Adaptive Hybrid Control of Cooperative Parallel Manipulators

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***Abstract* — In this paper, a new method to control the position and internal force of a cooperative parallel-manipulator system is proposed. The cooperative system includes two parallel 6-dof Stewart manipulators and a rigid object grasped by the manipulators. Combining dynamic equations of Stewart platforms and the object, the dynamic equation of the cooperative system is derived. A new adaptive strategy using feedback linearization method is suggested to control the object position in existence of parametric uncertainties in the dynamic model. To control the internal force, a linear controller composed of feedforward and integral parts, is used. Simulation results show the convergence of the object position to the desired path while the internal forces remain in the allowable bound.**

***Index Terms* — Cooperative Robots, Adaptive Control, Parallel Manipulators, Hybrid Control, Stewart Platform.**

I. Introduction

Cooperative multi-robot systems have attracted considerable attention since 1980s, because utilization of such systems provide greater lifting and manipulation capability and higher flexibility in assembly tasks or object manipulation. Approximately, in all these works, only conventional serial manipulators have been considered in the cooperative system. These serial-manipulator cooperative systems lack the strength to precisely manipulate heavy and large objects. Using parallel manipulators in the cooperative system is the best way to overcome this problem due to their advantages compared with serial manipulators. High accuracy and strength are two important characteristics offered by parallel manipulators. Therefore, an accurate manipulation and assembly capability of heavy and large objects can be obtained through using parallel manipulators in cooperative multi-robot systems.

Several approaches such as hybrid control, robust control and impedance control, have been used in the position and force control of cooperative serial-manipulator systems [1]-[3]. Among these, adaptive control is the most popular method used in the existence of parametric uncertainties in the dynamic model. Uzmay, Burkan and Sarikaya [4] presented a study on application of adaptive control methods to a cooperative manipulation system, which was developed for handling an object by two-link planar manipulators. Zribi and Ahmad [5] proposed an adaptive controller that ensured asymptotic convergence of the load position to their desired values and boundedness of the internal forces. They also considered the effects of bounded disturbances on the multi-robot system. Yao and Tomizuka [6] obtained a set of transformed dynamic equations in the joint space. Based on particular properties of these reformulated equations, an adaptive algorithm was developed with unknown parameters updated by both motion and force tracking error.

The cooperative system in all above works includes only serial manipulators. In this paper, two 6-dof Stewart platforms, the most popular parallel manipulators, are considered in handling a rigid object in a desired path. Proposed adaptive strategy uses feedback linearization to determine the control law and gradient method to derive adaptation law. To control the internal forces feedforward and integral controllers are utilized. Simulation results show convergence of the object position to the desired path and boundedness of internal forces.

II. Dynamic Modeling

The cooperative system including two Stewart Manipulators and a rigid object is shown in Fig. 1.

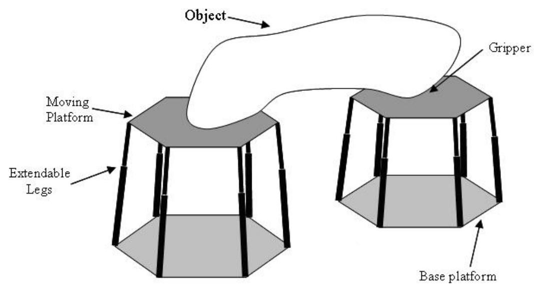

Fig. 1. Cooperative Stewart platform-based system

A. Multi-Robot Dynamic Equations

Dynamic equations of two Stewart platforms can be stated in the following compact form [7],

$$J\ddot{x}_e + \eta = HF + F_e \tag{1}$$

Where $J \in R^{12\times12}$ is a diagonal inertia matrix whose diagonal elements are the inertia matrices of the each manipulator, $\ddot{x}_e \in R^{12}$ is linear/angular acceleration vector of the two Stewart moving platforms, $\eta \in R^{12}$ is the vector of centrifugal, coriolis and gravity force/moment vector of two manipulators, $H \in R^{12}$ is a diagonal matrix with force transformation matrix of each manipulator as its diagonal elements, $F \in R^{12}$ is the input force vector in Stewart legs and $F_e \in R^{12}$ is external force/moment vector applied to the object at points of contact with manipulators.

