Bottom-Up Parsing

- Attempts to traverse a parse tree bottom up (**post-order traversal**)
- Reduces a sequence of tokens to the start symbol
- At each reduction step, the RHS of a production is replaced with LHS
- A reduction step corresponds to the **reverse of a rightmost derivation**
- Example: given the following grammar

$$
\begin{array}{lcl}
E & \rightarrow & E + T \mid T \\
T & \rightarrow & T * F \mid F \\
F & \rightarrow & (E) \mid \mathbf{id}
\end{array}
$$

A rightmost derivation for **id + id * id** is shown below:

$$
\begin{aligned}
E &\Rightarrow_{rm} E + T \Rightarrow_{rm} E + T * F \Rightarrow_{rm} E + T * \mathbf{id} \\
&\Rightarrow_{rm} E + F * \mathbf{id} \Rightarrow_{rm} E + \mathbf{id} * \mathbf{id} \Rightarrow_{rm} T + \mathbf{id} * \mathbf{id} \\
&\Rightarrow_{rm} F + \mathbf{id} * \mathbf{id} \Rightarrow_{rm} \mathbf{id} + \mathbf{id} * \mathbf{id}
\end{aligned}
$$

Handles

- If $S \Rightarrow^{+}_{rm} \alpha$ then α is called a **right sentential form**
- A **handle** of a right sentential form is:
  - A substring β that matches the RHS of a production $A \rightarrow \beta$
  - The reduction of β to A is a step along the reverse of a rightmost derivation
- If $S \Rightarrow^{+}_{rm} \gamma Aw \Rightarrow_{rm} \gamma\beta w$, where w is a sequence of tokens then
  - The substring β of $\gamma\beta w$ and the production $A \rightarrow \beta$ make the handle
- Consider the reduction of **id + id * id** to the start symbol E

Sentential Form	*Production*	*Sentential Form*	*Production*
id + id * id	$F \rightarrow$ **id**	$E + T *$ **id**	$F \rightarrow$ **id**
F **+ id * id**	$T \rightarrow F$	$E + T * F$	$T \rightarrow T * F$
T **+ id * id**	$E \rightarrow T$	$E + T$	$E \rightarrow E + T$
$E +$ **id * id**	$F \rightarrow$ **id**	E	
$E + F$ *** id**	$T \rightarrow F$		

Stack Implementation of a Bottom-Up Parser

- A bottom-up parser uses an explicit stack in its implementation
- The main actions are **shift** and **reduce**
  - A bottom-up parser is also known as as **shift-reduce** parser
- Four operations are defined: **shift**, **reduce**, **accept**, and **error**
  - **Shift**: parser shifts the next token on the parser stack
  - **Reduce**: parser reduces the RHS of a production to its LHS
    - The handle always appears on top of the stack
  - **Accept**: parser announces a successful completion of parsing
  - **Error**: parser discovers that a syntax error has occurred
- The parser operates by:
  - Shifting tokens onto the stack
  - When a handle β is on top of stack, parser reduces β to LHS of production
  - Parsing continues until an error is detected or input is reduced to start symbol

Example on Bottom-Up Parsing

- Consider the parsing of the input string **id + id * id**

Stack	Input	Action
$	**id + id * id $**	**shift**
$**id**	**+ id * id $**	**reduce** $F \rightarrow$ **id**
$*F*	**+ id * id $**	**reduce** $T \rightarrow F$
$*T*	**+ id * id $**	**reduce** $E \rightarrow T$
$*E*	**+ id * id $**	**shift**
$*E* +	**id * id $**	**shift**
$*E* + **id**	*** id $**	**reduce** $F \rightarrow$ **id**
$*E* + *F*	*** id $**	**reduce** $T \rightarrow F$
$*E* + *T*	*** id $**	**shift**
$*E* + T *	**id $**	**shift**
$*E* + T * **id**	**$**	**reduce** $F \rightarrow$ **id**
$*E* + *T* * *F*	**$**	**reduce** $T \rightarrow T * F$
$*E* + *T*	**$**	**reduce** $E \rightarrow E + T$
$*E*	**$**	**accept**

