Optimal Integrated Control and Filtering Approach for Dynamic Weighing Systems Performance Improvement

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***Abstract* — In this paper an optimal integrated control and filtering approach is investigated for performance improvement of a weighcell based dynamic weighing system. The weighcell is a controllable weighing device that operates according to the principle of electromagnetic force compensation. The two main aims for improvement are: (1) to increase the speed of weighing and (2) to achieve good measurement accuracy. These goals are contradictory and are addressed through an integrated control and filtering approach by employing Linear Quadratic Gaussian (LQG) design method. The method is further blended with classical control scheme creating a frequency shaped LQG approach. In order to use LQG technique a mathematical model of the weighing system is developed. Moreover, an analytical solution for weight filter is derived. Finally, obtained results are compared to the results of controllers employed in the contemporary dynamic weighing systems.**

***Index Term* — Checkweighing systems, Dynamic weighing systems, Instrumentation, LQG controller, Measurement, Modelling, Optimal control and filtering.**

I. Introduction

In the area of mass production, products are weighed dynamically using dynamic weighing systems, also called checkweighing systems. The weight of an article is estimated while it has been carried by a transport system over a weightable mounted on the weighing sensor. There are several types of weighing sensors and all of them are divided into two groups [1]: (i) uncontrollable weighing devices termed as load cells, and (ii) controllable weighing devices termed as weighing cells.

The load cell is based on a strain gauge, which makes use of the property of a conductive material that changes the electrical resistance in response to deformation by a mechanical load [2]. As such, the displacement of the weighing table, however it is small, tends to amplify the environmental noise and the accuracy of dynamic checkweighers based on load cells goes up to 0.02% at most [3]. Load cells, however, offer a cheaper solution. Within the weighing cell the weight of an article causes a motion of a beam balance. This motion is sensed by a position sensor and then electrodynamically compensated to as near as zero [4]. Consequently, the weighing cell actively compensate for vibration noise and the accuracy of dynamic checkweighers based on weighing cells goes up to 0.0004% [3].

The main objectives of a dynamic weighing system are to increase accuracy and throughput rate of article weighing. The accuracy of a dynamic weighing system based on the weighing cell depends on the performance of the position control loop, in particular steady-state error. At the same time, however, high performance of the position control loop slows down the overall transient response, hence decreasing throughput rate. Current solutions are primarily based on Proportional Integral Derivative (PID) controller to stabilise the system and to provide fast transient response [4]. Additional filters are often used to further increase accuracy [5] and to provide stable results. However, these solutions based on control and filtering approach have not been able to keep in step with the current weighcell, the resolution of which has been improved up to 10^5-10^7 steps.

In this paper a novel method for improvement of performance of the positional control loop of a checkweighing system based on the Linear Quadratic Gaussian (LQG) approach is presented. The performance was experimentally verified and compared with published results.

II. Modelling

In order to apply the LQG method, a model of weighing cell based dynamic checkweigher is needed. A dynamic weighing cell used in this study is based on a controllable electromagnetic compensated weighing device employing a beam balance (Fig. 1), which can be described by a second order differential equation as

$$\left(w(t)+m_1+m_8+m_2+\frac{m_6+m_7}{3}+\frac{\Theta_b}{2}\right)\frac{a^2}{d'}\ddot{\theta}(t)+ \quad (1)$$
$$+k\frac{a^2}{d'}\dot{\theta}(t)+c\frac{a^2}{d'}\theta(t)=w(t)ga-F_c(t)$$

and with compensation force produced by the coil

$$F_c(t)=Bli_c(t) \quad (2)$$

Combining equations (1) and (2) gives

$$\frac{\Theta_{tot}}{d'}\ddot{\theta}(t)+\frac{ka^2}{d'}\dot{\theta}(t)+c\frac{a^2}{d'}\theta(t)= \quad (3)$$
$$=w(t)ga-Bli_c(t)b$$

Where w(t) is the mass of the article to be weighted, m_1-m_8 are the masses of the mechanical structure, k is damping coefficient, c is spring constant, a, b, c, d, d', e are particular dimensions of mechanical structure, F_6, F_7, F* are reaction forces in the joints of the mechanical structure, F_c is compensation force produced by electromechanical compensator, Θ is position of beam balance, Θ_b, Θ_k are inertia moments of weighcell, Θ_{tot} is total inertia moment of weighcell, Bl is the coil constant and i_c is the control current through the coil.

