Throughput Analysis of STS-based CDMA System with Variable Spreading Factor in Non-Frequency Selective Rayleigh Fading Channel

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***Abstract* — This paper investigates the application of Variable Spreading Factor (VSF) method in Space-Time Spreading (STS)-based Code Division Multiple Access (CDMA) system to improve the data throughput of the system. In this contribution an analytical approach to compute the throughput enhancement of the STS-based CDMA system with VSF method is proposed for non-frequency-selective Rayleigh fading channel. The Bit Error Rate (BER) performance of the system against pre-despreading Signal-to-Noise-Ratio (SNR) is also calculated for different values of spreading factor. Simulation and analytical results demonstrate that using VSF method in STS-based CDMA system improves the throughput of the system by keeping the BER performance at a target level.**

***INDEX TERMS* — CDMA, Space-Time Spreading, Throughput, Variable Spreading Factor.**

I. INTRODUCTION

The object of recent studies in wireless communication is to increase the capacity and data rate in order to improve the performance of these systems. One interesting approach is spatial diversity in which more than one antenna is used in the transmitter, receiver, or both. In this paper an open loop transmit diversity method called STS which has been proposed for CDMA system is considered [1]. In addition, in order to overcome the problems of wireless channels variable rate transmission techniques have been proposed [2, 8]. As CDMA is suitable to support these techniques, adaptive rate transmission methods are used to increase the data throughput of CDMA systems. In our previous work it was shown that using adaptive modulation for STS-based CDMA system results in increasing the data throughput of this system in different fading channels [9]. In the other work we computed the throughput enhancement of the STS-based CDMA system with adaptive modulation in non-frequency-selective Rayleigh fading channel [10]. In other words, using space diversity technique results in increasing the capacity of CDMA system and applying adaptive rate transmission to this system enhances the data throughput of each user.

In this paper the Variable Spreading Factor as another adaptive rate technique is used for STS-based CDMA system to increase the data throughput of the system in non-frequency-selective Rayleigh fading channel. First, an analytical approach is proposed to compute the BER performance of the STS-based CDMA system against pre-despreading SNR for different values of spreading factor. Afterwards, the throughput enhancement of the VSF system is calculated.

The rest of the paper is organized as follows. Section II describes the applied Space-Time Spreading scheme. In section III Variable Spreading factor is considered for STS-based CDMA system. Then in section IV an analytical approach to compute the BER performance of STS-based CDMA system and throughput enhancement of VSF system is proposed. In section V the simulation and analytical results are presented. Finally section VI includes the conclusion.

II. SPACE-TIME SPREADING

Considering a CDMA system with two transmitter antennas, the Space-Time Spreading method proposed by [1] first splits the data stream of each user into two sub streams $\{b_1\}$ and $\{b_2\}$. The transmitted signal on each of the two antennas for the real data symbols is given by:

$$\begin{aligned} \underline{t}_1 &= \tfrac{1}{\sqrt{2}}(b_1\underline{c}_1 + b_2\underline{c}_2) \\ \underline{t}_2 &= \tfrac{1}{\sqrt{2}}(b_2\underline{c}_1 - b_1\underline{c}_2) \end{aligned} \qquad (1)$$

where $\underline{c}_1$ and $\underline{c}_2$ are $2N \times 1$ orthogonal unit-norm spreading codes and $\underline{c}_1^T \underline{c}_2 = 0$. One possible choice of $\underline{c}_1$ and $\underline{c}_2$ is

$$\begin{aligned} \underline{c}_1 &= \begin{bmatrix} \underline{c}_o^T & \underline{c}_o^T \end{bmatrix}^T \\ \underline{c}_2 &= \begin{bmatrix} \underline{c}_o^T & -\underline{c}_o^T \end{bmatrix}^T \end{aligned} \qquad (2)$$

where $\underline{c}_0$ is an $N \times 1$ spreading code, specified to each user. At the receiver part after despreading the received signal with $\underline{c}_1$ and $\underline{c}_2$, the signals d_1 and d_2 from which b_1 and b_2 could be estimated are:

$$
\begin{aligned}
d_1 &= \tfrac{1}{\sqrt{2}}(h_1 b_1 + h_2 b_2) + \underline{c}_1^T\,\underline{n}\\
d_2 &= \tfrac{1}{\sqrt{2}}(-h_2 b_1 + h_1 b_2) + \underline{c}_2^T\,\underline{n}
\end{aligned}
\tag{3}
$$

where h_1 , h_2 are channel coefficients and $\underline{n}$ is due to AWGN.
Defining

$$
\underline{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \quad , \quad \underline{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad , \quad H = \begin{bmatrix} h_1 & h_2 \\ -h_2 & h_1 \end{bmatrix} \quad , \quad \underline{v} = \begin{bmatrix} \underline{c}_1^T\,\underline{n} \\ \underline{c}_2^T\,\underline{n} \end{bmatrix}
$$

then,

$$
\underline{d} \;=\; \tfrac{1}{\sqrt{2}} H \;\underline{b} \;+\; \underline{v} \tag{4}
$$

The real part of $H^H\,\underline{d}$ includes sufficient information to estimate b_1 and b_2 :

$$
\mathrm{Re}\left[H^H\,\underline{d}\right] = \tfrac{1}{\sqrt{2}}\begin{bmatrix} (|h_1|^2 + |h_2|^2) b_1 \\ (|h_1|^2 + |h_2|^2) b_2 \end{bmatrix} + \mathrm{Re}\left[H^H\,\underline{v}\right] \tag{5}
$$

in which H^H is the complex conjugate transpose of H .
In the case of complex data symbols, the transmitted signals are selected so that:

$$
\begin{aligned}
\underline{t}_1 &= \tfrac{1}{\sqrt{2}}(b_1 \underline{c}_1 + b_2^* \underline{c}_2)\\
\underline{t}_2 &= \tfrac{1}{\sqrt{2}}(b_2 \underline{c}_1 - b_1^* \underline{c}_2)
\end{aligned}
\tag{6}
$$

Therefore, the received signals after despreading with $\underline{c}_1$ and $\underline{c}_2$ are

$$
\begin{aligned}
d_1 &= \tfrac{1}{\sqrt{2}}(h_1 b_1 + h_2 b_2) + \underline{c}_1^T\,\underline{n}\\
d_2 &= \tfrac{1}{\sqrt{2}}(-h_2 b_1^* + h_1 b_2^*) + \underline{c}_2^T\,\underline{n}
\end{aligned}
\tag{7}
$$

Considering $\underline{d} = \begin{bmatrix} d_1 \\ d_2^* \end{bmatrix}$ and $H = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}$ the received signal equation is written as:

$$
\underline{d} = \tfrac{1}{\sqrt{2}} H \underline{b} + \begin{bmatrix} \underline{c}_1^T\,\underline{n} \\ (\underline{c}_2^T\,\underline{n})^* \end{bmatrix} \tag{8}
$$

Multiplying $\underline{d}$ on the left by H^H gives:

$$
H^H\,\underline{d} = \tfrac{1}{\sqrt{2}}\begin{bmatrix} (|h_1|^2 + |h_2|^2) b_1 \\ (|h_1|^2 + |h_2|^2) b_2 \end{bmatrix} + H^H \begin{bmatrix} \underline{c}_1^T\,\underline{n} \\ (\underline{c}_2^T\,\underline{n})^* \end{bmatrix} \tag{9}
$$

Using this result, b_1 and b_2 can be estimated. The received signal also could be stated as follows:

$$
\hat{b} = \begin{bmatrix} \widehat{b}_1 \\ \widehat{b}_2 \end{bmatrix} = \frac{1}{\sqrt{2}} H^H H b + H^H \eta \tag{10}
$$

$$
b = \begin{bmatrix} b_1 \\ b_1 \end{bmatrix} \quad , \quad \eta = \begin{bmatrix} \eta_1 = \underline{c}_1^T\,\underline{n} \\ \eta_2 = (\underline{c}_2^T\,\underline{n})^* \end{bmatrix} \tag{11}
$$

$$
\hat{b} = \frac{1}{\sqrt{2}}\begin{bmatrix} (|h_1|^2 + |h_2|^2) b_1 \\ (|h_1|^2 + |h_2|^2) b_2 \end{bmatrix} + \begin{bmatrix} h_1^* \eta_1 - h_2 \eta_2 \\ h_2^* \eta_1 + h_1 \eta_2 \end{bmatrix} \tag{12}
$$