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (\ E\ ) \mid \mathbf{id}$$

We use $ to mark the bottom of the stack as well as the end of input

LR Parsing

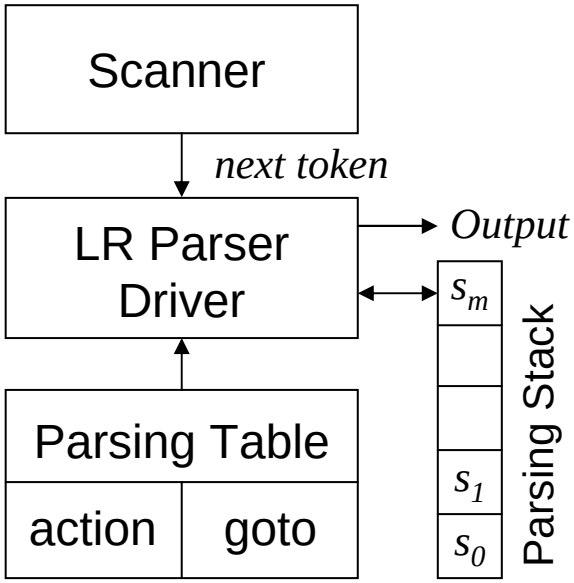
- To have an operational shift-reduce parser, we must determine:
  - Whether a handle appears on top of the stack
  - The reducing production to be used
  - The choice of actions to be made at each parsing step
- LR parsing provides a solution to the above problems
  - Is a general and efficient method of shift-reduce parsing
  - Is used in a number of automatic parser generators
- The LR(*k*) parsing technique was introduced by Knuth in 1965
  - L is for Left-to-right scanning of input
  - R corresponds to a Rightmost derivation done in reverse
  - *k* is the number of lookahead symbols used to make parsing decisions

LR Parsing – cont'd

- LR parsing is attractive for a number of reasons …
  - Is the most general deterministic parsing method known
  - Can recognize virtually all programming language constructs
  - Can be implemented very efficiently
  - The class of LR grammars is a proper superset of the LL grammars
  - Can detect a syntax error as soon as an erroneous token is encountered
  - A LR parser can be generated by a parser generating tool
- Four LR parsing techniques will be considered
  - LR(0) : LR parsing with no lookahead token to make parsing decisions
  - SLR(1) : Simple LR, with one token of lookahead
  - LR(1) : Canonical LR, with one token of lookahead
  - LALR(1) : Lookahead LR, with one token of lookahead
- LALR(1) is the preferable technique used by parser generators

LR Parsers

- An LR parser consists of …
  - Driver program
    - Same driver is used for all LR parsers
  - Parsing stack
    - Contains state information, where s_i is *state i*
    - States are obtained from grammar analysis
  - Parsing table, which has two parts
    - Action section: specifies the parser actions
    - Goto section: specifies the successor states



- The parser driver receives tokens from the scanner one at a time
- Parser uses top state and current token to lookup parsing table
- Different LR analysis techniques produce different tables

LR Parsing Table Example

- Consider the following grammar G_1 …

  1: $E \rightarrow E + T$ 3: $T \rightarrow$ **ID**

  2: $E \rightarrow T$ 4: $T \rightarrow$ **(** E **)**

- The following parsing table is obtained after grammar analysis

State	Action					Goto	
	+	**ID**	**(**	**)**	**$**	*E*	*T*
0		S1	S2			G4	G3
1	R3			R3	R3		
2		S1	S2			G6	G3
3	R2			R2	R2		
4	S5				A		
5		S1	S2				G7
6	S5			S8			
7	R1			R1	R1		
8	R4			R4	R4		

Entries are labeled with …

S*n*: Shift token and goto state *n* (call scanner for next token)

R*n*: Reduce using production *n*

G*n*: Goto state *n* (after reduce)

A: Accept parse (terminate successfully)

blank : Syntax error

LR Parsing Example

Stack	Symbols	Input	Action
0	$	**id + (id + id) $**	S1
0 1	$ **id**	**+ (id + id) $**	R3, G3
0 3	$ *T*	**+ (id + id) $**	R2, G4
0 4	$ *E*	**+ (id + id) $**	S5
0 4 5	$ *E* +	**(id + id) $**	S2
0 4 5 2	$ *E* + (	**id + id) $**	S1
0 4 5 2 1	$ *E* + (**id**	**+ id) $**	R3, G3
0 4 5 2 3	$ *E* + (*T*	**+ id) $**	R2, G6
0 4 5 2 6	$ *E* + (*E*	**+ id) $**	S5
0 4 5 2 6 5	$ *E* + (*E* +	**id) $**	S1
0 4 5 2 6 5 1	$ *E* + (*E* + **id**	**) $**	R3, G7
0 4 5 2 6 5 7	$ *E* + (*E + T*	**) $**	R1, G6
0 4 5 2 6	$ *E* + (*E*	**) $**	S8
0 4 5 2 6 8	$ *E* + **(*E*)**	**$**	R4, G7
0 4 5 7	$ *E + T*	**$**	R1, G4
0 4	$ *E*	**$**	A

