Efficient Algorithms for Pulse Parameter Estimation, Pulse Peak Localization And Pileup Reduction in Gamma Ray Spectroscopy

M.W.Raad[1] , *L. Cheded*[2]

[1]Computer Engineering Department, [2]Systems Engineering Department
King Fahd University of Petroleum and Minerals,
Dhahran 31261, Saudi Arabia,
{mwraad, cheded}@ccse.kfupm.edu.sa

ABSTRACT

A fast waveform sampling facility has been recently developed and integrated into the VAX-based data acquisition system at the Energy Research Laboratory(ERL). The study reported in this paper uses the above facility to develop algorithms for digitally determining the basic pulse parameters and tackling the problem of pulse pile up in Gamma-Ray spectroscopy. A number of parameter estimation and digital online peak localization algorithms are developed here including a 3-point deconvolution and polynomial fitting of the pulse model. The set-up was also tested with random signals from a ^{137}CS test source. Gamma pulses from a 3" Na(TI) scintillation detector were captured as single and double pulses for the purpose of testing the peak detection algorithms. Two finite input *deconvolution* filters with 3 and 4 coefficients have been tested successfully to resolve pile-up to an average percentage of 93% *pileup* free. A polynomial of degree 8 or higher has been found to fit the Gamma Ray pulse very well giving a small Chi-square goodness-of-fit of approximately 1.2. The 3-point deconvolution filter gives an almost four-fold enhancement in the maximum throughput of the pulse analyzer, thus yielding a fast, reliable and cost-effective pulse analysis scheme.

Keywords: Pulse Pileup, Peak Detection, Deconvolution Filter, Gamma Ray Spectroscopy.

1. PULSE PILE UP IN GAMMA-RAY SPECTROSCOPY

A common problem in nuclear spectroscopy is pulse pileup caused by the non-zero response time of the detection system. For germanium detectors, the time required to collect all the ionization current associated with an event ranges from 0.5 to 6.0 µs [1]. The fact that pulses from a radiation detector are randomly spaced in time can lead to interfering effects between pulses when counting rates are not low. These effects are generally called pileup and can be minimized by making the total width of the pulses as small as possible [2]. Pileup phenomena are of two types. The first type is known as tail pileup and involves the superposition of pulses on the long duration tail from a preceding pulse (see Figure 1). Tails can persist for relatively long periods of time so that tail pileup can be significant even at relatively low counting rates. A second type of pileup is the peak pileup, which occurs when two pulses are sufficiently close to each other so as to be treated as a single pulse by the analysis system [3].

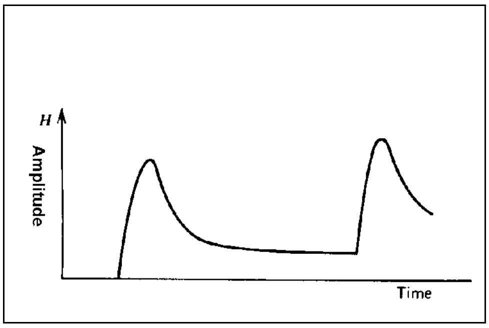

Figure 1. Pileup effect on a pulse peak from the tail of a preceding pulse

From a practical point of view, we have found this pulse spacing to be best taken as approximately equal to the peaking time of the Gamma event. The peak pileup available in real Gamma pulses amounts to approximately 16% of the Gamma pulses captured. Researchers often simply reject pileup even when it is recognized as such [4], thus leading to an unnecessary loss of information. In our case, we do not wish to lose the information associated with such events, so we use a 3-point deconvolution to separate the pulses.

2. Peak detection using deconvolution

In many applications we are given an output signal from a system whose characteristics are known and are requested to determine the input signal to this system. For instance, in nuclear spectrometry the semi-Gaussian shaping