III. VARIABLE SPREADING FACTOR

Variable Spreading Factor is a technique that attempts to increase the average throughput of the system by switching between spreading codes with different spreading factor, depending on the quality of channel [6-8, 11]. In VSF system the chip rate of the CDMA users is kept constant throughout the transmission, while bit rate is varied by using spreading codes with different Spreading Factor (SF). When the channel quality is high, a low SF is used in order to increase the throughput and conversely, when the channel is hostile, a high SF is employed for maintaining a target BER performance.

Analogously to adaptive modulated STS-based CDMA system proposed in [9, 10] a metric corresponding to near-instantaneous channel quality is required. In this work we suppose that the number of interfering users is constant and under the assumption of perfect interference cancellation, we use the instantaneous output SNR as the adaptation metric. Comparing this metric by the pre-determined thresholds, the transmitter decides about the appropriate spreading code for the next symbol. Plotting the BER curves versus output SNR for STS-based CDMA system in conjunction with different SFs, implies no difference in BER performances of the system with various SFs. This is due to the fact that, in order to exploit the channel fluctuation when SF varies, E_s/N_0 varies accordingly [11]. Therefore, similar to [8, 11] we use the BER curves versus pre-despreading SNR to determine the switching thresholds l_{1p}, l_{2p}. Pre-despreading SNR (γ_p)

is the system SNR before despreading the received sequence by the spreading codes. Since despreading results in reducing the noise power by a factor of SF, γ_p has the relation of $\gamma = SF \times \gamma_p$ with γ i.e. the output SNR.

IV. THROUGHPUT ANALYSIS OF STS-BASED CDMA SYSTEM WITH VSF IN NON-FREQUENCY-SELECTIVE RAYLEIGH FADING CHANNEL

The first important step in improving the throughput is to determine the thresholds by which the selected metric would be compared. Therefore, the BER performance of the STS-based CDMA system against pre-despreading SNR should be initially computed for different values of SF. In order to compute the corresponding expressions in non-frequency-selective Rayleigh fading channel, it should be considered that the fading impacts are reflected in the variations of γ_p. The average BER of the system then becomes $\int_0^\infty P(\gamma_p) f'(\gamma_p) d\gamma_p$, where $p(\gamma_p)$ is the probability of error of the performed modulation in AWGN channel which is equal to $Q(\sqrt{SF.\gamma_p})$ for QPSK modulation. $f'(\gamma_p)$ is the probability density function of γ_p and should be calculated for the STS-based CDMA system in the desired environment. Since $\gamma_p = \frac{1}{SF}\gamma$, by using the approach proposed in [12], $f'(\gamma_p)$ could be computed based on the probability density function of the instantaneous SNR; $f(\gamma)$:

$$f'(\gamma_p) = f(SF.\gamma_p)\frac{d}{d\gamma_p}\gamma = SF.f(SF.\gamma_p) \tag{13}$$

In our previous work, the overall instantaneous SNR of the system is obtained as bellow [10]:

$$\gamma = \frac{1}{2}\frac{\varepsilon_b |h_1|^2}{\sigma^2} + \frac{1}{2}\frac{\varepsilon_b |h_2|^2}{\sigma^2} \tag{14}$$

where $|h_1|$ and $|h_2|$ are the envelopes of the channel transfer functions and are Rayleigh distributed. Therefore, considering [13], the recently obtained instantaneous SNR has a Chi-square probability function with 2*L* degrees of freedom; where L is the number of diversity paths and is 2 in our system. As calculated in the previous work, $f(\gamma)$ is given as follows [10].

$$f(\gamma) = \frac{4\gamma e^{-2\gamma/\bar{\gamma}}}{\bar{\gamma}^2} \tag{15}$$

where $\bar{\gamma} = E[\gamma] = \frac{\varepsilon}{N_0}(2\alpha_0^2)$ and α_0 is Rayleigh distribution parameter:

$$C(\alpha) = (\frac{\alpha}{\alpha_0^2})e^{\frac{-\alpha^2}{2\alpha_0^2}} \tag{16}$$