1: $E \rightarrow E + T$

2: $E \rightarrow T$

3: $T \rightarrow$ **id**

4: $T \rightarrow$ **(** E **)**

Grammar symbols do not appear on the parsing stack

They are shown here for clarity

LR Parser Driver

- Let *s* be the parser stack top state and *t* be the current input token
- If *action*[*s*,*t*] = **shift** *n* then
  - Push state *n* on the stack
  - Call scanner to obtain next token
- If *action*[*s*,*t*] = **reduce** $A \rightarrow X_1 X_2 \ldots X_m$ then
  - Pop the top *m* states off the stack
  - Let *s'* be the state now on top of the stack
  - Push *goto*[*s'*, *A*] on the stack (using the goto section of the parsing table)
- If *action*[*s*,*t*] = **accept** then return
- If *action*[*s*,*t*] = **error** then call error handling routine
- All LR parsers behave the same way
  - The difference depends on how the parsing table is computed from a CFG

LR(0) Parser Generation – Items and States

- LR(0) grammars can be parsed looking only at the stack
- Making shift/reduce decisions without any lookahead token
- Based on the idea of an **item** or a **configuration**
- **An LR(0) item consists of a production and a dot**

$$A \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_n$$

- The dot symbol • may appear anywhere on the right-hand side
  - Marks how much of a production has already been seen
  - $X_1 \ldots X_i$ appear on top of the stack
  - $X_{i+1} \ldots X_n$ are still expected to appear
- An LR(0) **state** is a set of LR(0) items
  - It is the **set of all items that apply** at a given point in parse

LR(0) Parser Generation – Initial State

- Consider the following grammar *G1*:

  1: $E \rightarrow E + T$ 3: $T \rightarrow$ **ID**

  2: $E \rightarrow T$ 4: $T \rightarrow$ **(** E **)**

- For LR parsing, grammars are **augmented** with a . . .
  - New start symbol *S*, and a
  - New start production 0: $S \rightarrow E$ \$
- The input should be reduced to *E* followed by \$
  - We indicate this by the item: $S \rightarrow \bullet E$ \$
- The initial state (numbered 0) will have the item: $S \rightarrow \bullet E$ \$
- An LR parser will start in state 0
- State 0 is initially pushed on top of parser stack

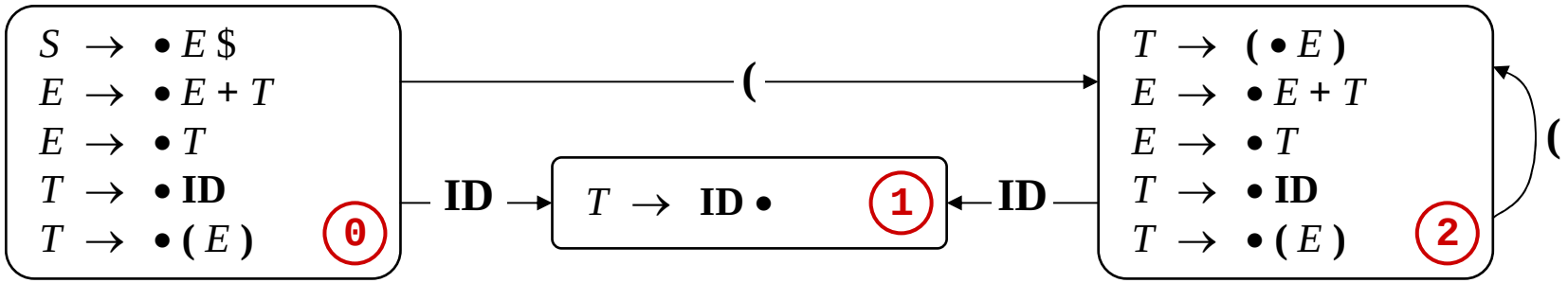
Identifying the Initial State

- Since the dot appears before E, an E is expected
  - There are two productions of E: $E \rightarrow E + T$ and $E \rightarrow T$
  - Either E+T or T is expected
  - The items: $E \rightarrow \bullet\ E + T$ and $E \rightarrow \bullet\ T$ are added to the initial state
- Since T can be expected and there are two productions for T
  - Either **ID** or **(** E **)** can be expected
  - The items: $T \rightarrow \bullet$ **ID** and $T \rightarrow \bullet$ **(** E **)** are added to the initial state
- The initial state (0) is identified by the following set of items