B. Object Dynamic Equation

We assume that the object is rigidly grasped by the manipulators. So, the equations of motion of the object is obtained from Newton-Euler approach as,

$$M_o \ddot{z} + M_o g = \sum_{i=1}^{2} f_{ei} \tag{2}$$

$$I_o \dot{\omega} + \omega \times (I_o \omega) = \sum_{i=1}^{2} (\tau_{ei} + r_i \times f_{ei}) \tag{3}$$

Where $M_o \in R^{3\times3}$ is a diagonal mass matrix whose diagonal elements are the mass of the object, $I_o \in R^{3\times3}$ is the inertia matrix of the object, $g \in R^3$ is the gravity force vector, $f_{ei} \in R^3$ and $\tau_{ei} \in R^3$ are contact forces and moments, respectively. $z \in R^3$ is the position vector of the object center of mass and $\omega \in R^3$ is the angular velocity of the object. $r_i = [r_{ix}, r_{iy}, r_{iz}]^T$ represents the displacement vector from the center of mass of the object and the contact point of the object and ith manipulator.

Defining $\dot{x} = [\dot{z}^T, \omega^T]^T$, (2) and (3) can be written in the following compact form,

$$M\ddot{x} + N\dot{x} + G_l = GF_e \tag{4}$$

Where,

$$M = \begin{bmatrix} M_o & 0 \\ 0 & I_o \end{bmatrix},\ N\dot{x} = \begin{bmatrix} 0 \\ \omega \times (I_o \omega) \end{bmatrix},\ G_l = \begin{bmatrix} M_o g \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} I_3 & 0 & I_3 & 0 \\ \Omega_1 & I_3 & \Omega_2 & I_3 \end{bmatrix},\ \Omega_i = \begin{bmatrix} 0 & -r_{iz} & r_{iy} \\ r_{iz} & 0 & -r_{ix} \\ -r_{iy} & r_{ix} & 0 \end{bmatrix} \tag{5}$$

$G \in R^{6\times12}$ is called grasp matrix and I_3 is the identity matrix.

C. Kinematics of the Cooperative System

Using (4), the resultant force/moment vector, $F_o \in R^6$, in the mass center of the object is obtained as,

$$F_o = GF_e \tag{6}$$

From (6) and using Duality principle between the forces and velocities [8], we can write the following equation,

$$G^T \dot{x} = \dot{x}_e \tag{7}$$

Differentiating (7) with respect to time, yields to the following equation,

$$\ddot{x}_e = \frac{d}{dt}(G^T \dot{x}) = \frac{d}{dt}(G^T)\dot{x} + G^T \ddot{x} \tag{8}$$

D. Dynamic Model of Cooperative Multi-Robot System

Calculating F_e From (1), and replacing it in (4) results the following equation,

$$M\ddot{x} + N\dot{x} + G_l = G(J\ddot{x}_e + \eta - HF) \tag{9}$$

Using kinematic relations in (7) and (8) and simplifying the resulted equation, dynamic model of the cooperative system is obtained in terms of the object variables as follows,

$$\begin{aligned}&(G\bar{J}G^T - M)\ddot{x} + \left[G\bar{J}(\frac{d}{dt}(G^T)) - N \right]\dot{x} \\ &+ G\bar{\eta} - G_l = G\bar{H}F\end{aligned} \tag{10}$$

Where $\bar{J}$, $\bar{\eta}$ and $\bar{H}$ are expressed in terms of object vectors, x and $\dot{x}$.

