Top-Down Parsing

- A parser is top-down if it discovers a parse tree top to bottom
  - A top-down parse corresponds to a preorder traversal of the parse tree
  - A leftmost derivation is applied at each derivation step
- Top-down parsers come in two forms
  - **Predictive Parsers**
    - Predict the production rule to be applied using lookahead tokens
  - **Backtracking Parsers**
    - Will try different productions, backing up when a parse fails
- Predictive parsers are much faster than backtracking ones
  - Predictive parsers operate in linear time – will be our focus
  - Backtracking parsers operate in exponential time – will not be considered
- Two kinds of top-down parsing techniques will be studied
  - Recursive-descent parsing
  - LL parsing

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Top-Down Parsing by Recursive-Descent

- We view a nonterminal *A* as a definition of a procedure *A*
  - Procedure *A* will match the token sequence generated by nonterminal *A*
- The RHS of a production of *A* specifies the code for procedure *A*
  - Terminals are matched against input tokens
  - Nonterminals produce calls to corresponding procedures
- If multiple production rules exist for a nonterminal *A*
  - One of them is predicted based on a **lookahead token**
    - The lookahead token is the next input token that should be matched
  - The predicted rule is the only one that applies
    - Other rules will NOT be tried
    - This is a predictive parsing technique, not a backtracking one
- A syntax error is detected when
  - Next token in the input sequence does NOT match the expected token

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Example on Recursive-Descent Parsing

- Consider the following grammar for expressions in EBNF notation

  $expr \rightarrow term$ { **addop** $term$ }

  $term \rightarrow factor$ { **mulop** $factor$ }

  $factor \rightarrow$ '(' $expr$ ')' | **id** | **num**

- Since three nonterminals exist, we need three parsing procedures
  - The curly brackets expressing repetition is translated into a while loop
  - The vertical bar expressing alternation is translated into a case statement

```
procedure expr( )
begin
  term( );
  while token = ADDOP do
    match(ADDOP);
    term( );
  end while;
end expr;
```

```
procedure term( )
begin
   factor( );
   while token = MULOP do
      match(MULOP);
      factor( );
   end while;
end term;
```

```
procedure factor( )
begin
  case token of
   '(':     match('('); expr( ); match(')');
   ID:      match(ID);
   NUM:     match(NUM);
   else     syntax_error(token);
  end case;
end factor;
```

Lookahead Token and Match Procedure

- The recursive-descent procedures use a *token* variable
- The *token* variable is the **lookahead token**
  - Keeps track of the next token in the input sequence
  - Is initialized to the first token before parsing begins
  - Is updated after every call to the match procedure
- The *match* procedure matches lookahead *token* with its parameter
  - Is called to match an **expected token** on the RHS of a production
  - Match succeeds if expected token = lookahead token and fails otherwise
  - Match calls scanner function to update the lookahead token

```
procedure match ( ExpectedToken )
begin
    if token = ExpectedToken then token := scan( ) ;
    else syntax_error( token , ExpectedToken ) ;
    end if;
end match ;
```

Syntax Tree Construction for Expressions

- A recursive-descent parser can be used to construct a syntax tree

  SyntaxTree := *expr* ( ) ; Calling parser function for start symbol

- Parsing functions allocate and return pointers to syntax tree nodes
- Construction of a syntax tree for simple expressions is given below
  - **New** *node* allocates a tree node and returns a pointer to it

```
function expr( ) : TreePtr
begin
   left := term( );
   while token = ADDOP do
      op := ADDOP.op ; match(ADDOP);
      right := term( );
      left := new node(op, left, right);
   end while;
   return left;
end expr;
```

```
function term( ) : TreePtr
begin
   left := factor( );
   while token = MULOP do
      op := MULOP.op;  match(MULOP);
      right := factor( );
      left := new node(op, left, right);
   end while;
   return left;
end term;
```


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Syntax Tree Construction – cont'd

- For a *factor*, we have the following parsing function
  - *symtable.lookup*(ID.*name*) searches a symbol table for a given *name*
  - *lookup* function returns a pointer to an identifier symbol in *symtable*
  - Identifiers are inserted into symbol table when parsing a declaration
  - The NUM.*ptr* is a pointer to a literal symbol in the literal table
  - Literal constants are inserted into the literal table when scanned

```
function factor( ) : TreePtr
begin
   case token of
      '(':     match('('); ptr := expr( ); match(')');
      ID:      ptr := symtable.lookup(ID.name); match(ID);
      NUM:  ptr := NUM.ptr; match(NUM);
      else     syntax_error(token, "Expecting a number, an identifier, or (" );
   end case;
   return ptr;
end factor;
```


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Node Structure for Expression Trees

- A syntax tree node for expressions should have at least:
  - **Node operaror**: + , – , * , / , etc. Different for each operator
    - For symbol table entries, the node operator is **ID**
    - For literal table entries, the node operator is **NUM**
    - Other node operators can be added to statements and various types of literals
  - **Left Pointer**: pointer to left child expression tree
    - Can point to a tree node, to a symbol node, or to a literal node
  - **Right Pointer**: pointer to right child expression tree
    - Can point to a tree node, to a symbol node, or to a literal node
- The following fields are also important:
  - **Line, Pos**: keeps track of line and position of each tree node
  - **Type**: associates a type with each tree node
    - Type information is used to check the type of expressions