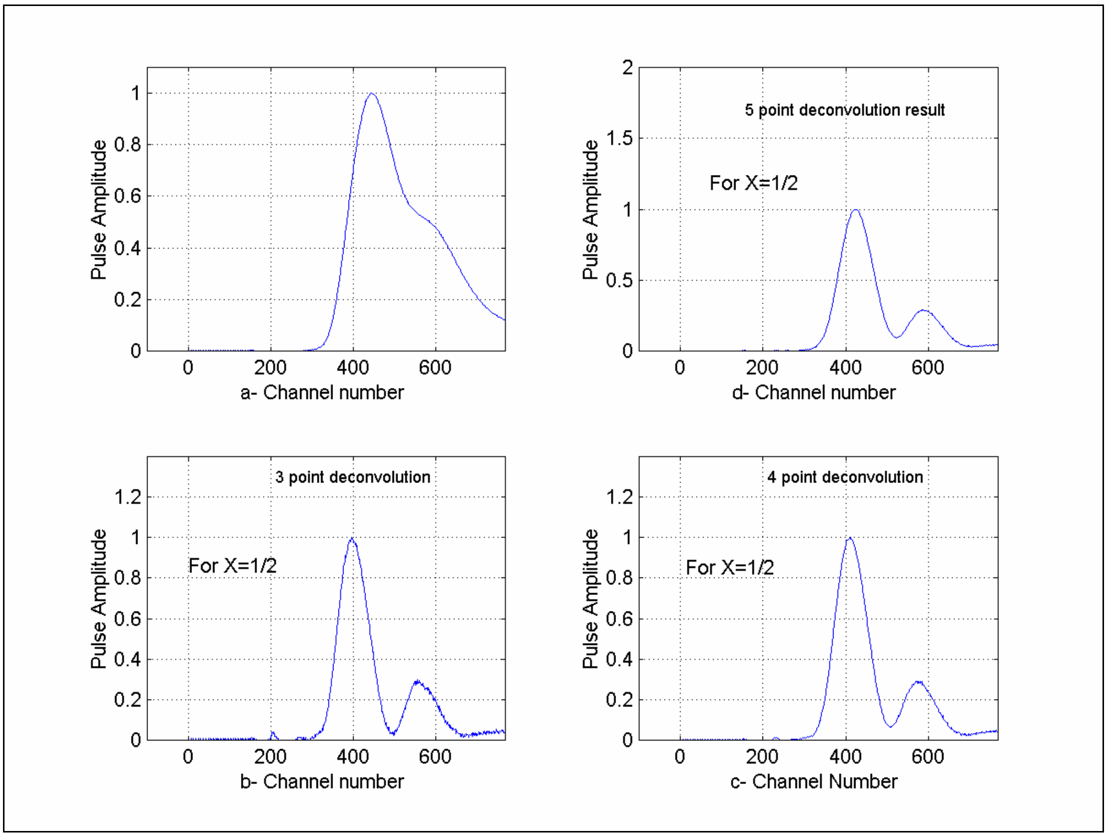

Figure 2. a) Gamma-ray pileup captured at ERL for sampling interval=20 nsec. b) 3-point deconvolution result for x=1/2, SNR=24.3 db. c) 4-point deconvolution result for x=1/2, SNR= 37.41 db, d) 5-point deconvolution result for x=1/2, SNR= 43.18 db

amplifier acts as a convolving filter processing the signal coming from the preamplifier and thus causes the shaped signal to have an elongated tail, which makes it prone to pile-up. In such a case, the problem is to design a corrective system which, when cascaded with the shaping system, produces an output that, in some sense, removes the effect of the shaper. The corrective system is called an inverse system and the operation is called deconvolution [2].

The deconvolution algorithm uses knowledge of the pulse shape to help locate and separate overlapping pulses. The pulse shape or the impulse response of the system has the following expression:

$$v(t) = t*\exp(-t/\tau) \qquad (1)$$

where τ is the shaping time constant. We have confined our system to be of the CR-RC type to make its transfer function as simple as possible. It has been found that the inverse system of this preamplifier CR-RC shaper has the following three weights [5]

$$w1=(1/x)*\exp(x-1) \qquad (2a)$$
$$w2=(-2/x)*\exp(-1) \qquad (2b)$$
$$w3=(1/x)*\exp(-x-1) \qquad (2c)$$

where $x = \Delta t/\tau$ and (Δ t) is the sampling interval. Given the input voltage samples $\{v_k\}$, The deconvolved output has the following expression:

$$S_k = w1*v_k + w2*v_{k-1} + w3*v_{k-2} \quad (3)$$

This means that a filter performing this operation can be constructed by forming the weighted sum of three consecutive voltage samples in time. We have verified this fact by computing the inverse of the impulse response matrix, which showed that only three weights of the inverse matrix are non-zero. A robust algorithm which maximises the signal amplitude at the expense of some loss in time resolution is to make a sum of two deconvolved samples. This is equivalent to using an algorithm with 4 weights which are easily calculated from the three original weights [6]. It can be easily shown that using a similar procedure will generate a 5-point deconvolution.

