Design of FIR Filters Using Identical Subfilters of Even Length

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***Abstract*—This article presents an analytic development for the design of linear-phase FIR digital filters with reduced computational and hardware complexity. The proposed approach is based on a frequency transformation implemented by replacing a subfilter in a prototype filter. The previous approach forced subfilters and prototype filters to be of odd length, while this approach supports subfilters and prototype filters of even length. Depending on the specifications of the filter, either the previous method or our proposed method can give the optimal solution for the design of the required filter. It has been shown by means of an example that the overall composite FIR filter with the proposed approach contributes to a saving of 22%, 22% and 22% in the number of adders, delays and multipliers respectively compared to the previous approach.**

I. Introduction

It is often of interest to employ finite impulse response (FIR) filters in a wide range of applications such as digital audio, video, and mobile telephony. This is due to their guaranteed stability, absence of limit cycles, and linearity of phase. However, FIR filters generally require more computation and hardware compared to infinite impulse response (IIR) filters with equivalent frequency responses. During the past several years, many design methodologies have been proposed to reduce the realization complexity of the FIR filters. One approach to the reduction of computational and hardware complexity is to design the filter by interconnecting a number of identical subfilters with the aid of a few additional adders and multipliers. This approach was suggested by Kaiser and Hamming in [1] and has been extended in [2] to allow for a piecewise linear desired amplitude change function (ACF). Another approach is the tapped cascaded interconnections of identical subfilters [3] with relatively relaxed frequency responses which results in an overall composite filter with desired superior characteristics. This method has been generalized in [4] with comprehensive analysis given. The design scheme consists of applying a frequency transformation for decomposing the overall composite filter into a prototype filter joining some identical subfilters. The passband and stopband edges of the subfilter are the same as those of the overall composite filter while the passband ripple and stopband attenuation of the prototype filter and those of the overall composite filter are the same. On the other side, the passband ripple and stopband attenuation of the subfilter and the passband and stopband edges of the prototype filter are relatively relaxed compared to those of the overall composite filter. In other words, the computational and hardware complexity of the overall composite FIR filter splits into the design of two FIR filters, one with relaxed ripple and attenuation and the other with relaxed transition band. The transfer function $H(z)$ of the overall composite linear-phase FIR filter can be expressed as

$$H(z) = \sum_{n=0}^{N} a(n) z^{-(N-n)M} \left[H_M(z)\right]^n, \tag{1a}$$

where

$$H_M(z) = \sum_{r=0}^{2M} h_{sub}(r) z^{-r}, \tag{1b}$$

is a linear-phase filter of length $2M+1$. The frequency transformation determines the subfilter $H_M(z)$, where as the prototype filter determines the $a(n)$ coefficients. An implementation of the transfer function (1) is depicted in Fig. 1a.

The architecture in [4] has several interesting features. The overall composite filter has been constructed in such a way as to permit the frequency response to be controlled by only changing the tap coefficients of the prototype filter or the coefficients of the subfilter implying a filter with adjustable frequency response. Moreover, filter's modular structure gives the ability to reuse a subfilter to save area by

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means of scheduling and resource sharing for lower frequency applications.

The major drawback of this approach is that it does not support subfilters and prototype filters with even lengths. To illustrate the point, consider that a prototype filter (subfilter) of length $2N$ ($2M$) can meet the desired frequency response. Having known that the even length is not allowed, the designer should overdesign the filter and use the prototype filter (subfilter) with the length $2N+1$ ($2M+1$). Our proposed approach is a complement to the method in [4] which makes the design of even length prototype filters and subfilters possible. Depending on the specifications of the filter, either the method in [4] or our proposed method can give the optimal solution for the design of the required filter.

This paper is organized as follows: Our analytic derivations are presented in Section II. Details of design of an FIR filter with the given specifications are described in Section III. Finally, the measured results, and conclusions are included in Sections IV and V, respectively.

