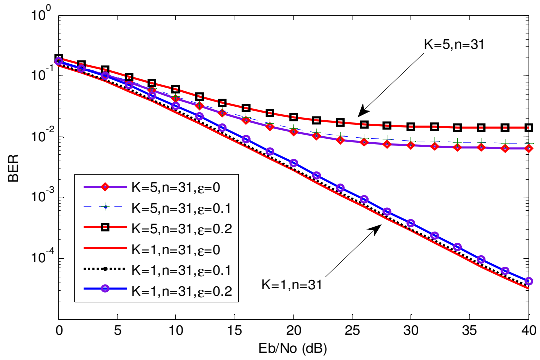


Fig.2 BER of MC-CDMA, flat fading, Є=0, 0.1, 0.2, K=1, 5, n=31.

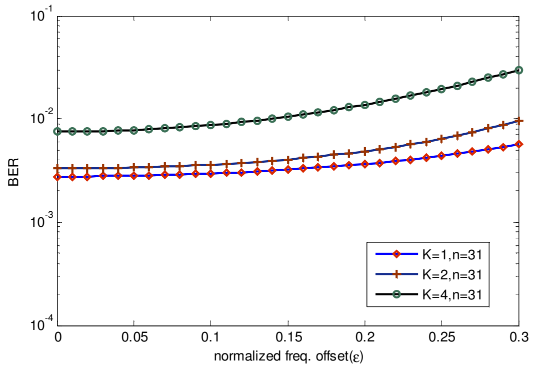


Fig.3 BER of MC-CDMA, flat fading, K=1,2,4, n=31. E_b / N_o=20 dB.

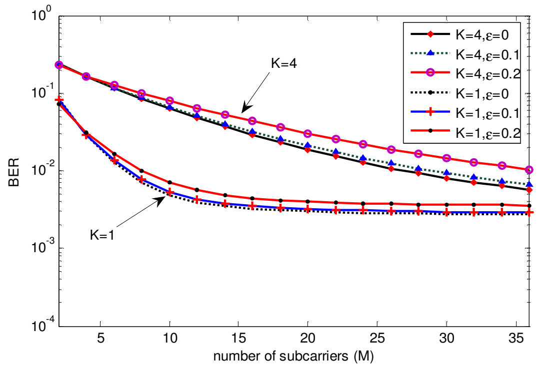


Fig.4 BER of MC-CDMA, flat fading,Є=0,0.1,0.2,K=1,4, E_b / N_o =20dB.

Fig.5 shows the Gaussian approximation result of MC-CDMA for the system loading (*K/n*) equals to 1/4, 1/6 and 3/32. The number of interferers is equal to 1, 2 3 and 6, and the spreading factor is taken as 4, 8, 32 and 64. We can see that for the same ratio of the number of interferers to the spreading factor (system loading), the effect of frequency offset is shown to be similar.

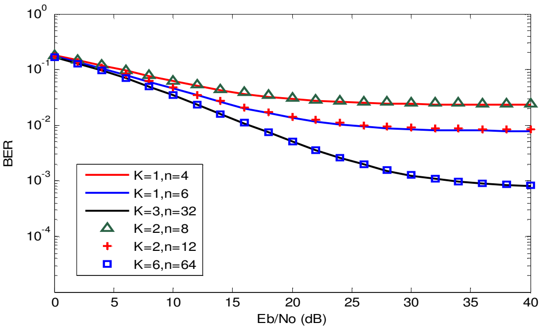


Fig5. BER of MC-CDMA, flat fading, Є=0.1, K/n=1/4, 1/6,3/32.

b) Frequency Selective Fading Channel: Fig. 6 shows the BER in frequency selective fading channels for one interferer with 31 chips/bit. Significant performance degradation due to larger number of users can be observed in Fig. 7. In frequency selective fading channels, each carrier experiences fading that may be jointly correlated. For simplicity, we employ an independent fading at each sub-carrier in Figs. 6 and 7. In calculating Eq. (19), we approximate the fading random variables in the denominator with the average of the random variables due to the difficulties in performing the multiple integrations in order to obtain the numerical value. Using large number of sub-carriers will enhance the system performance but this will deteriorate the overall performance by the frequency offset increase, as shown in Fig. 8 with 4 and 8 interferers. The BER of MC-CDMA system is similar for the same system loading ratio; this is shown in Fig. 9 for different ratios of system loading. The results show that the Gaussian approximation is accurate in both channels. These results can be verified for AWGN channel as shown in [10].

VI. CONCLUSIONS& FUTURE WORK

In this paper, the BER performance of synchronous MC-CDMA systems that employ random spreading sequences in downlink frequency selective fading channels have been analyzed. The pdf of the MAI in CDMA was obtained and the BER calculations were verified by numerical results. Our result is extended to the pdf of the MAI and ICI in MC-CDMA to determine the BER performance in terms of the number of users, the spreading factor, the number of sub-carriers, frequency offset and fading parameters. To reduce the computational complexity involved in calculating the BER, we have derived a Gaussian approximation which provides the approximate BER. The effects of frequency offset vary with the system loading, i.e., the ratio of the number of interferers to the spreading factor. These effects are less significant for large number of users; this is because in this case the dominant part of interference is the MAI. In the future, the analysis will be applied with the same procedures to evaluate the effects of FO and MAI on the performance of MC-DS-CDMA.