Substituting equation (15) in equation (13) results in:

$$f'(\gamma_p) = SF\,\frac{4SF\gamma_p e^{\frac{-2SF\gamma_p}{\bar{\gamma}}}}{\bar{\gamma}^2} \tag{17}$$

which could also be written as below, if $\bar{\gamma}$ is substituted with $SF \times \bar{\gamma}_p$:

$$f'(\gamma_p, \bar{\gamma}_p) = \frac{4\gamma_p e^{\frac{-2\gamma_p}{\bar{\gamma}_p}}}{\bar{\gamma}_p^{\,2}} \tag{18}$$

Consequently, the BER of STS-based CDMA system with respect to pre-despreading SNR, in non-frequency-selective Rayleigh fading channel is obtained as:

$$P_{QPSK}(\bar{\gamma}_p) = \int_0^\infty Q(\sqrt{SF.\gamma_p})\, f'(\gamma_p, \bar{\gamma}_p)\, d\gamma_p \tag{19}$$

In a triple-mode VSF scheme at a specific value of γ when SF is switched between 64, 32 and 16, γ_p is the lowest when SF=64 is employed. Hence in VSF-CDMA scheme γ_p is maintained at the value associated with the highest spreading factor i.e. SF=64, and the transmitted power is multiplied by a factor of $1/f$ for the two other cases; in which *f* is 2 for SF=32 and 4 for SF=16 [11]. Therefore, in VSF system the SNR associated with the highest SF is used in SF adaptation procedure which is performed as expression (20).

$$\begin{array}{llll} \gamma < l_1 & \rightarrow \quad SF = 64 \;, & f = 1 \;, & N_s = 1 \\ l_1 \le \gamma < l_2 & \rightarrow \quad SF = 32 \;, & f = 2 \;, & N_s = 2 \\ \gamma \ge l_2 & \rightarrow \quad SF = 16 \;, & f = 4 \;, & N_s = 4 \end{array} \qquad (20)$$

In the above equation N_s is the number of transmitted symbols in each transmission course and $l_1 = l_{1p} + 10\log 64$ and $l_2 = l_{2p} + 10\log 64$.

The average throughput of the system as the average number of transmitted bits in each transmission course when QPSK modulation is used can be computed as follows.

$$B(\bar{\gamma}, \{l_i\}) = 2p(0 < \gamma < l_1) + 4p(l_1 \le \gamma < l_2) + 8p(l_2 < \gamma < \infty)$$

$$p(\alpha < \gamma < \beta) = \int_{\alpha}^{\beta} f(\gamma)d\gamma \qquad (21)$$

V. RESULTS AND DISCUSSION

In this section some simulation results are provided comparing with the analytical results obtained by numerically computing the corresponding expressions. Figure 1 shows the BER curves versus output SNR for STS-based CDMA system with QPSK modulation in conjunction with different SFs in non-frequency-selective Rayleigh channel with $\alpha_0 = 0.5$. As stated in section III, this figure implies no difference in BER performances of the system with various SFs. Therefore in figure 2 the BER curves are plotted with respect to pre-despreading SNR to determine the switching thresholds for VSF procedure. In this figure, the simulation results are compared with the analytical calculation of expression (19). Selecting the threshold levels in a way that the BER of the system remains below 1%, the average number of Bits transmitted per Symbol (BPS) as a measure of the average throughput of the VSF STS-based CDMA system, is given in figure 3 through both simulation and numerical computation of equation (21). The matching of the simulation and analytical results is obvious in the figures. It proves that the analytical expressions proposed in this paper can be used to evaluate the throughput performance of the VSF STS-based CDMA system in non-frequency-selective Rayleigh channel. BER performance of VSF system versus output SNR is shown in figure 4, in comparison with the simulation and analytical results of the non-VSF system with SF=64. Since the BER performance at the output of the system, as shown in figure 1, is independent of SF, the BER curves in this figure are nearly the same.