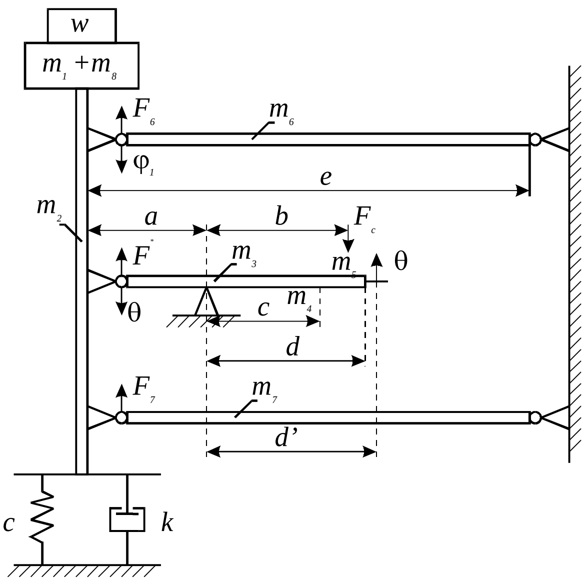

Fig. 1. Mechanical structure of the weighing cell

Defining the state variables as $x_1(t) = \Theta(t)$ and $x_2(t) = \dot{\theta}(t)$, the state differential equations of the system from equation (3) are

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = = \begin{bmatrix} 0 & 1 \\ -\dfrac{ca^2}{\Theta_{tot}} & -\dfrac{ka^2}{\Theta_{tot}} \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + + \begin{bmatrix} 0 \\ \dfrac{d' ga}{\Theta_{tot}} \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ -\dfrac{d' Blb}{\Theta_{tot}} \end{bmatrix} i_c(t) \tag{4}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + v(t)$$

where $x_n(t)$ (n=1,2) are states of the systems, y(t) is the output of the system and v(t) is the measurement noise, Gaussian with zero mean and variance R_v. For the measured parameters of the weighing cell given in [6], the state equations (4) became

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -87.658 & -2.124 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + + \begin{bmatrix} 0 & 0 \\ 3.985 & -11.709 \end{bmatrix} \mathbf{u}(t) + z(t) \tag{5}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + v(t)$$

Where $u(t) = \begin{bmatrix} w(t) \\ i_c(t) \end{bmatrix}$ is the system input, and z is the system noise, Gaussian system noise with zero mean and variance Rz. In discretised state space form, the equations (5) becomes

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \mathbf{Ax}(k) + \mathbf{Bu}(k) + \mathbf{z}(k) = = \begin{bmatrix} 0.9998 & 0.0020 \\ -0.1749 & 0.9956 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + + \begin{bmatrix} 0 & 0 \\ 0.0080 & -0.0234 \end{bmatrix} \begin{bmatrix} w(k) \\ i_c(k) \end{bmatrix} + \mathbf{z}(k) \tag{6}$$

$$y(k) = \mathbf{Cx}(k) + \mathbf{v}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \mathbf{v}(k)$$