E. Dynamic Model Considering Internal Force

The force/moment applied to the object by two manipulators is composed of two parts; an effective part in the object motion, $\hat{F}_o$, and internal force/moment part, F_{int}, calculated from following equations,

$$\hat{F}_o = G^+ F_o \quad , \quad F_{\text{int}} = (I - G^+ G)\varepsilon_{\text{int}} \tag{11}$$

$\varepsilon_{\text{int}} \in R^{12}$ is an arbitrary vector determining the internal force/moment vector and G^+ is the pseudo-inverse matrix of G. Combining $\hat{F}_o$ and F_{int} and using (4) and (6) results in,

$$F_e = G^+(M\ddot{x} + N\dot{x} + G_l) + (I - G^+ G)\varepsilon_{\text{int}} \tag{12}$$

Finally, (12) is replaced in dynamic equation of multi-robot system, (1) and kinematic relations are used to obtain following simplified equation,

$$\begin{aligned}&(\bar{J}G^T - G^+ M)\ddot{x} + (\bar{J}\frac{d}{dt}(G^T) - G^+ N)\dot{x} + \bar{\eta} - G^+ G_l = \\ &\bar{H}F + (I - G^+ G)\varepsilon_{\text{int}}\end{aligned} \tag{13}$$

(13) is the dynamic model of the cooperative system which will be used in the control of internal forces.

III. Control Design

A. Position Control

Considering parametric uncertainties in the object and manipulators, dynamic model in (10) can be written in the following form,

$$\begin{aligned}\ddot{x} &= (G\hat{\bar{J}}G^T - \hat{M})^{-1}\{-[G\hat{\bar{J}}(\frac{d}{dt}(G^T)) - \hat{N}]\dot{x} \\ &-G\hat{\bar{\eta}} + \hat{G}_l\} + \{(G\hat{\bar{J}}G^T - \hat{M})^{-1}G\bar{H}\}F\end{aligned} \tag{14}$$

Where $\hat{\bar{J}}$, $\hat{M}$, $\hat{N}$, $\hat{\bar{\eta}}$ and $\hat{G}_l$ are the estimated unknown variables. (14) is in the known companion form [9]. Therefore, position control law can be derived using feedback linearization method as,

$$\begin{aligned}F_m &= \left((G\hat{\bar{J}}G^T - \hat{M})^{-1}(G\bar{H})\right)^+ \\ &\{v - (G\hat{\bar{J}}G^T - \hat{M})^{-1}[-(G\hat{\bar{J}}(\frac{d}{dt}(G^T)) - \hat{N})\dot{x} \\ &-G\hat{\bar{\eta}} + \hat{G}_l]\}\end{aligned} \tag{15}$$

Where v is defined as,

$$v = \ddot{x}_{des} - 2\lambda\,\dot{e} - \lambda^2 e \ , \ \dot{e} = \dot{x} - \dot{x}_{des} \ , \ e = x - x_{des} \tag{16}$$

x_{des} represents the object desired path and λ is a positive control gain. F_m is the part of the control input force which contributes to the motion control of the object.

The gradient method is used to derive the adaptation law in order to on-line estimation of the unknown parameters. Physical parameters of the object and manipulators should be selected such that dynamic model in (10) can be expressed in the following linear form,

$$\begin{aligned}&(G\bar{J}G^T - M)\ddot{x} + \left[G\bar{J}(\frac{d}{dt}(G^T)) - N\right]\dot{x} \\ &+G\bar{\eta} - G_l = Y(x,\dot{x},\ddot{x})\ \theta = F^*\end{aligned} \tag{17}$$