$$
\begin{array}{lcl}
S & \rightarrow & \bullet\ E\ \$ \\
E & \rightarrow & \bullet\ E + T \\
E & \rightarrow & \bullet\ T \\
T & \rightarrow & \bullet\ \textbf{ID} \\
T & \rightarrow & \bullet\ \textbf{(}\ E\ \textbf{)}
\end{array}
\qquad (0)
$$

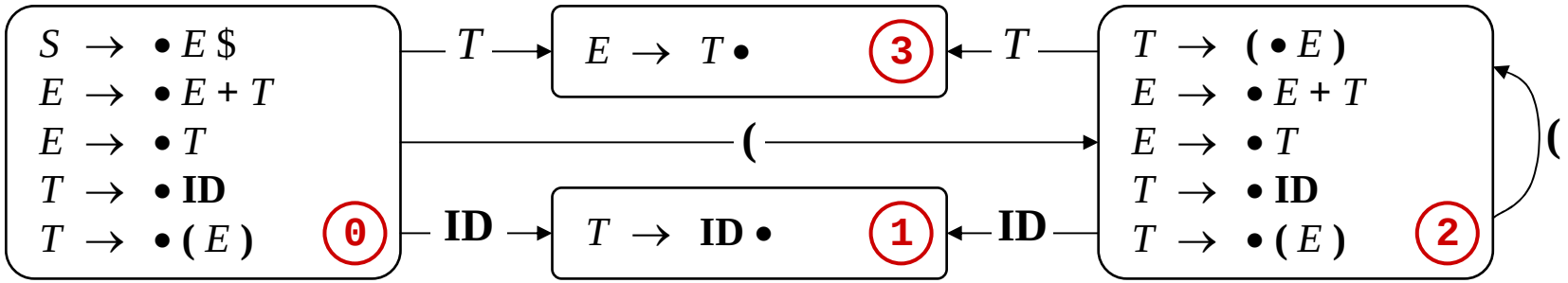
Shift Actions

- In state 0, we can shift either an **ID** or a left parenthesis
  - If we shift an **ID**, we shift the dot past the **ID**
  - We obtain a new item $T \rightarrow \mathbf{ID} \bullet$ and a new state (state 1)
  - If we shift a left parenthesis, we obtain $T \rightarrow \mathbf{(} \bullet E \mathbf{)}$
  - Since the dot appears before E, an E is expected
  - We add the items $E \rightarrow \bullet E + T$ and $E \rightarrow \bullet T$
  - Since the dot appears before T, we add $T \rightarrow \bullet \mathbf{ID}$ and $T \rightarrow \bullet \mathbf{(} E \mathbf{)}$
  - The new set of items forms a new state (state 2)
- In State 2, we can also shift an **ID** or a left parenthesis as shown