III. LQG CONTROL DESIGN

It becomes apparent from equation (6) that the checkweighing system based on weighing cell has two inputs, and one output. One input is the weight, w(k), which is uncontrollable input, and the other is the current $i_c(k)$ which is a controllable input. In general, an optimal linear state-variable-feedback-control law is an attempt to find the optimal input **u**(t) (=-**L**(k)**x**(k)), which minimizes the quadratic cost function

$$\mathbf{J} = \mathbf{x}^T(N)\mathbf{S}_N\mathbf{x}(N) + \sum_{k=0}^{N-1}[\mathbf{x}^T(k)\mathbf{Q}(k)\mathbf{x}(k) + \mathbf{u}^T(k)\mathbf{R}(k)\mathbf{u}(k)] \tag{7}$$

where **L** is a feedback gain matrix, $\mathbf{S}_N$ is a non-negative symmetric matrix, **Q** is a non-negative symmetric weighing matrix that determines how much weight is attached to each of the components of the state and **R** is a positive definite symmetric weighing matrix that determines how much weight is attached to each of the components of the input. Following the procedure given in ref. [7], the optimal feedback gain matrix **L**(k) is found as

$$\mathbf{L}(k) = \{\mathbf{R}(k) + \mathbf{B}^T(k)[\mathbf{Q}(k+1) + \mathbf{S}(k+1)]\mathbf{B}(k)\}^{-1} * \\ * \mathbf{B}^T(k)[\mathbf{Q}(k+1) + \mathbf{S}(k+1)]\mathbf{A}(k) \tag{8}$$

where the symmetric non-negative definite matrix **S**(k) satisfies the matrix Riccati equation

$$\mathbf{S}(k) = \mathbf{A}^T(k)[\mathbf{Q}(k+1) + \mathbf{S}(k+1)][\mathbf{A}(k) - \mathbf{B}(k)\mathbf{L}(k)] \\ \mathbf{S}(N) = \mathbf{S}_N \tag{9}$$

Basic structure of an LQG controller applied to checkweighing systems is presented in Fig. 2. The approach adopted in this paper was to treat the weight as an unknown disturbance and let the input current i_c of the system neutralise it over time. In this case, the overall problem is refereed to as regulator. Furthermore, a separate weight filter was designed to get an estimate of the weight. This weight filter is connected to the LQG controller as shown in Fig. 2**.**

From a control point of view, this is SISO system, and we found that the appropriate cost function is of the form

$$\mathbf{J} = \sum_{k=0}^{N}[\mathbf{x}(k)^T \mathbf{Q}\mathbf{x}(k) + i_c(k)^T R i_c(k)] \tag{10}$$

where, $\mathbf{Q} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix}$ and R is the positive scalar value. The actual range of Q_{22} values depend on mechanical characteristics of a particular weighcell.

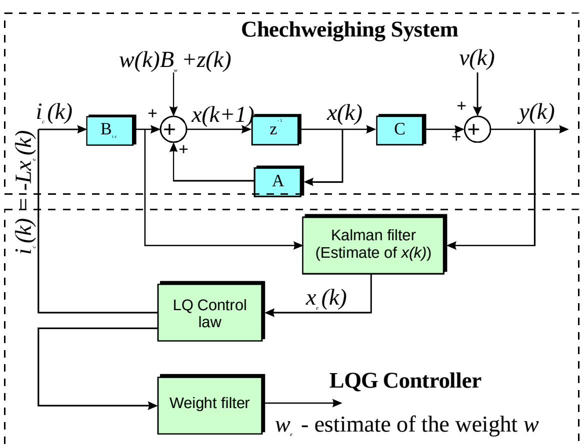

Fig. 2. LQG controller for the system in discrete time state space description

The value of R directly effects the value of the control input i_c(k): smaller the value of R, the larger is the current i_c(k). Therefore, in reality the value of R is heavily dependent on the electrical characteristics of weighcells. In order to get the optimal value of the control input i_c(k) (=-**Lx**(k)), which minimises the cost function given in (10), the feedback gain matrix **L** was calculated according to equation (8). Once the feedback gain matrix had been determined, equations for the closed loop were obtained by substituting values for the current i_c(k) into equation (6) that gives

$$\mathbf{x}(k+1) = \\ = \mathbf{A}\mathbf{x}(k) - \mathbf{B}_{i_c} Lx(k) + \mathbf{B}_w w(k) + \mathbf{z}(k) = \\ = (\mathbf{A} - \mathbf{B}_{i_c}\mathbf{L})\mathbf{x}(k) + \mathbf{B}_w w(k) + \mathbf{z}(k) \tag{11}$$