II. Proposed Approach

We express the transfer function of a linear-phase FIR filter with an impulse response $h(n)$ of length $2N$ with symmetry of $h(2N-1-n)=h(n)$ as given in (2):

$$\hat{H}(Z) = Z^{-(\frac{2N-1}{2})}\hat{H}_0(Z), \qquad (2a)$$

where,

$$\hat{H}_0(Z) = \sum_{n=0}^{N-1} h(N-n-1)(Z^{\frac{2n+1}{2}} + Z^{-\frac{2n+1}{2}}). \qquad (2b)$$

By using the following equivalence, (2) is reduced to (4).

$$Z^{\frac{2n+1}{2}} + Z^{-\frac{2n+1}{2}} = 2T_{2n+1}\left[\frac{Z^{\frac{1}{2}} + Z^{-\frac{1}{2}}}{2}\right], \qquad (3)$$

where $T_{2n+1}(x)$ is a Chebyschev polynomial of $(2n+1)$th order,

$$\hat{H}_0(Z) = \sum_{n=0}^{N-1} a(n)\left(\frac{Z^{\frac{1}{2}} + Z^{-\frac{1}{2}}}{2}\right)^{2n+1}. \qquad (4)$$

The basic approach to reducing the complexity of the filter is to apply a proper frequency transformation to $\hat{H}_0(Z)$. Using the transformation (5) in (4) and by a substitution of variables (6) is implied, where the amplitude characteristics of the filter are preserved while scaling the frequency axis.

$$\frac{Z^{\frac{1}{2}} + Z^{-\frac{1}{2}}}{2} = \sum_{r=0}^{M-1} A(r)\left(\frac{z^{\frac{1}{2}} + z^{-\frac{1}{2}}}{2}\right)^{2r+1}, \qquad (5)$$

$$H_0(z) = \sum_{n=0}^{N-1} a(n).\left[\sum_{r=0}^{M-1} A(r)\left(\frac{z^{\frac{1}{2}} + z^{-\frac{1}{2}}}{2}\right)^{2r+1}\right]^{2n+1}, \qquad (6)$$

where $H_0(z)$ is an $(2M-1)(2N-1)$th degree polynomial of $(z^{1/2}+z^{-1/2})$. Note that coefficients $A(r)$ can be determined by either using some optimization procedures similar to those of [4] or utilizing an application specific prescribed subfilter (e.g. CIC filters in this study). By using the equivalence (3) once more, this time returning from right side of Chebyschev equivalence to the left, (6) is reduced to (7),

$$H_0(z) = \sum_{n=0}^{N-1} a(n).\left[\sum_{r=0}^{M-1} h_{sub}(M-r-1)(z^{\frac{2r+1}{2}} + z^{-\frac{2r+1}{2}})\right]^{2n+1}. \qquad (7)$$

The transfer function $H(z)$ of the overall linear-phase filter is given by (8).

$$\begin{aligned} H(z) &= z^{-(\frac{2M-1}{2})(2N-1)} H_0(z) \\ &= \sum_{n=0}^{N-1} a(n).z^{-(2M-1)(N-n-1)}.\left[H_M(z)\right]^{2n+1}, \end{aligned} \qquad (8a)$$

where,

$$H_M(z) = \sum_{r=0}^{2M-1} h_{sub}(r) z^{-r}. \qquad (8b)$$

$H_M(z)$ is a $2M$ length FIR filter. An implementation of 8(a) general transfer function is depicted in Fig. 1b. Note that the zero-phase prototype filter $\hat{H}_0(Z)$ of length $2N$ as given by (4), determines the tap coefficients $a(n)$, where the transformation in (5) gives the subfilter $H_M(z)$ of (8b).