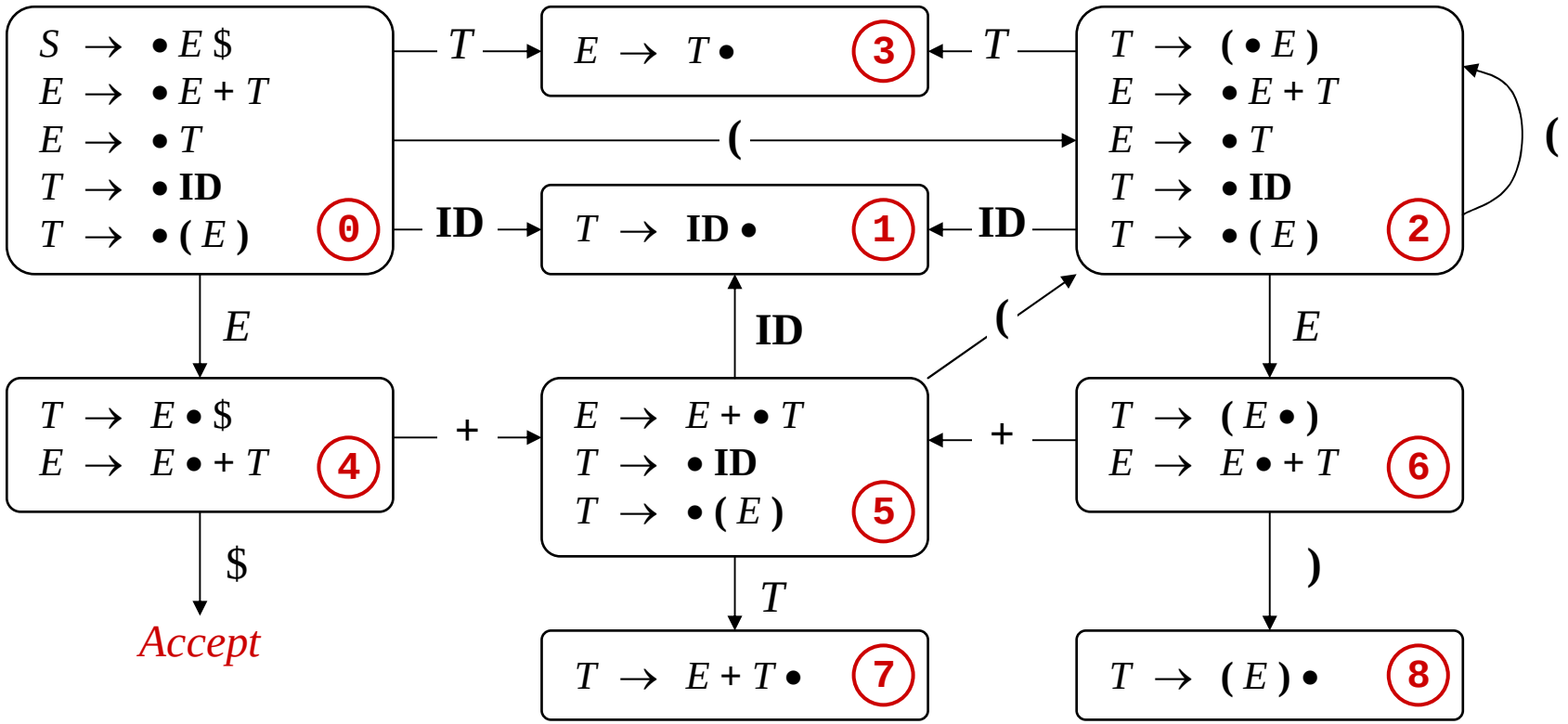
Reduce and Goto Actions

- In state 1, the dot appears at the end of item $T \rightarrow \mathbf{ID} \bullet$
  - This means that **ID** appears on top of stack and can be reduced to T
  - When $\bullet$ appears at end of an item, the parser can perform a reduce action
- If **ID** is reduced to T, what is the next state of the parser?
  - **ID** is popped from the stack; Previous state appears on top of stack
  - T is pushed on the stack
  - A new item $E \rightarrow T \bullet$ and a new state (state 3) are obtained
  - If top of stack is state 0 and we push a T, we go to state 3
  - Similarly, if top of stack is state 2 and we push a T, we go also to state 3



DFA of LR(0) States

- We complete the state diagram to obtain the DFA of LR(0) states
- In state 4, if next token is **$**, the parser **accepts** (successful parse)



LR(0) Parsing Table

- The LR(0) parsing table is obtained from the LR(0) state diagram
- The rows of the parsing table correspond to the LR(0) states
- The columns correspond to tokens and non-terminals
- For each state transition $i \rightarrow j$ caused by a token x …
  - Put **Shift** j at position [i, x] of the table
- For each transition $i \rightarrow j$ caused by a nonterminal A …
  - Put **Goto** j at position [i, A] of the table
- For each state containing an item $A \rightarrow \alpha \bullet$ of rule n …
  - Put **Reduce** n at position [i, y] for every token y
- For each transition $i \rightarrow Accept$ …
  - Put **Accept** at position [i, \$] of the table

LR(0) Parsing Table – cont'd

- The LR(0) table of grammar *G1* is shown below
  - For a shift, the token to be shifted determines the next state
  - For a reduce, the state on top of stack specifies the production to be used