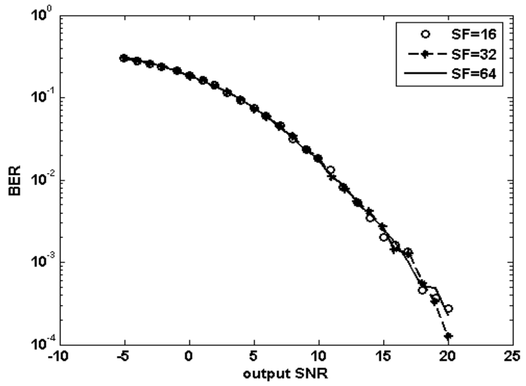

Fig. 1. BER performance of STS-based CDMA system versus output SNR in non-frequency-selective Rayleigh channel with different SFs.#

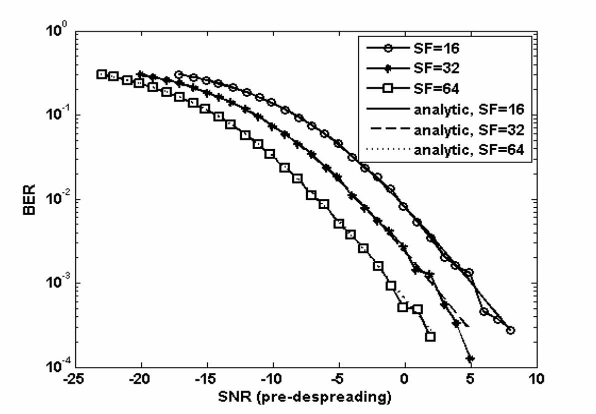

Fig. 2. BER performance of STS-based CDMA system with respect to pre-despreading SNR in non-frequency-selective Rayleigh channel with different SFs (Comparing the simulation and analytical results).#

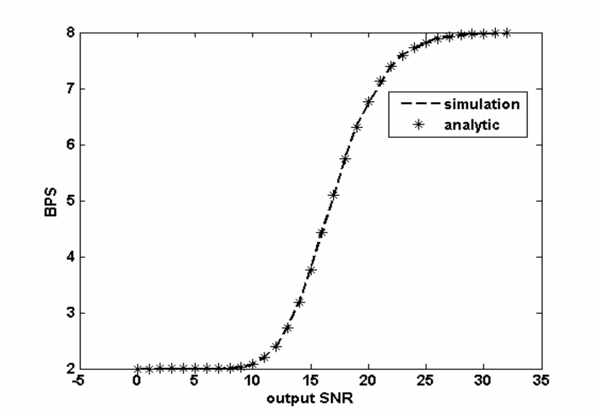

Fig. 3. BPS as the average throughput of the VSF STS-based CDMA system in non-frequency-selective Rayleigh channel (Comparing the simulation and analytical results).#

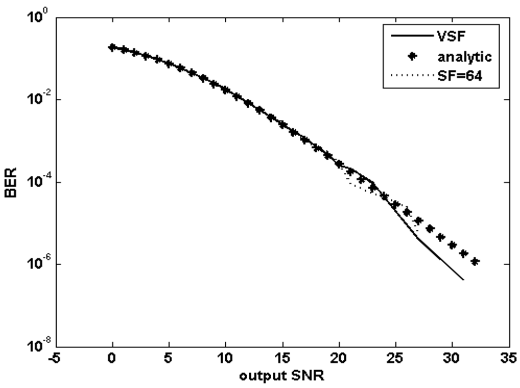

Fig. 4. The BER performance of the VSF STS-based CDMA system in non-frequency-selective Rayleigh channel (Comparing with the simulation and analytical results of the non-VSF system with SF=64).#

VI. CONCLUSION

In this paper we investigated the application of Variable Spreading Factor method in STS-based CDMA system to improve the data throughput of the system. The BER performance of the system with respect to pre-despreading SNR was calculated in non-frequency-selective Rayleigh fading channel, for different values of spreading factor. Using the thresholds obtained in this way and under the assumption of constant interfering users and perfect interference cancellation, Variable Spreading Factor method was performed on STS-based CDMA system to enhance the average throughput of the system in the same environment. An analytical approach was then proposed to compute the throughput enhancement of the system. The matching between the results of simulation and numerical computation of the corresponding expressions proved that the proposed expressions can be used to evaluate the performance of the VSF STS-based CDMA system in non-frequency-selective Rayleigh channel.

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