Where $\theta \in R^{\mathrm{p}}$ is the vector of exact parameters and $Y(x,\dot{x},\ddot{x}) \in R^{6\times\mathrm{p}}$ is the regressor matrix whose elements are combinations of the elements of inertia, centrifugal/coriolis and gravity vectors. F^* is the input vector to the plant and from (10), it is obtained as,

$$F^* = G\bar{H}F_m \tag{18}$$

Similarly, the estimated dynamic model can be written as,

$$\begin{aligned}&(G\hat{\bar{J}}G^T - \hat{M})\ddot{x} + \left[G\hat{\bar{J}}(\frac{d}{dt}(G^T)) - \hat{N}\right]\dot{x} \\ &+G\hat{\bar{\eta}} - \hat{G}_l = Y(x,\dot{x},\ddot{x})\ \hat{\theta}\end{aligned} \tag{19}$$

$\hat{\theta}$ is the vector of estimated parameters. Estimation error is defined as,

$$E = Y\hat{\theta} - F^* \tag{20}$$

In fact, E represents the difference between the current estimated output of the plant, $Y\,\hat{\theta}$, and plant input F^* which is determined by preceding information of the system. Gradient method [9] is utilized to derive adaptation law as follows,

$$\dot{\hat{\theta}} = -\frac{1}{2}\gamma\frac{\partial(E^T E)}{\partial\hat{\theta}} \tag{21}$$

Where γ is a positive constant called estimation gain. Replacing (20) in (21) and differentiating with respect to $\hat{\theta}$ yields to,

$$\dot{\hat{\theta}} = -\gamma\ Y^T(Y\hat{\theta} - F^*) \tag{22}$$

The unknown parameters are estimated on-line through adaptation law in (22).

To prove the stability of the adaptive controller, consider a continuous and positive semi-definite function such as $V = 1/2\ \delta^T\delta$ where $\delta = \hat{\theta} - \theta$ is the parameter error vector. Differentiating V and using (17) and (22) results in,

$$\begin{aligned}\dot{V} &= \delta^T\dot{\delta} = \delta^T\dot{\hat{\theta}} \\ &= \delta^T(-\gamma\ Y^T E) = -\gamma\ (\delta^T Y^T)E \\ &= -\gamma\ E^{\mathrm{T}}E \le 0\end{aligned} \tag{23}$$

Therefore, $\dot{V}$ is negative semi-definite. It can be concluded that V and the following integral are bounded,

$$\left|\int_0^\infty \dot{V}\,dt\right| < \infty \ \Rightarrow \ \int_0^\infty (E^T E)\,dt < \infty \tag{24}$$

Assuming that $E^T E$ is continuous, differentiable and positive, the following is inferred from (24),

$$\lim_{t\to\infty}\ (E^T E) = 0 \tag{25}$$

which results in,

$$\lim_{t\to\infty}\ E(t) = 0 \quad \Rightarrow \quad \lim_{t\to\infty}\ (Y\hat{\theta} - F^*) = 0 \tag{26}$$

Adding and subtracting F^* to right side of (19), the following equation is obtained,

$$(G\hat{\bar{J}}G^T - \hat{M})\ddot{x} + \left[G\hat{\bar{J}}(\frac{d}{dt}(G^T)) - \hat{N} \right]\dot{x} + G\hat{\bar{\eta}} - \hat{G}_l = F^* + (Y\hat{\theta} - F^*) \tag{27}$$

Using (15) and (18) to replace F^* in (27) and simplifying the resulted Eq. yields to,

$$\ddot{e} + 2\lambda\dot{e} + \lambda^2 = (Y\hat{\theta} - F^*) = E(t) \tag{28}$$

Taking the Laplace transform of (28),

$$\bar{e}(s) = \frac{\bar{E}(s)}{s^2 + 2\lambda s + \lambda^2} \tag{29}$$

Where $\bar{e}(s)$ and $\bar{E}(s)$ are Laplace Transforms of $e(t)$ and $E(t)$, respectively. From (29), we can write the following equations,

$$\lim_{s\to 0} s\,\bar{e}(s) = \lim_{s\to 0} \frac{s\,\bar{E}(s)}{s^2 + 2\lambda s + \lambda^2} = 0 \ , \quad \lim_{s\to 0} s^2\,\bar{e}(s) = \lim_{s\to 0} \frac{s^2\,\bar{E}(s)}{s^2 + 2\lambda s + \lambda^2} = 0 \tag{30}$$

From (30) and using Final-Value Theorem [10], convergence of tracking error to zero is ensured,

$$\lim_{t\to\infty} e(t) = 0 \quad , \quad \lim_{t\to\infty} \dot{e}(t) = 0 \tag{31}$$

Therefore, the proposed adaptive control system including the control law of (15) and the adaptation law of (22) guarantees convergence of the object position to the desired path.