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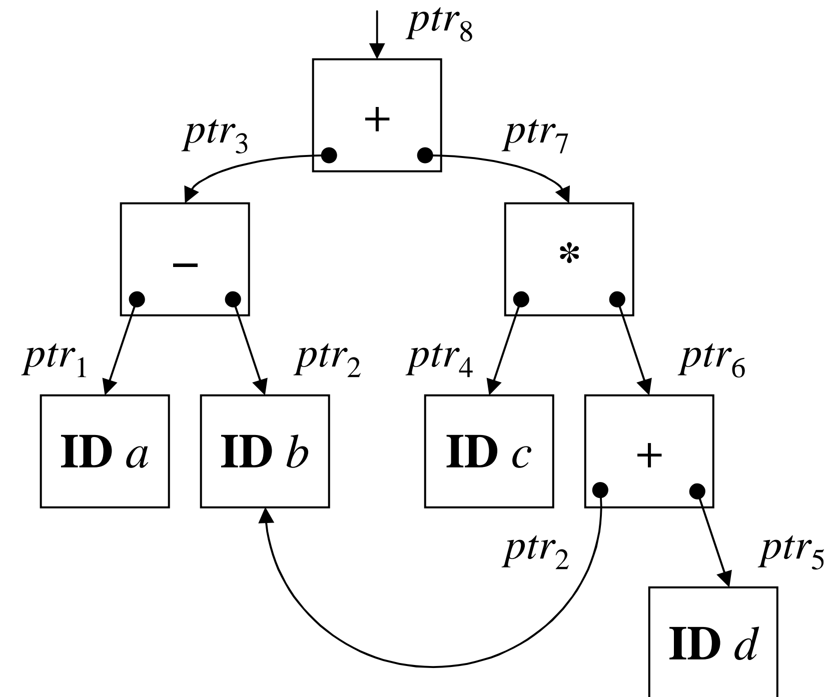
Tracing the Construction of a Syntax Tree

- Although recursive-descent is a top-down parsing technique …
  - The construction of the syntax tree for expressions is bottom up
  - Tracing verifies the precedence and associativity of operators
- The tree construction of $a - b + c * (b + d)$ is given below
  - $ptr_1 \leftarrow$ *symtable.lookup*(a)
  - $ptr_2 \leftarrow$ *symtable.lookup*(b)
  - $ptr_3 \leftarrow$ **new** *node*('–' , ptr_1 , ptr_2)
  - $ptr_4 \leftarrow$ *symtable.lookup*(c)
  - $ptr_2 \leftarrow$ *symtable.lookup*(b)
  - $ptr_5 \leftarrow$ *symtable.lookup*(d)
  - $ptr_6 \leftarrow$ **new** *node*('+' , ptr_2 , ptr_5)
  - $ptr_7 \leftarrow$ **new** *node*('*' , ptr_4 , ptr_6)
  - $ptr_8 \leftarrow$ **new** *node*('+' , ptr_3 , ptr_7)



Syntax Tree Construction for **if** Statements

- An Extended BNF grammar for **if** statements with optional **else**:

  if-stmt → **if** *expr* **then** *stmt* [**else** *stmt*]

- A parsing function can eliminate the ambiguity of **else**
  - By matching an **else** token as soon as encountered
- Syntax tree of **if** stmt is constructed bottom-up

```
function ifstmt( ) : TreePtr
begin
   match(IF);  exprptr := expr( ); match(THEN);  thenptr := stmt( );
   if token = ELSE then
      match(ELSE); elseptr := stmt( );
      elseptr := new node(ELSE, thenptr, elseptr);       // ELSE node
      ifptr := new node(IF, exprptr, elseptr);           // IF node points to ELSE node
   else  ifptr := new node(IF, exprptr, thenptr); end if; // No ELSE node
   return ifptr;
end ifstmt;
```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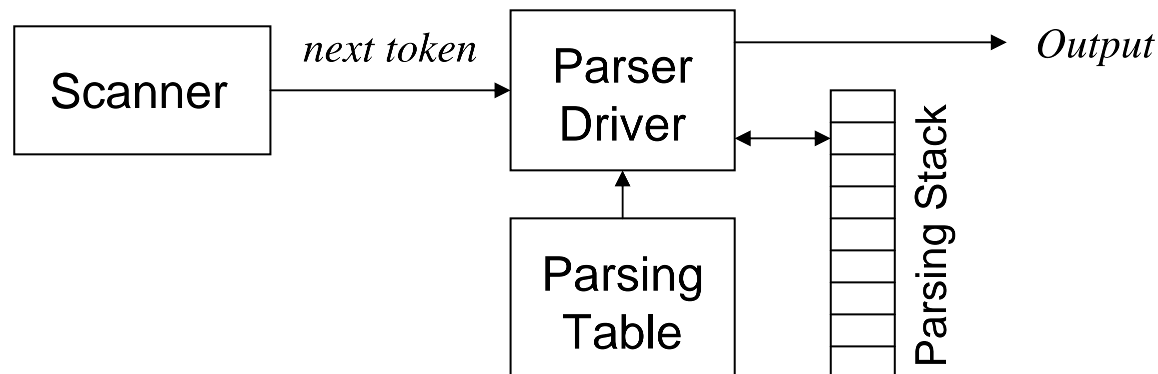
LL Parsing