State	Action					Goto	
	+	**ID**	(	)	$	*E*	*T*
0		S1	S2			G4	G3
1	R3	R3	R3	R3	R3		
2		S1	S2			G6	G3
3	R2	R2	R2	R2	R2		
4	S5				A		
5		S1	S2				G7
6	S5			S8			
7	R1	R1	R1	R1	R1		
8	R4	R4	R4	R4	R4		

Entries are labeled with …

S*n*: Shift token and goto state *n* (call scanner for next token)

R*n*: Reduce using production *n*

G*n*: Goto state *n* (after reduce)

A: Accept parse (terminate successfully)

blank : Syntax error

LR(0) Closure and Goto Functions

- To construct the LR(0) DFA, we need *closure*(I) and *goto*(I, X)
  - I is a set of items and X is a grammar symbol
- *Closure*(I) returns the complete set of items for an LR(0) state
  - New items are added to a state when dot appears before a non-terminal
- *Goto*(I, X) determines next state for a given state I and symbol X
  - The dot is moved past X and the *closure* of new set of items is obtained

function *closure*(I)
  $J := I$
  forall item $A \rightarrow \alpha \bullet B\,\gamma$ in J **do**
    forall production $B \rightarrow \beta$ **do**
      if $B \rightarrow \bullet\,\beta \notin J$ **then**
        $J := J \cup \{B \rightarrow \bullet\,\beta\}$
  return J
end function *closure*

function *goto*(I, X)
  $J := \varnothing$
  forall item $A \rightarrow \alpha \bullet X\,\gamma$ in I **do**
    $J := J \cup \{A \rightarrow \alpha\,X \bullet \gamma\}$
  return *closure*(J)
end function *goto*

Constructing the DFA of LR(0) States

- The following algorithm builds an LR(0) DFA for a grammar G
  - The algorithm determines the set of LR(0) states and the set of transitions
  - The algorithm ends when no additional states or transitions can be added

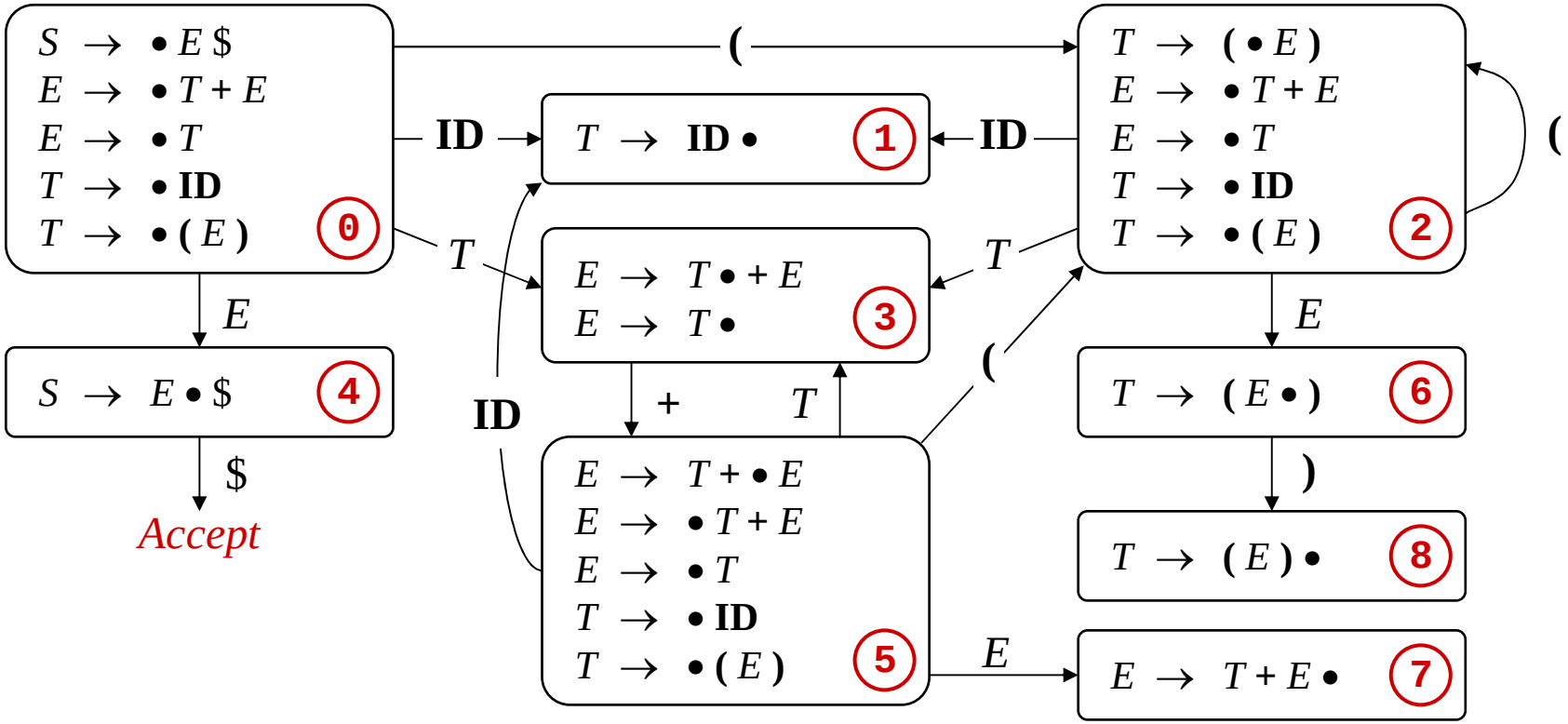
function *LR0_DFA*(G)
  $States := \{closure(\{S' \rightarrow \bullet\ S\ \$\})\}$ -- Set of all LR(0) states
  $Edges := \varnothing$ -- Set of all transitions
  forall state I in $States$ **do**
    forall item $A \rightarrow \alpha \bullet X\ \gamma$ in I **do**
      $J := goto(I, X)$
      if $J \notin States$ **then** $States := States \cup \{J\}$
      $Edges := Edges \cup \{\ I \xrightarrow{X} J\ \}$
  return *States*, *Edges*
end function *LR0_DFA*