B. Internal force control

The part of the control law which contributes in controlling the internal force/moment vector in (13) is considered as,

$$F_f = \bar{H}^{-1}(-F_{\text{int}\,d} + K_f \int e_f dt) \tag{32}$$

Where $e_f = F_{\text{int}} - F_{\text{int}\,d}$ is the internal force error, $F_{\text{int}\,d}$ is the desired internal force, and K_f is a positive constant gain.

Total control law is obtained by combining control forces in (15) and (32). Substituting this control law in dynamic model of (13) and simplifying the resulted equation yields to,

$$(\bar{J}G^T - G^+M)(\ddot{e} + 2\lambda\,\dot{e} + \lambda^2 e) = (\tilde{\bar{J}}G^T - G^+\tilde{M})v + (\tilde{\bar{J}}\frac{d}{dt}(G^T) - G^+\tilde{N})\dot{x} + \tilde{\bar{\eta}} - G^+\tilde{G}_l + e_f + K_f \int e_f dt \tag{33}$$

Where □=□−□ . Using (17) and (26), the following equation is concluded,

$$\lim_{t\to\infty} \tilde{\bar{J}} = 0 \ , \quad \lim_{t\to\infty} \tilde{M} = 0 \ , \quad \lim_{t\to\infty} \tilde{N} = 0 \ , \quad \lim_{t\to\infty} \tilde{\bar{\eta}} = 0 \ , \quad \lim_{t\to\infty} \tilde{G}_l = 0 \tag{34}$$

Assuming that $(\bar{J}G^T - G^+M)$ is a non-singular matrix and using (31), (33) and (34), convergence of the internal forces to the desired values is inferred,

$$\lim_{t\to\infty} (e_f + K_f \int e_f dt) = 0 \ \Rightarrow \ \lim_{t\to\infty} e_f = 0 \tag{35}$$

IV. Simulation Results

In order to prevent computational problems, it is assumed in this simulation that all parameters of two Stewart manipulators are known and uncertainties only exist in the object parameters. These seven unknown parameters are the mass and the inertia matrix elements of the object. Real parameters of the object is considered as,

$$m_o = 200 \ , \ I_o = \begin{bmatrix} 38 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 54 \end{bmatrix} \tag{36}$$

Whereas, initial estimated parameters is selected as,

$$m_o = 195 \ , \ I_o = \begin{bmatrix} 35 & -2 & 1 \\ -2 & 15 & 0.5 \\ 1 & 0.5 & 56 \end{bmatrix} \tag{37}$$

The desired path of the object must be chosen such that each Stewart platform moves in the manipulator workspace and also, the path does not enter the singularity manifolds. In this simulation, following time functions has been considered as a desired path,

$$x_d = \frac{1}{2}\sin(t) \ , \ y_d = \frac{1}{2}\cos(t) \ , \ z_d = 1 + \frac{1}{2}\sin(\frac{t}{2}) \qquad \gamma_d = 0 \ , \ \beta_d = 0 \ , \ \alpha_d = 0 \tag{38}$$

Where x_d, y_d and z_d are the desired components of the position vector of the object center of mass and γ_d, β_d and α_d are Z-Y-X Euler angles representing the desired orientation of the object. Desired internal force/moment vector between the object and each manipulator is considered as zero.