III. Case Study

The specifications of a low-pass filter are as follows: passband edge (ω_p)=0.0085π, stopband edge (ω_s)=0.9115π, passband ripple (δ_p)=0.0025 dB, and minimum stopband attenuation (δ_s)=105 dB. The design of the overall composite filter meeting the above specifications can be split into two parts, the design of the prototype filter and the design of the subfilter. Note that the passband and stopband edges of the subfilter are: ω_p =0.0085π and ω_s=0.9115π, while the passband ripple and stopband attenuation of the subfilter can be relatively relaxed. Alternatively, the passband ripple and stopband attenuation of the prototype filter are: δ_p=0.0025 dB and δ_s=105 dB, where the passband and stopband edges of the prototype filter are somewhat relaxed compared to those of the resulting composite filter. We have chosen a subfilter transfer function specified in (9):

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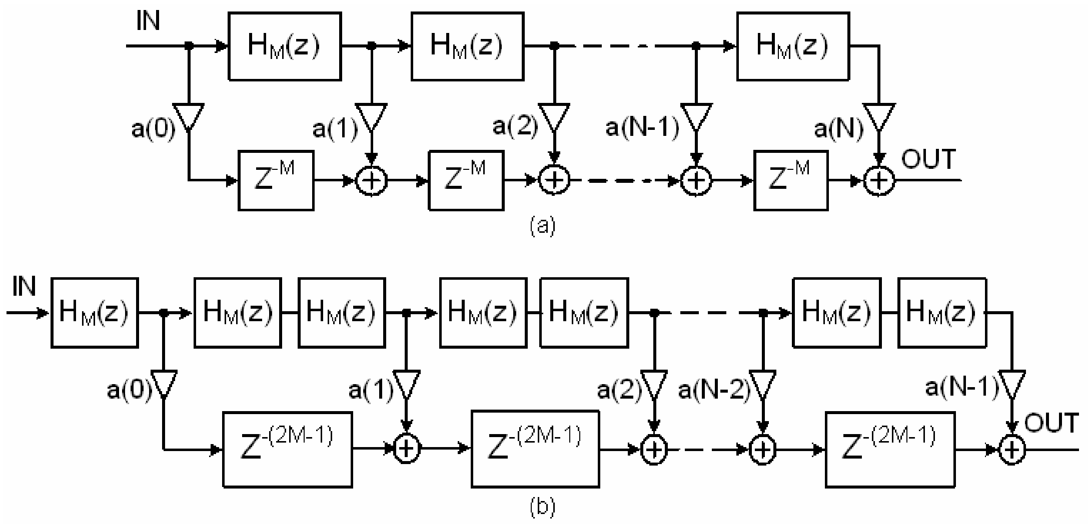

Fig. 1. a) Implementation of the filter in [4],
b) Implementation of our proposed structure

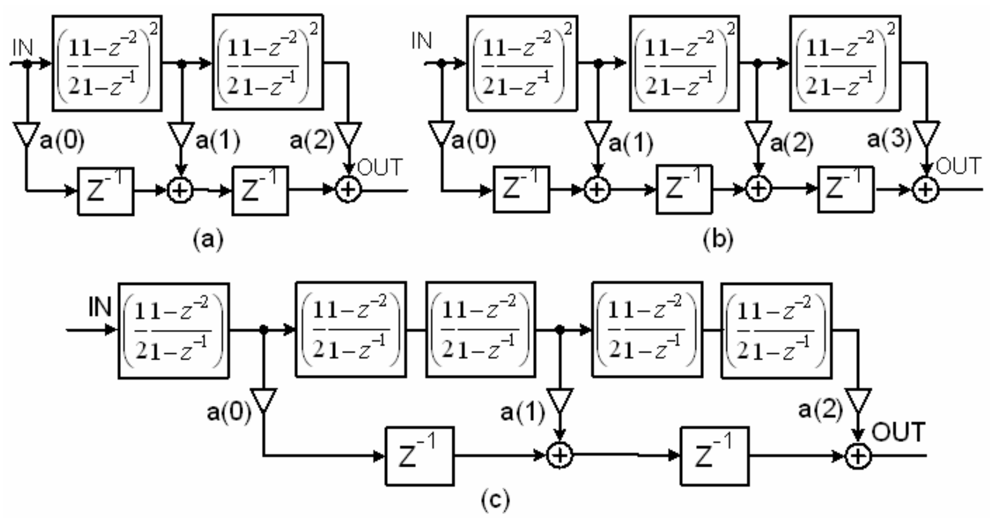

Fig. 2. Implementation of the three filters in the case study. a) Underdesigned filter (length 5, using [4]), b) Overdesigned filter (length 7, using [4]), c) Our proposed filter (length 6, this approach).