The 3-point deconvolution technique applied to tail pile-up events digitized at the ERL facility, succeeded in resolving pileup by 93% on average, which means that after *deconvolution*, 93% of the events were pileup-free. However, deconvolution was not able to recover peak pileup, which had to be rejected. Gamma Ray records captured at the ERL, showed a total pulse width of approximately 16μs, or 16τ, where τ is the pulse shape time constant. This means that the next event should not arrive within a period of 16τ, otherwise a signal pileup will occur. Hence, for this particular case, the maximum count rate or the maximum throughput of any pileup classifier is 1/(16τ). For Na(Ti) detector this translates to a maximum throughput .06 Mcps [7].

In our study, we have used simulated data based on the impulse response of the CR-RC amplifier/shaper, discussed earlier, and different types of white Gaussian noise are added to the simulated waveform to evaluate the accuracy of the different methods. The random statistical fluctuations were introduced by adding 25,000 sequences or iterations of white Gaussian noise to the simulated pulse signal. Applying the 3-point deconvolution on single Gamma pulses has resulted in the reduction of pulse width from 16τ to approximately 4τ,. This amounts to almost a four-fold enhancement of the maximum throughput of a pulse analyzer. Figure 2 depicts the results of deconvolution for various values of SNR (Signal-to-noise ratio) on real pile-up cases. It is clear from figure2 that the 5-point and 4-point deconvolution filters have a larger SNR than the 3-point deconvolution filter due to their wider processing rectangular weighting functions, but have less capability in resolving pileups than their 3-point counterpart. Hence there is always a trade-off between SNR and pileup-resolving power, to be considered when choosing these deconvolution filters. The sharp fluctuations shown in the background of the deconvolved pulse do not degrade the performance of the pulse analyzer because they are outside the pulse window under study.

3. Polynomial fitting of Gamma Ray Pulses

To reduce distortion in the Gamma Ray pulses used while retaining noise attenuation, simple polynomials can be fitted to data before applying the peak search routine as part of the pulse classification technique. One particular polynomial fit of interest to us here that deals with noisy and windowed data is based on the Savitzky-Golay digital filtering technique. This technique involves obtaining a least-squares fit of a polynomial to a set of an odd number (m) of adjacent noisy data points, and taking the value of the fitted polynomial at the central point as the smoothed value. The filter is essentially implemented as a finite-impulse response (FIR) filter. Using higher-order polynomials should allow fitting data that change rapidly within the filter window. As long as the noise continues to change more rapidly than the data, good noise rejection will be possible. By using low-order polynomials and increasing the size of the window, more noise is filtered out, but at the cost of signal distortion [8].

As a general rule, the filter window length should be selected as no more than a third of the resolution of the best-featured data. A Savitzky-Golay (SG) filter is a time-domain FIR filter. As the initial window, the SG filter takes the first (2m+1) points and fits, by least squares, the corresponding polynomial of order M to these data. The fitted value for the point in position m replaces the measured value. Next, the window is shifted by one point and the process is repeated until the the entire pulse signal is processed. The method uses tabulated coefficients A_i in such a way that the fitted value for the centre point of the window is computed as the new point:

$$Y_k = \sum_{i=-m}^{m} \frac{A_i y_{k+i}}{Norm} \quad (4)$$

where $Y_{k,}$ represents the fitted value at the centre point of the filter window; A_i is the appropriate coefficient value for each point and “Norm” is a normalizing constant. The user or designer of the Savitzky-Golay filter must decide the size of the filter window (2m+1) and the order M of the polynomial to be used. The Savitzky-Golay filter outperforms the moving-average filter in that it preserves the higher order moments up to (M-1), where M is the order of the polynomial used, whereas the moving-average filter only preserves the zero-order moment, which is the mean peak position. The Savitzky-Golay filter also preserves the spectral line resolution or width.

Through extensive simulation, we showed how well a polynomial of higher order can fit or approximate well the gamma -ray pulse model. Although the shape of the pulse is already known from the non-linear model, expressed as $t*\exp(-t/\tau)$, we have found from our own experience that we could approximate our nonlinear model with a polynomial of degree M with a satisfactory performance. Despite the fact that the pulse model for the gamma- ray event is well established, a thorough investigation revealed that a polynomial of 9^{th} degree fitted the real gamma-ray event with a chi-square goodness-of-fit=7.73. Referring to chi-square tables and using 9 degrees of freedom, we found that a chi^2 value 7.73 is far below the chi-square threshold for a 5% significance level. Hence, the polynomial of 9^{th} degree fits very well the gamma-ray pulse as shown below in figure 3.