Constant control gains in (16), (22) and (32) are selected as $\lambda = 5, K_f = 8, \gamma = 0.05$. The simulation results are shown in Figs. 2-7. In Figs. 2-5, six components of the object position vector have been shown. To compare the object trajectory with the desired path, they are plotted in the same figure. The results show that the object properly tracks the desired path. Internal force/moment vector between the object and manipulator 1 is shown in Fig. 6. Although there are some variations around the desired values, the internal forces are in an allowable bound. Finally, the control input forces in prismatic joints of the Stewart legs is illustrated in Fig. 7 for manipulator 2. Regarding the geometric characteristics of Stewart manipulators, these forces can be provided by the manipulator's actuators and therefore, they are acceptable.

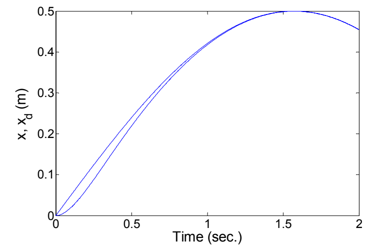

Fig. 2. Control output and desired path for x-component of the object position

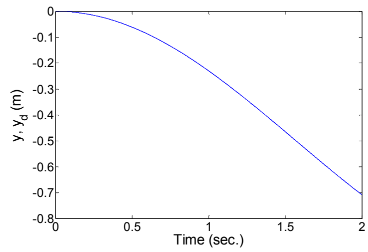

Fig. 3. Control output and desired path for y-component of the object position

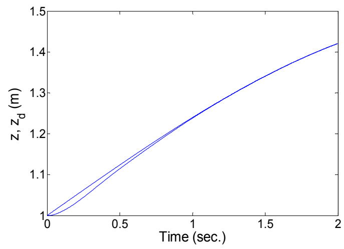

Fig. 4. Control output and desired path for z-component of the object position

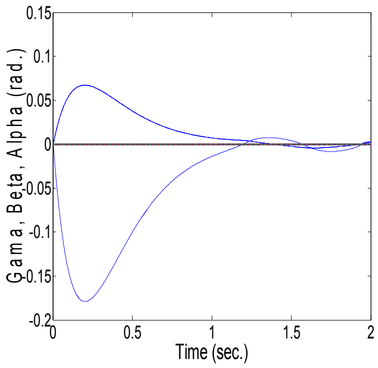

Fig. 5. Euler angles of the object

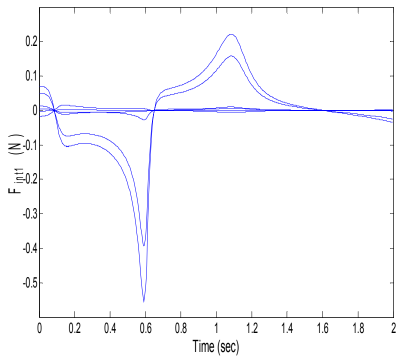

Fig. 6. Internal forces and moments between the object and manipulator 1

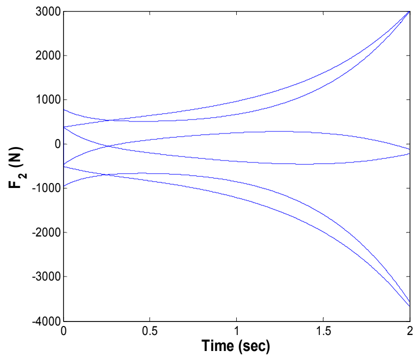

Fig. 7. Control input forces for manipulator 2

V. Conclusion

In this paper, two parallel Stewart manipulators handling a rigid object has been considered as a cooperative multi-robot system. A new adaptive control system using Feedback linearization and gradient method, has been proposed to make the grasped object track a desired path. Moreover, a linear controller including have been utilized to control the internal force. The stability analysis ensures the convergence of the object position and internal forces to their desired values. Simulation results show that the object properly tracks the desired path and internal forces remain in an allowable bound while input control forces are in an acceptable range.

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