Limits of the LR(0) Parsing Method

- Consider grammar *G2*, with right-recursive production 1

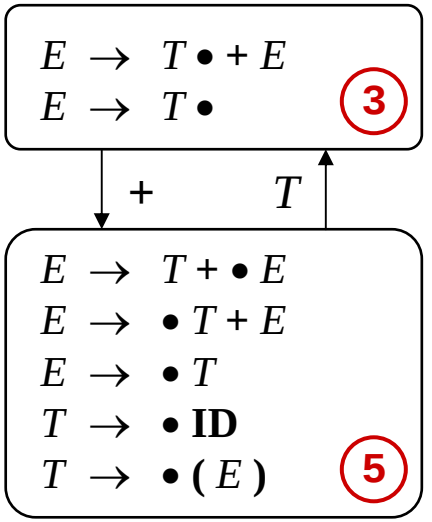
  0: $S \rightarrow E\ \$$ 1: $E \rightarrow T + E$ 2: $E \rightarrow T$ 3: $T \rightarrow \mathbf{ID}$ 4: $T \rightarrow (\ E\ )$

- The LR(0) DFA of grammar *G*2 is shown below



Conflicts

- In state 3, the parser encounters a conflict …
  - It can shift state 5 on stack when next token is +
  - It can reduce using production 2: $E \rightarrow T$
  - This is a called a **shift-reduce** conflict
  - This conflict appears in the LR(0) table under [3,+]
- Two kinds of conflicts may arise
  - **Shift-reduce** and **reduce-reduce**



- **Shift-reduce** conflict

  Parser can shift and can reduce
- **Reduce-reduce** conflict

  Two (or more) productions can be reduced

State	Action					Goto	
	+	ID	(	)	$	*E*	*T*
0		S1	S2			G4	G3
1	R3	R3	R3	R3	R3		
2		S1	S2			G6	G3
3	S5,R2	R2	R2	R2	R2		
…							

LR(0) Grammars

- The shift-reduce conflict in state 3 indicates that *G2* is not LR(0)
- A grammar is LR(0) if and only if each state is either …
  - A **shift state**, containing only shift items
  - A **reduce state**, containing only a single reduce item
- If a state contains $A \rightarrow \alpha \bullet x\ \gamma$ then it cannot contain $B \rightarrow \beta \bullet$
  - Otherwise, parser can shift *x* and reduce $B \rightarrow \beta \bullet$ (shift-reduce conflict)
- If a state contains $A \rightarrow \alpha \bullet$ then it cannot contain $B \rightarrow \beta \bullet$
  - Otherwise, parser can reduce $A \rightarrow \alpha \bullet$ and $B \rightarrow \beta \bullet$ (reduce-reduce conflict)
- LR(0) lacks the power to parse programming language grammars
  - Because they do not use the lookahead token in making parsing decisions