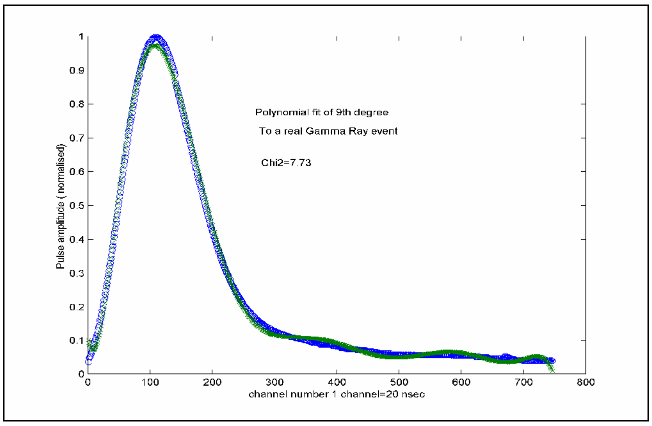

Figure 3. Excellent 9-degree polynomial fitting to the gamma-ray event with a very small chi-square goodness-of- fit at a 5% significance level.

An extensive simulation work was carried out to investigate using polynomial filters to fit both the deconvolved results and the real Gamma event by using the Chi-square goodness-of-fit and the least-squares methods. Due to the high computational cost of the least-squares fit to the established Gamma Ray model, we found that a polynomial fit of 8^{th}-9^{th} degree approximates very well the Gamma Ray event and can therefore be applied to the deconvolved events to give a very small Chi-square goodness-of-fit of approximately 1.2. Figure 4 shows the relation between the degree of the fitting polynomial and the sum of error squares. We observe from figure 4 that a fitting polynomial of degree 8 represents a small percent sum-of-error-squares of about 5% which is an excellent polynomial fitting performance level. Fitting to degrees less than 6 is not acceptable at all.

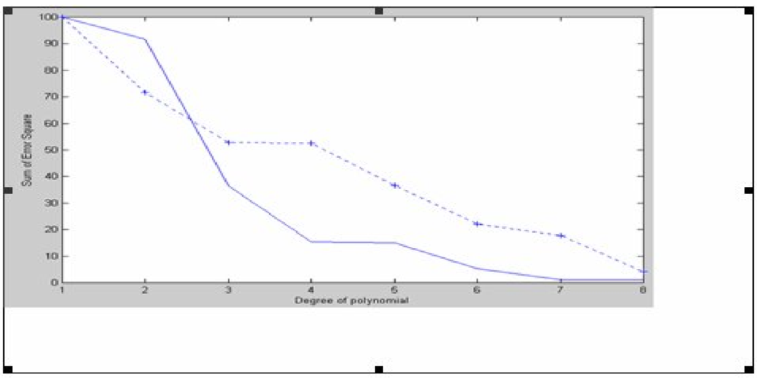

Figure 4. Effect of degree of polynomial fit to 3-point deconvolution output on the sum of error squares. The solid line shows fit to a typical Gamma Ray, and dotted line fit to the result of 3-point Deconvolution.

This result shows that a polynomial of degree higher than 8 does not only fit well the Gamma Ray pulse model, but fits extremely well the deconvolved events. It is also clear from figure 4 that the polynomial fitting method, itself based on the least-squares method, provides a better fitting method for cases where deconvolution is not used than with cases where deconvolution is used, since for polynomial orders shown higher than 3, the sum-of-error-squares, for cases without deconvolution (solid line) are less than their counterparts for cases with deconvolution (dashed line). However, as mentioned earlier, deconvolution is required if pileups occur and no information is to be lost.

4. Conclusion

A number of parameter estimation and digital online peak localisation algorithms have been developed and proposed in this paper, for the purpose of Gamma Ray pulse identification in the absence of noise and *pileup*. A 3-point and 4-point *deconvolution* techniques have been successfully tested and compared with each other and were shown to resolve pileups up to 93% of the cases studied here. The 3-point deconvolution yielded a maximum throughput of the pulse analyzer of almost 0.25 MCPS, therefore amounting to a four-fold enhancement when compared to the case of no deconvolution being used. The pulse shape of the Gamma ray was found to be very well modelled by a polynomial of degree 8 or higher, leading to a Chi-square-goodness-of-fit of approximately 1.2. This excellent pulse modeling performance in a noisy environment is due to our use of the powerful Saviztky-Golay digital filtering technique. Work is currently in progress to implement the parameter estimation algorithms in real time using a DSP processor.

Acknowledgments: Support by the EE department University of Bradford UK, the Computer Engineering Department and the Research Institute of King Fahd University of Petroleum and Minerals is gratefully acknowledged.

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