Consequently, the closed-loop characteristic values can be calculated as

$$\mathbf{ZI\text{-}A\text{+}B}i_c\mathbf{L}=0 \tag{12}$$

Different loci of the closed-loop characteristic values can be obtained by varying the control gain matrix L. The matrix is completely controllable by weighting matrices **Q** and *R*. Therefore, by changing the value of matrices **Q** and *R*, the system response and steady-state error change.

IV. Experimental results

In the regulator problem, the control input, current, i_c(k) which keeps the position of beam balance close to zero, follows the changes in the value of the forcing function. Therefore, the current i_c(k) is used for estimation of a product weight. The fact that an estimator of Kalman filter is used for estimation of the states (Fig. 2) means that the value for current i_c(k) is already filtered. Figure 3 shows the values for the control input for the case without estimator, as well as with estimator.

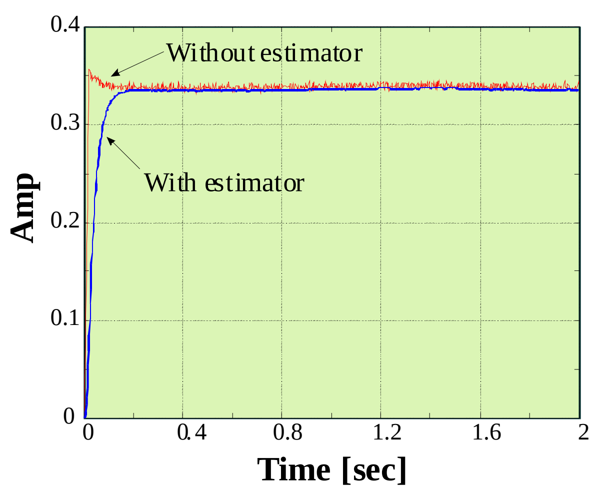

Fig. 3. Control input of the closed-loop system

However, an additional weight filter, as shown in Figure 2, is needed for further performance improvement. The algorithm for the weight filter was derived as follows. From equation (6), the states of the system are:

$$x_1(k+1) = 0.9998x_1(k) + z_1(k) \\ x_2(k+1) = -0.1749x_1(k) + 0.9956x_2(k) + \\ + 0.008w(k) - 0.0234i_c(k) + z_2(k) \tag{13}$$

In the steady state, with no movement of the balance beam, $x_2=0$. Consequently, equations (13) become

$$\begin{aligned} x_1(k+1) &= 0.9998x_1(k) + z_1(k) \\ 0 &= -0.1749x_1(k) + 0.008w(k) - \\ &\quad - 0.0234i_c(k) + z_2(k) \end{aligned} \tag{14}$$

and the control input current $i_c(k)$ is calculated as

$$i_c(k) = -L_1 x_1(k) = -L_1 x_{e1}(k) \tag{15}$$

where x_{e1} is the estimate of x_1, which takes into account the system and measurement noise. Substituting values for $i_c(k)$ from equation (15) into equation (14), the expression for the weight filter is obtained as

$$w_e(k) = [21.875 - 2.925L_1]x_{e1}(k) \tag{16}$$

In order to reduce the steady state error further, an integrating action was included in LQG design method as shown in **Fig. 4**. The outputs with and without integral action are shown in Figure 5 indicating that the output of the system controlled with additional integral action is balanced to zero, whilst the output of the system controlled without integral action exhibits a steady state error of 0.0008 V.