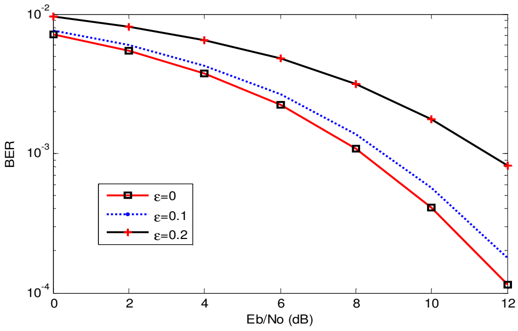


Fig. 6. BER of MC-CDMA, frequency selective fading, Є=0, 0.1, 0.2, K=1,n=31.

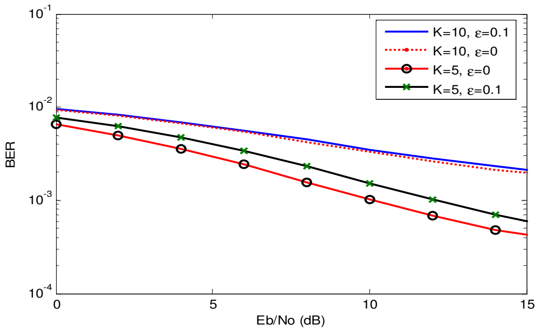


Fig. 7. BER of MC-CDMA, frequency selective fading, Є=0, 0.1, K=5, 10,n=31.

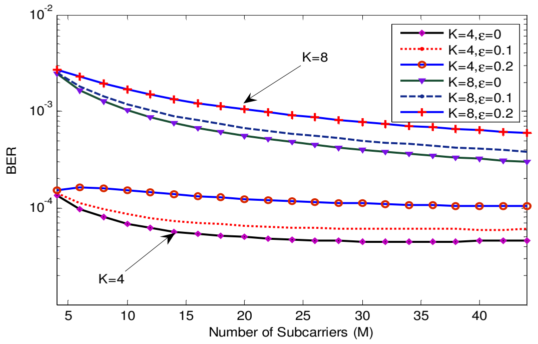


Fig.8 BER of MC-CDMA, frequency selective fading, Є=0,0.1,0.2,K=4,8 E_b / N_o =20dB.

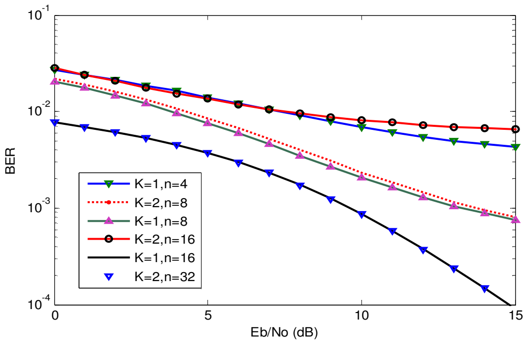
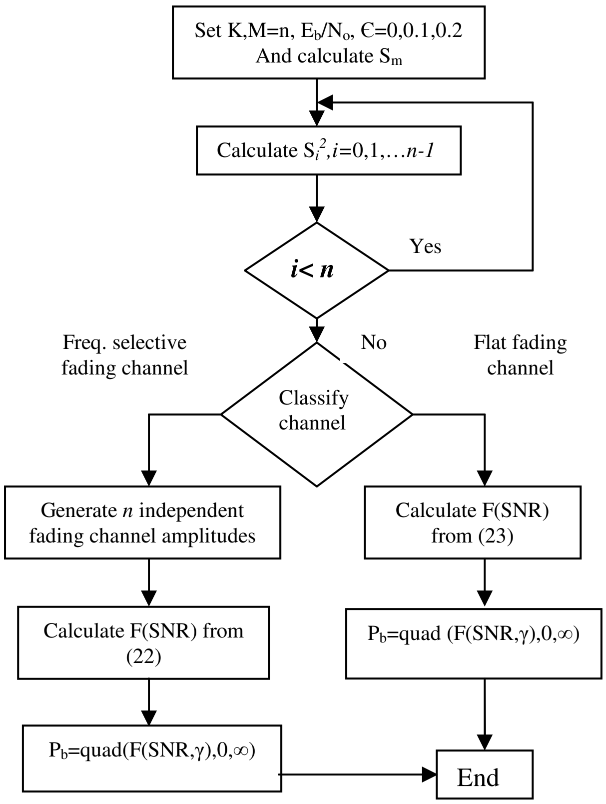


Fig.9, BER of MC-CDMA, frequency selective fading, Є=0.1, K/n=1/4,1/8,1/16.

APPENDEX

NUMERICAL EVALUATION OF (20) & (21)



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