- Uses an explicit stack rather than recursive calls to perform a parse
- LL(k) parsing means that k tokens of lookahead are used
  - The first L means that token sequence is read from left to right
  - The second L means a leftmost derivation is applied at each step
- An LL parser consists of
  - **Parser stack** that holds grammar symbols: non-terminals and tokens
  - **Parsing table** that specifies the parser action
  - **Driver function** that interacts with parser stack, parsing table and scanner



LL Parsing Actions

- The LL parsing actions are:
  - **Match**: to match top of parser stack with next input token
  - **Predict**: to predict a production and apply it in a derivation step
  - **Accept**: to accept and terminate the parsing of a sequence of tokens
  - **Error**: to report an error message when matching or prediction fails
- Consider the following grammar: $S \rightarrow (S) S \mid \varepsilon$

Parsing of (( ))

Stack grows backward from right to left

Parser Stack	Input	Parser Action
S	(( )) $	Predict $S \rightarrow (S) S$
$(S) S$	(( )) $	Match (
$S) S$	( )) $	Predict $S \rightarrow (S) S$
$(S) S) S$	( )) $	Match (
$S) S) S$	)) $	Predict $S \rightarrow \varepsilon$
$) S) S$	)) $	Match)
$S) S$	) $	Predict $S \rightarrow \varepsilon$
$) S$	) $	Match)
S	$	Predict $S \rightarrow \varepsilon$
Empty	$	Accept

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Grammar Analysis: Nonterminals that Derive ε

- Grammar analysis is necessary to
  - Determine whether a grammar can be used in LL parsing
  - Construct the LL parsing table that defines the actions of an LL parser
- A common analysis is to determine which nonterminals derive ε
  - Nonterminals that derive ε are called **nullable**
- To determine which nonterminals derive ε …
  - We use an iterative marking algorithm
  - First, nonterminals that derive ε directly in one step are marked
  - Nonterminals that derive ε in two, three, … steps are found and marked
  - Continue until no more nonterminals can be marked as deriving ε
- Consider the following grammar

$A \rightarrow B\ C\ D$

$B \rightarrow \mathbf{b}\ C \mid \varepsilon$

$C \rightarrow \mathbf{c}\ D \mid \varepsilon$

$D \rightarrow \mathbf{d} \mid \varepsilon$

A, B, C, and D are all **nullable**

B, C, and D derive ε directly

A derives ε indirectly: $A \Rightarrow B\ C\ D \Rightarrow C\ D \Rightarrow D \Rightarrow \varepsilon$

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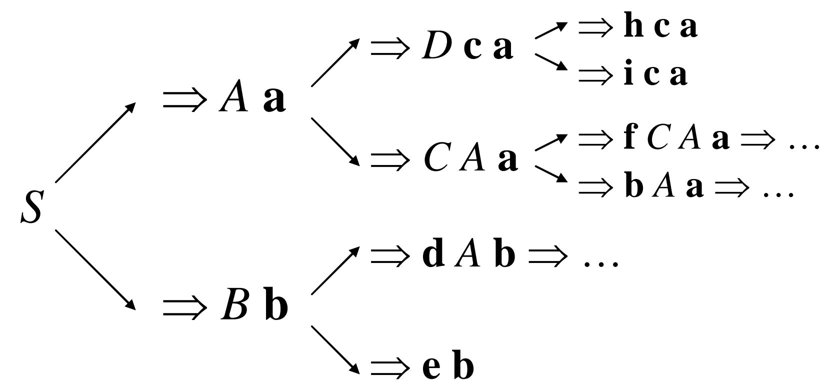
Grammar Analysis: The First Set

- Suppose we have the following grammar:
  - The RHS of the productions of S do not begin with terminals
  - Parser has no immediate guidance which production to apply to expand S
  - We may follow all possible derivations of S as shown below

$$
\begin{aligned}
S &\rightarrow A\,\mathbf{a} \mid B\,\mathbf{b} \\
A &\rightarrow D\,\mathbf{c} \mid C\,A \\
B &\rightarrow \mathbf{d}\,A \mid \mathbf{e} \\
C &\rightarrow \mathbf{f}\,C \mid \mathbf{b} \\
D &\rightarrow \mathbf{h} \mid \mathbf{i}
\end{aligned}
$$



- We predict $S \rightarrow A\,\mathbf{a}$ when
  - First token is **h**, **i**, **f**, or **b**. First(A**a**) = {**h**, **i**, **f**, **b**}
- We predict $S \rightarrow B\,\mathbf{b}$ when
  - First token is **d** or **e**. First(B**b**) = {**d**, **e**}
- Otherwise, we have an error

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Grammar Analysis: Determining the First Set

- Formally, $\text{First}(\alpha) = \{\mathbf{a} \in T \mid \alpha \Rightarrow^* a\beta\}$
  - $\text{First}(\alpha)$ is the set of all terminals that can begin a sentential form of α
- If $\alpha \Rightarrow^* \varepsilon$ then $\varepsilon \in \text{First}(\alpha)$
- To calculate $\text{First}(\alpha)$ we apply the following rules
  - $\text{First}(\varepsilon) = \{\varepsilon\}$ — $\alpha = \varepsilon$
  - $\text{First}(\mathbf{a}\beta) = \{\mathbf{a}\}$ — $\alpha = \mathbf{a}\beta$
  - $\text{First}(A) = \text{First}(\beta_1) \cup \text{First}(\beta_2) \cup \ldots$ — $\alpha = A$ and $A \rightarrow \beta_1 \mid \beta_2 \mid \ldots$
  - $\text{First}(A\beta) = \text{First}(A)$ — $\alpha = A\beta$ and A is NOT nullable
  - $\text{First}(A\beta) = (\text{First}(A) - \{\varepsilon\}) \cup \text{First}(\beta)$ — $\alpha = A\beta$ and $A \Rightarrow^+ \varepsilon$
- Consider the following grammar:

$S \rightarrow A\ B\ C\ \mathbf{d}$ $\text{First}(A) = \{\mathbf{e}, \mathbf{f}, \varepsilon\}$

$A \rightarrow \mathbf{e} \mid \mathbf{f} \mid \varepsilon$ $\text{First}(B) = \{\mathbf{g}, \mathbf{h}, \varepsilon\}$

$B \rightarrow \mathbf{g} \mid \mathbf{h} \mid \varepsilon$ $\text{First}(C) = \{\mathbf{p}, \mathbf{q}\}$

$C \rightarrow \mathbf{p} \mid \mathbf{q}$ $\text{First}(S) = \text{First}(ABC\mathbf{d}) = (\text{First}(A) - \{\varepsilon\}) \cup (\text{First}(B) - \{\varepsilon\}) \cup \text{First}(C\mathbf{d})$

$= \{\mathbf{e}, \mathbf{f}\} \cup \{\mathbf{g}, \mathbf{h}\} \cup \{\mathbf{p}, \mathbf{q}\} = \{\mathbf{e}, \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{p}, \mathbf{q}\}$

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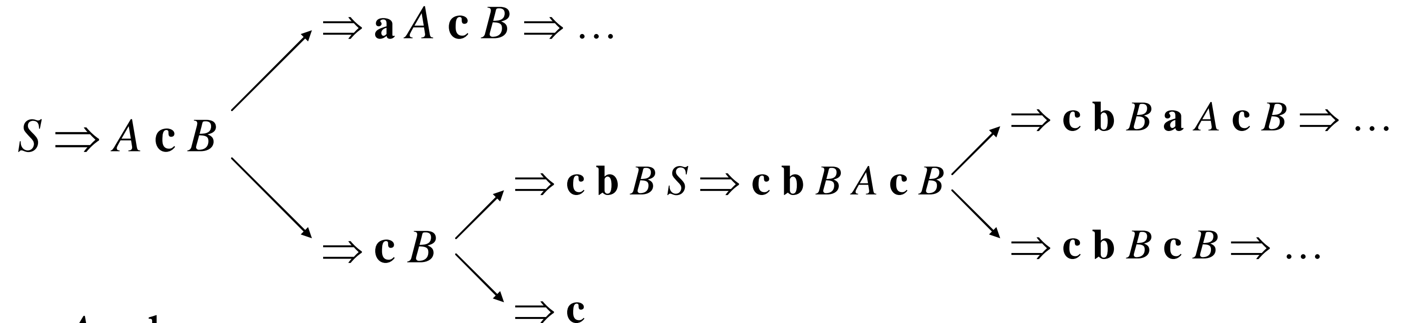
Grammar Analysis: The Follow Set

- Suppose we have the following grammar
  - We follow derivations of S as shown below …

$S \rightarrow A\ \mathbf{c}\ B$
$A \rightarrow \mathbf{a}\ A$
$A \rightarrow \varepsilon$
$B \rightarrow \mathbf{b}\ B\ S$
$B \rightarrow \varepsilon$



- We predict $A \rightarrow \mathbf{a}\ A$ when
  - Next token is **a** because First(**a**A) = {**a**}
- We predict $A \rightarrow \varepsilon$ when
  - Next token is **c** because Follow(A) = {**c**}
- Similarly, we predict $B \rightarrow \mathbf{b}\ B\ S$ when
  - Next token is **b** because First(**b** B S) = {**b**}
- We predict $B \rightarrow \varepsilon$ when
  - Next token is **a**, **c**, or **$** (**end-of-file token**) because Follow(B) = {**a**, **c**, **$**}

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Grammar Analysis: Determining the Follow Set

- Formally, Follow(A) = $\{\mathbf{a} \in T \mid S \Rightarrow^+ \alpha\, A\, \mathbf{a}\, \beta\}$
  - Follow(A) is the set of terminals that may follow A in any sentential form
- If $S \Rightarrow^+ \alpha\, A$ then **$** $\in$ Follow(A)
  - If A is not followed by any terminal then it is followed by the end of file
  - The **$** represents the end-of-file token
- We compute Follow(A) using the following rules:
  - If A is the start symbol then **$** $\in$ Follow(A)
  - Inspect RHS of productions for all occurrences of A

    Let a typical production be $B \to \alpha\, A\, \beta$
    - If β is NOT nullable then add First(β) to Follow(A)
      - Any token that can begin a sentential form of β can follow A
    - If β is ε or derives ε then add (First(β) – $\{\varepsilon\}$) $\cup$ Follow(B) to Follow(A)
      - If β vanishes in a given derivation then what follows A is what follows B
      - $\ldots \Rightarrow \delta\, B\, \gamma \Rightarrow \delta\, \alpha\, A\, \beta\, \gamma \Rightarrow \delta\, \alpha\, A\, \gamma \Rightarrow \ldots$ what follows A is what follows B is First(γ)

Examples on the First and Follow Sets

Example 1:

$S \rightarrow A\ \mathbf{c}\ B$
$A \rightarrow \mathbf{a}\ A$
$A \rightarrow \varepsilon$
$B \rightarrow \mathbf{b}\ B\ S$
$B \rightarrow \varepsilon$

Nonterminals that derive ε are A and B

First(S) = First($A\mathbf{c}B$) = (First(A)–{ε}) ∪ First($\mathbf{c}B$) = {**a**, **c**}
First(A) = First($\mathbf{a}A$) ∪ First(ε) = {**a**, ε}
First(B) = First($\mathbf{b}BS$) ∪ First(ε) = {**b**, ε}

Follow(S) = {**$**} ∪ Follow($B$)
Follow(A) = {**c**}
Follow(B) = Follow(S) ∪ First(S) = {**$**, **a**, **c**}
Follow(S) = {**$**, **a**, **c**}

Example 2:

$E \rightarrow T\ Q$
$Q \rightarrow +\ T\ Q \mid -\ T\ Q \mid \varepsilon$
$T \rightarrow F\ R$
$R \rightarrow *\ F\ R \mid /\ F\ R \mid \varepsilon$
$F \rightarrow (\ E\ ) \mid \mathbf{id}$

Nonterminals that derive ε are Q and R

First(E) = First(TQ) = First(T) = First(FR) = First(F) = {(, **id**}
First(Q) = {+ , – , ε}
First(R) = {* , / , ε}

Follow(E) = {**$** ,)}
Follow(Q) = Follow(E) = {**$** ,)}
Follow(T) = (First(Q)–{ε}) ∪ Follow(E) ∪ Follow(Q) = {+, –, **$**,)}
Follow(R) = Follow(T) = {+, –, **$**,)}
Follow(F) = (First(R)–{ε}) ∪ Follow(T) ∪ Follow(R) = {*, /, +, –, **$**,)}

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Grammar Analysis: Determining the Predict Set

- The predict set of a production $A \rightarrow \alpha$ is defined as follows:
  - If α is **NOT nullable** then Predict($A \rightarrow \alpha$) = First(α)
  - If α is **Nullable** then Predict($A \rightarrow \alpha$) = (First(α) – {ε}) $\cup$ Follow(A)
  - This is the set of lookahead tokens that will cause the selection of $A \rightarrow \alpha$
- Example on determining the predict set:

Production	Predict Set
$E \rightarrow T\ Q$	Predict $E \rightarrow T\ Q$ = First(TQ) = First(T) = {(, **id**}
$Q \rightarrow +\ T\ Q$	Predict $Q \rightarrow +\ T\ Q$ = First($+TQ$) = { + }
$Q \rightarrow -\ T\ Q$	Predict $Q \rightarrow -\ T\ Q$ = First($-TQ$) = { – }
$Q \rightarrow \varepsilon$	Predict $Q \rightarrow \varepsilon$ = Follow(Q) = {**$** ,)}
$T \rightarrow F\ R$	Predict $T \rightarrow F\ R$ = First(FR) = First(F) = {(, **id**}
$R \rightarrow *\ F\ R$	Predict $R \rightarrow *\ F\ R$ = First($*FR$) = { * }
$R \rightarrow /\ F\ R$	Predict $R \rightarrow /\ F\ R$ = First($/FR$) = { / }
$R \rightarrow \varepsilon$	Predict $R \rightarrow \varepsilon$ = Follow(R) = {+ , – , **$** ,)}
$F \rightarrow (\ E\ )$	Predict $F \rightarrow (\ E\ )$ = { (}
$F \rightarrow$ **id**	Predict $F \rightarrow$ **id** = { **id** }

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LL(1) Grammars

- Not all context-free grammars are suitable for LL parsing
- CFGs suitable for LL(1) parsing are called **LL(1) Grammars**
- A grammar is LL(1) if for productions with the same LHS A

  $A \to \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$

  $\text{Predict}(A \to \alpha_i) \cap \text{Predict}(A \to \alpha_j) = \varnothing$ for all $i \neq j$

  The predict sets of productions with same LHS are pairwise disjoint

- The following grammar is LL(1)

Production	Predict set
$S \to A\ \mathbf{c}\ B$	
$A \to \mathbf{a}\ A$	$\text{Predict}(A \to \mathbf{a}\ A) = \{\mathbf{a}\}$
$A \to \varepsilon$	$\text{Predict}(A \to \varepsilon) = \text{Follow}(A) = \{\mathbf{c}\}$
$B \to \mathbf{b}\ B\ S$	$\text{Predict}(B \to \mathbf{b}\ B\ S) = \{\mathbf{b}\}$
$B \to \varepsilon$	$\text{Predict}(B \to \varepsilon) = \text{Follow}(B) = \text{Follow}(S) \cup \text{First}(S) = \{\$\} \cup \text{Follow}(B) \cup \{\mathbf{a}, \mathbf{c}\} = \{\$, \mathbf{a}, \mathbf{c}\}$

Disjoint (Predict($A \to \mathbf{a}\ A$) and Predict($A \to \varepsilon$))

Disjoint (Predict($B \to \mathbf{b}\ B\ S$) and Predict($B \to \varepsilon$))

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Constructing the LL(1) Parsing Table

- The predict sets can be represented in an LL(1) parse table
  - The rows are indexed by the nonterminals
  - The columns are indexed by the tokens
- If A is a nonterminal and **tok** is the lookahead token then
  - ***Table***[A][**tok**] indicates which production to predict
  - If no production can be used ***Table***[A][**tok**] gives an error value
- ***Table***[A][**tok**] $= A \rightarrow \alpha$ iff **tok** $\in$ predict($A \rightarrow \alpha$)
- Example on constructing the LL(1) parsing table:

Production	Predict set
1: $S \rightarrow A\ \mathbf{c}\ B$	Predict(1) = {**a**, **c**}
2: $A \rightarrow \mathbf{a}\ A$	Predict(2) = {**a**}
3: $A \rightarrow \varepsilon$	Predict(3) = {**c**}
4: $B \rightarrow \mathbf{b}\ B\ S$	Predict(4) = {**b**}
5: $B \rightarrow \varepsilon$	Predict(5) = {**$**, **a**, **c**}

	a	**b**	**c**	**$**
S	1		1	
A	2		3	
B	5	4	5	5

Empty slots indicate error conditions

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Constructing the LL(1) Parsing Table – cont'd

❖ Here is a second example on constructing the parsing table

	Production	Predict set
1:	$E \rightarrow T\ Q$	Predict(1) = { (, **id** }
2:	$Q \rightarrow +\ T\ Q$	Predict(2) = { + }
3:	$Q \rightarrow -\ T\ Q$	Predict(3) = { – }
4:	$Q \rightarrow \varepsilon$	Predict(4) = { $,) }
5:	$T \rightarrow F\ R$	Predict(5) = { (, **id** }
6:	$R \rightarrow *\ F\ R$	Predict(6) = { * }
7:	$R \rightarrow /\ F\ R$	Predict(7) = { / }
8:	$R \rightarrow \varepsilon$	Predict(8) = {+, –, $,)}
9:	$F \rightarrow (\ E\ )$	Predict(9) = { (}
10:	$F \rightarrow$ **id**	Predict(10) = { **id** }

	+	–	*	/	(	)	**id**	$
E					1		1	
Q	2	3				4		4
T					5		5	
R	8	8	6	7		8		8
F					9		10	

❖ Because the above grammar is LL(1)

- A unique production number is stored in a table entry

❖ Blank entries correspond to error conditions

- In practice, special error numbers are used to indicate error situations

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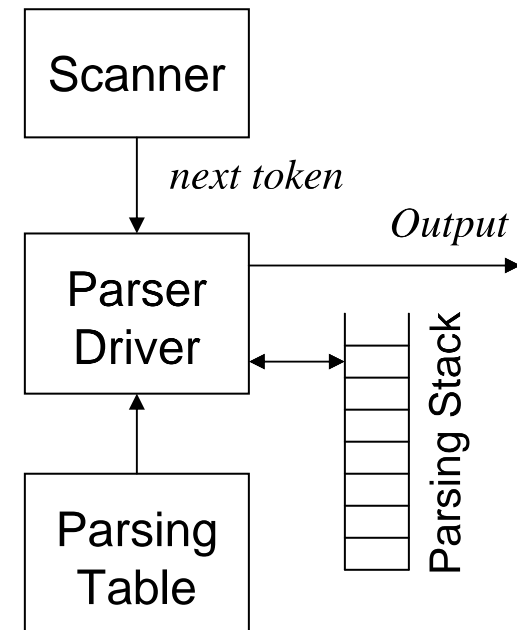
LL(1) Parser Driver Algorithm

- The LL(1) parser driver algorithm can be described as follows:

Token := *scan*( )
Stack.push(*StartSymbol*)
while not *Stack.empty*( ) **do**
  X := *Stack.pop*( )
  if *terminal*(*X*)
    if *X* = *Token* **then** *Token* := *scan*( )
    else process a syntax error at *Token* **end if**
  else (* X is a nonterminal *)
    Rule := *Table*[*X*][*Token*]
    if *Rule* = $X \rightarrow Y_1\ Y_2\ \ldots\ Y_n$ **then**
      for *i* **from** *n* **downto** 1 **do** *Stack.push*(Y_i) **end for**
    else process a syntax error at *Token* **end if**
  end if
end while
if *Token* = $ **then** accept parsing
else report a syntax error at *Token* **end if**



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Tracing an LL(1) Parser

Consider the parsing of **id * (id + id) $**

1: $E \rightarrow T\ Q$
2: $Q \rightarrow +\ T\ Q$
3: $Q \rightarrow -\ T\ Q$
4: $Q \rightarrow \varepsilon$
5: $T \rightarrow F\ R$
6: $R \rightarrow *\ F\ R$
7: $R \rightarrow /\ F\ R$
8: $R \rightarrow \varepsilon$
9: $F \rightarrow (\ E\ )$
10: $F \rightarrow$ **id**

	+	–	*	/	(	)	id	$
E					1		1	
Q	2	3				4		4
T					5		5	
R	8	8	6	7		8		8
F					9		10	