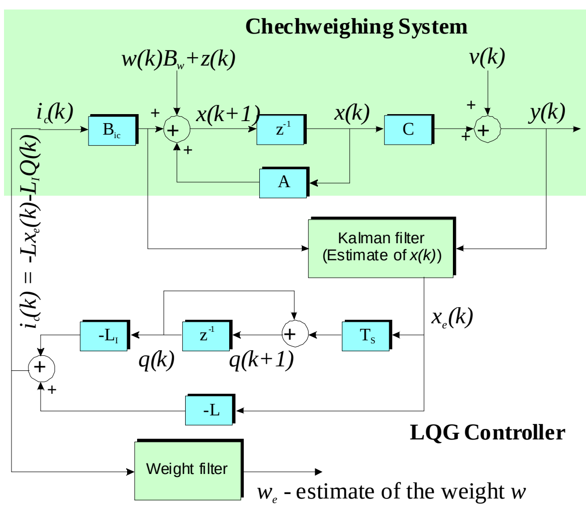

Fig. 4. LQG controller with integral action

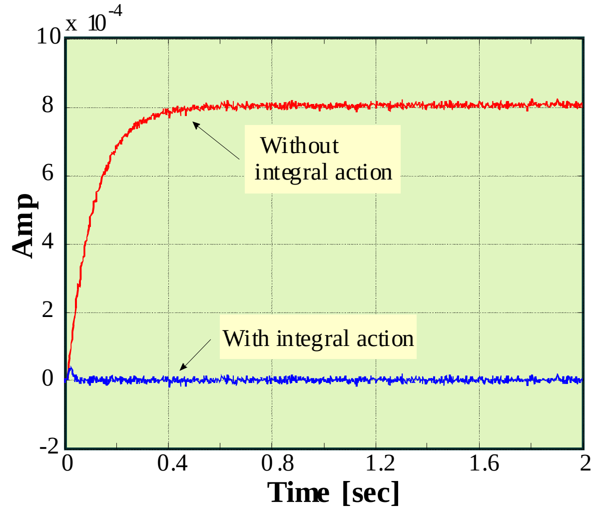

Fig. 5. Output of the controlled system with and without integral action included

The performance of a dynamic weighing system is specified by accuracy and throughput rate. This can be translated as the steady state error and transient response of the controlled system, respectively. To evaluate the performance of the developed LQG controller with respect to existing alternatives, a comparison was made to the PD controllers designed in ref.[6]. Performance of the controllers were analysed by using the step response, and the results are summarised in Table 1.

TABLE I
COMPARATIVE PERFORMANCE OF LQG CONTROLLER

		tr in ms		Overshoot (%)		tss=1% ms	
Mass kg		0.1	1.5	0.1	1.5	0.1	1.5
Ref[6]		1.4	2.4	0.128	0.162	112.4	111.4
		1.4	1.4	0.128	0.143	8.4	9.4
LQG Controller		1.2	1.2	0.130	0.150	2.2	2.9

$$I_4 = \int t\left|i_{kd}(t)\right|dt$$

$$I_5 = \int t\left|i_{kd}(t)\right|dt$$

The performance of the controller is characterized in terms of rise time t_r, overshoot, and the time taken for the output to come within a defined percentage of its final value, $t_{ss}=X$ %. The output signal used is a step response to a noise-free input. i_{kd} corresponds to i_c in this paper.

It becomes apparent from Table 1 that the LQG controller performance exceeds the performances of both controllers designed in ref. [6]. However, the overshoot is slightly higher than the one obtained by a controller tuned using function I_5.

V. CONCLUSION

This paper investigates the suitability of using LQG design method in weighcell based checkweighing systems. The method is based on a linear optimal control law, which takes state variables of a system as its inputs. The regulating aspect of control problem, with included stochastic disturbances, was presented. The known mathematical model of checkweighing system was adapted for the required state space model representation. An LQG controller was designed and its characteristics presented. In order to reduce the steady state error further, an integrating action was included in LQG design method. Furthermore, an algorithm for weight filter was derived. Finally, this method was compared to the results of previous work. The comparison showed that additional performance improvement could be achieved by adopting the LQG design method

VI. References

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