$$H_{sub}(z) = \left(\frac{1}{P} \frac{1 - z^{-P}}{1 - z^{-1}} \right)^K , \tag{9}$$

where $H_{sub}(z)$ is the transfer function of a CIC (cascaded integrator-comb) filter, first proposed by Hogenauer [5]. Note that $1/P$ is a scaling factor and K and P are integers. By selecting P=2 (for downsampling of two), (9) reduces to (10),

$$H_{sub}(z) = \left(\frac{1 + z^{-1}}{2} \right)^K . \tag{10}$$

If we first use the algorithm in [4] to implement the design, the length of the subfilter is imposed to be odd. Since length of the $(1+z^{-1})^k$ polynomial is K+1, by choosing K=2, a polynomial of odd length (i.e. 3) is obtained. Also, the length of the prototype filter should be odd. If we employ the prototype filter of length five (Fig. 2(a)), the specifications will not be met. On the other hand, if the length is chosen to be seven (Fig. 2(b)), the filter would be overdesigned.

Next by employing our method, the length of the subfilter is restricted to be even. Therefore, we should choose an odd integer for K in (10), where the simplest is one. Then, the length of the prototype filter becomes six (Fig. 2(c)), which simultaneously meets the specifications and is not overdesigned.

IV. SIMULATED RESULTS AND COMPARISON

A lowpass FIR filter with the given specifications has been designed in MATLAB. The filter has been designed using our proposed method and the method in [4]. The implementation of these three filters are demonstrated in Fig. 2. The frequency responses of these three filters are shown in Fig. 3. Table 1 compares the frequency response specifications and hardware complexities of these filters. Our proposed approach contributes to a saving of 22%, 22% and 22% in the number of adders, delays and multipliers respectively compared to the approach [4].

V. CONCLUSION

Since the method in [4] imposes a restriction on the length of the decomposing subfilter and the prototype filter to be odd, depending on the specifications of the overall composite filter, the method sometimes overdesigns these filters and consumes more power and area. In this paper, an analytic development for decomposing a design into identical subfilters and a prototype filter of even length has been presented. We conclude that depending on the specifications of a filter, either the method in [4] or our proposed method can give the optimal solution with lowest possible area and power.

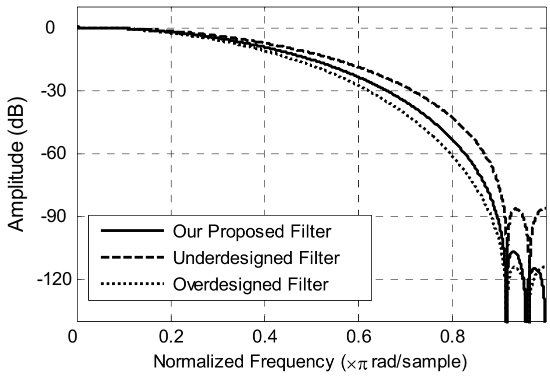

Fig. 3. Comparison of the frequency response of the three filters in the case study.

TABLE 1
COMPARISON BETWEEN THE THREE FILTERS IN THE CASE STUDY

Filter	Under Designed Filter	Over Designed Filter	Our proposed Filter	Saving
Adders	6	9	7	22%
Delays	6	9	7	22%
Multipliers	3	4	3	25%
Ripple (dB)	0.0030	0.0024	0.0019	
Attenuation (dB)	86	114	106	

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