Stack grows backwards from right to left

Parser Stack	Input	Parser Action
E	id*(id+id)$	Predict $E \rightarrow T\ Q$
$T\ Q$	id*(id+id)$	Predict $T \rightarrow F\ R$
$F\ R\ Q$	id*(id+id)$	Predict $F \rightarrow$ **id**
id $R\ Q$	id*(id+id)$	Match **id**
$R\ Q$	*(id+id)$	Predict $R \rightarrow *\ F\ R$
$*\ F\ R\ Q$	*(id+id)$	Match *
$F\ R\ Q$	(id+id)$	Predict $F \rightarrow (\ E\ )$
$(\ E\ )\ R\ Q$	(id+id)$	Match (
$E\ )\ R\ Q$	id+id)$	Predict $E \rightarrow T\ Q$
$T\ Q\ )\ R\ Q$	id+id)$	Predict $T \rightarrow F\ R$
$F\ R\ Q\ )\ R\ Q$	id+id)$	Predict $F \rightarrow$ **id**
id $R\ Q\ )\ R\ Q$	id+id)$	Match **id**
$R\ Q\ )\ R\ Q$	+id)$	Predict $R \rightarrow \varepsilon$
$Q\ )\ R\ Q$	+id)$	Predict $Q \rightarrow +\ T\ Q$
$+\ T\ Q\ )\ R\ Q$	+id)$	Match +
$T\ Q\ )\ R\ Q$	id)$	Predict $T \rightarrow F\ R$
$F\ R\ Q\ )\ R\ Q$	id)$	Predict $F \rightarrow$ **id**
id $R\ Q\ )\ R\ Q$	id)$	Match **id**
$R\ Q\ )\ R\ Q$	)$	Predict $R \rightarrow \varepsilon$
$Q\ )\ R\ Q$	)$	Predict $Q \rightarrow \varepsilon$
$)\ R\ Q$	)$	Match)
$R\ Q$	$	Predict $R \rightarrow \varepsilon$
Q	$	Predict $Q \rightarrow \varepsilon$
Empty	$	Accept

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The Problem of Left Recursion

- Left recursive grammars fail to be LL(1) or even LL(*k*)
  - A left recursive production puts an LL parser into infinite loop
  - If a left recursive production is predicted then
    - Nonterminal on LHS is replaced with RHS of production
    - The same nonterminal will appear again on top of parser stack
    - The same production is predicted again
    - Iteration goes forever
- Left recursion is commonly used to
  - Make an operation left associative
    - *Expr* → *Expr* **addop** *Term* | *Term*
  - Specify a list of identifiers, statements, etc.
    - *StmtList* → *StmtList* **;** *Statement* | *Statement*
- We need to eliminate left recursion to make a grammar LL(1)

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Eliminating Immediate Left Recursion

- The simplest case of left recursion is **immediate left recursion**
  - General Form: $A \rightarrow A\ \alpha \mid \beta$
  - The above productions of A generate strings of the form $\beta\alpha^n$, $n \geq 0$
  - We introduce a new nonterminal and use right recursion as follows:

$$A \rightarrow \beta\ Atail$$
$$Atail \rightarrow \alpha\ Atail \mid \varepsilon$$

- In general, if many immediate left recursive productions exist
  - General Form: $A \rightarrow A\ \alpha_1 \mid A\ \alpha_2 \mid \ldots \mid A\ \alpha_n \mid \beta_1 \mid \beta_2 \mid \ldots \mid \beta_m$
  - We introduce a new nonterminal and use right recursion

$$A \rightarrow \beta_1\ Atail \mid \beta_2\ Atail \mid \ldots \mid \beta_m\ Atail$$
$$Atail \rightarrow \alpha_1\ Atail \mid \alpha_2\ Atail \mid \ldots \mid \alpha_n\ Atail \mid \varepsilon$$

- For example: $Expr \rightarrow Expr + Term \mid Expr - Term \mid Term$ becomes:

$$Expr \rightarrow Term\ \ Exprtail$$
$$Exprtail \rightarrow +\ Term\ \ Exprtail \mid -\ Term\ \ Exprtail \mid \varepsilon$$

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Eliminating Indirect Left Recursion

- In some cases, **left recursion may be indirect**

  For example: $A \to B\ \beta \mid \ldots$ and $B \to A\ \alpha \mid \ldots$

- We can do substitutions to make left recursion immediate

- Consider the following grammar:

  $A \to B\ \mathbf{a} \mid A\ \mathbf{a} \mid \mathbf{c}$

  $B \to B\ \mathbf{b} \mid A\ \mathbf{b} \mid \mathbf{d}$

- First, we remove the immediate left recursion of A

  $A \to B\ \mathbf{a}\ Atail \mid \mathbf{c}\ Atail$ $Atail \to \mathbf{a}\ Atail \mid \varepsilon$

- Second, we eliminate the indirect left recursion of $B \to A\ \mathbf{b}$

  $B \to B\ \mathbf{b} \mid B\ \mathbf{a}\ Atail\ \mathbf{b} \mid \mathbf{c}\ Atail\ \mathbf{b} \mid \mathbf{d}$

- Finally, we remove the immediate left recursion of B

  $B \to \mathbf{c}\ Atail\ \mathbf{b}\ Btail \mid \mathbf{d}\ Btail$

  $Btail \to \mathbf{b}\ Btail \mid \mathbf{a}\ Atail\ \mathbf{b}\ Btail \mid \varepsilon$

Left Factoring of Common Prefixes

- Another problem to LL parsers is to have a **common prefix**
- An **if** statement may have 2 production with a common prefix:

  IfStmt → **if** *Expr* **then** *StmtList* **end if ;**
  IfStmt → **if** *Expr* **then** *StmtList* **else** *StmtList* **end if ;**

- An LL(1) parser cannot predict which production to apply
- The solution is use **left factoring** of the **common prefix**
  - General Form: $A \to \alpha\,\beta \mid \alpha\,\gamma \mid \ldots \mid \alpha\,\zeta$
  - Left Factoring solution:

    $A \to \alpha\ Atail$
    $Atail \to \beta \mid \gamma \mid \ldots \mid \zeta$

- The left factoring of the two **if** statement productions:

  IfStmt → **if** *Expr* **then** *StmtList* *IfStmtTail*
  IfStmtTail → **else** *StmtList* **end if ;** | **end if ;**

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Syntax Tree Construction in LL(1) Parsers

- To construct a syntax tree, we will need an additional stack
  - Called the **value stack** or the **attribute stack**
- Attribute values are pushed on the value stack while parsing
- Attribute values represent anything we choose
- For a syntax tree, we need the following attribute values
  - Pointers to syntax tree nodes
  - Pointers to symbols in the symbol and literal tables
  - Operator attributes as returned by the scanner
- **Semantic actions** are used to compute the attribute values
  - Semantic action **parameters** are popped from the value stack
  - Semantic action **results** are pushed on the value stack

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Action Symbols

- **Action symbols** are added to RHS of grammar productions
  - Begin with # to mark where and what semantic actions are required
- Example:

$E \rightarrow T\ Q$

$Q \rightarrow$ #P $+\ T$ #N Q | #P $-\ T$ #N Q | ε

$T \rightarrow F\ R$

$R \rightarrow$ #P $*\ F$ #N R | #P $/\ F$ #N R | ε

$F \rightarrow (\ E\ )$ | #L **id**

- Actions Symbols:

#P Push operator on value stack

#L Lookup *symtable* for **id.***name*; push symbol pointer on stack

#N Pop *right* pointer, *operator*, and *left* pointer from value stack
New node(*op*, *left*, *right*) and push node pointer on value stack

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Tracing Syntax Tree Construction in LL(1)

Parsing Stack	Input	Parser Action	Value Stack
E	a / $(b - c)\$$	Predict $E \rightarrow T\ Q$	Empty
$T\ Q$	a / $(b - c)\$$	Predict $T \rightarrow F\ R$	Empty
$F\ R\ Q$	a / $(b - c)\$$	Predict $F \rightarrow$ #L **id**	Empty
#L **id** $R\ Q$	a / $(b - c)\$$	Lookup a	**S**(a)
id $R\ Q$	a / $(b - c)\$$	Match **id**	**S**(a)
$R\ Q$	/ $(b - c)\$$	Predict $R \rightarrow$ #P / $F\ R$ #N	**S**(a)
#P / $F\ R$ #N Q	/ $(b - c)\$$	Push /	/ , **S**(a)
/ $F\ R$ #N Q	/ $(b - c)\$$	Match /	/ , **S**(a)
$F\ R$ #N Q	$(b - c)\$$	Predict $F \rightarrow (E)$	/ , **S**(a)
(E) R #N Q	$(b - c)\$$	Match (	/ , **S**(a)
E) R #N Q	$b - c)\$$	Predict $E \rightarrow T\ Q$	/ , **S**(a)
$T\ Q$) R #N Q	$b - c)\$$	Predict $T \rightarrow F\ R$	/ , **S**(a)
$F\ R\ Q$) R #N Q	$b - c)\$$	Predict $F \rightarrow$ #L **id**	/ , **S**(a)
#L **id** $R\ Q$) R #N Q	$b - c)\$$	Lookup b	**S**(b), / , **S**(a)
id $R\ Q$) R #N Q	$b - c)\$$	Match **id**	**S**(b), / , **S**(a)

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Tracing Syntax Tree Construction in LL(1)

Parsing Stack	Input	Parser Action	Value Stack
R Q) R #N Q	– c)$	Predict $R \to \varepsilon$	**S**(b) , / , **S**(a)
Q) R #N Q	– c)$	Predict $Q \to$ #P – T Q #N	**S**(b) , / , **S**(a)
#P – T Q #N) R Q	– c)$	Push –	– , **S**(b) , / , **S**(a)
– T Q #N) R Q	– c)$	Match –	– , **S**(b) , / , **S**(a)
T Q #N) R Q	c)$	Predict $T \to F\ R$	– , **S**(b) , / , **S**(a)
F R Q #N) R Q	c)$	Predict $F \to$ #L **id**	– , **S**(b) , / , **S**(a)
#L **id** R Q #N) R Q	c)$	Lookup c	**S**(c) , – , **S**(b) , / , **S**(a)
id R Q #N) R Q	c)$	Match **id**	**S**(c) , – , **S**(b) , / , **S**(a)
R Q #N) R #N Q	)$	Predict $R \to \varepsilon$	**S**(c) , – , **S**(b) , / , **S**(a)
Q #N) R #N Q	)$	Predict $Q \to \varepsilon$	**S**(c) , – , **S**(b) , / , **S**(a)
#N) R #N Q	)$	New Node	**N**(–, **S**(b), **S**(c)) , / , **S**(a)
) R #N Q	)$	Match)	**N**(–, **S**(b), **S**(c)) , / , **S**(a)
R #N Q	$	Predict $R \to \varepsilon$	**N**(–, **S**(b), **S**(c)) , / , **S**(a)
#N Q	$	New Node	**N**(/, S(a), **N**(–, **S**(b), **S**(c)))
Q	$	Predict $Q \to \varepsilon$	**N**(/, S(a), **N**(–, **S**(b), **S**(c)))
Empty	$	Accept	**N**(/, S(a), **N**(–, **S**(b), **S**(c)))

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