SLR(1) Parsing

- SLR(1), or simple LR(1), improves LR(0) by …
  - Making use of the lookahead token to eliminate conflicts
- SLR(1) works as follows …
  - It uses the same DFA obtained by the LR(0) parsing method
  - It puts reduce actions only where indicated by the FOLLOW set
- To reduce α to A in $A \rightarrow \alpha \bullet$ we must ensure that …
  - Next token may follow A (belongs to FOLLOW(A))
- We should not reduce $A \rightarrow \alpha \bullet$ when next token $\notin$ FOLLOW(A)
- In grammar $G2$ …

  0: $S \rightarrow E$ \$ 1: $E \rightarrow T + E$ 2: $E \rightarrow T$ 3: $T \rightarrow$ **ID** 4: $T \rightarrow$ **(** E **)**
  - FOLLOW(E) = {**$**, **)**} and FOLLOW($T$) = {**$**, **)**, **+**}
  - Productions 1 and 2 are reduced when followed by **$** or **)** only
  - Productions 3 and 4 are reduced when followed by **$**, **)**, or **+** only

SLR(1) Parsing Table

- The SLR(1) parsing table of grammar *G2* is shown below
- The shift-reduce conflict is now eliminated
  - The R2 action is removed from [3,+], because + does not follow *E*
  - S5 remains under [3,+]
- Grammar *G2* is SLR(1)
  - No conflicts
  - R1 and R2 for **)** and **$**
  - R3 and R4 for **+** , **)** and **$**

State	Action					Goto	
	+	**ID**	**(**	**)**	**$**	*E*	*T*
0		S1	S2			G4	G3
1	R3			R3	R3		
2		S1	S2			G6	G3
3	S5			R2	R2		
4					A		
5		S1	S2			G7	G3
6				S8			
7				R1	R1		
8	R4			R4	R4		

SLR(1) Grammars

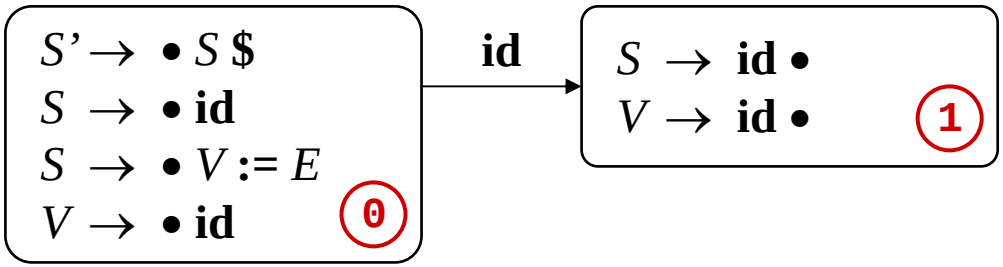
- SLR(1) parsing increases the power of LR(0) significantly
  - Lookahead token is used to make parsing decisions
  - Reduce action is applied more selectively according to FOLLOW set
- A grammar is SLR(1) if two conditions are met in every state …
  - If $A \rightarrow \alpha \bullet x\,\gamma$ and $B \rightarrow \beta \bullet$ then token $x \notin \text{FOLLOW}(B)$
  - If $A \rightarrow \alpha \bullet$ and $B \rightarrow \beta \bullet$ then $\text{FOLLOW}(A) \cap \text{FOLLOW}(B) = \varnothing$
- Violation of first condition results in **shift-reduce conflict**
  - $A \rightarrow \alpha \bullet x\,\gamma$ and $B \rightarrow \beta \bullet$ and $x \in \text{FOLLOW}(B)$ then …
  - Parser can shift x and reduce $B \rightarrow \beta$
- Violation of second condition results in **reduce-reduce conflict**
  - $A \rightarrow \alpha \bullet$ and $B \rightarrow \beta \bullet$ and $x \in \text{FOLLOW}(A) \cap \text{FOLLOW}(B)$
  - Parser can reduce $A \rightarrow \alpha$ and $B \rightarrow \beta$
- SLR(1) grammars are a superset of LR(0) grammars

Limits of the SLR(1) Parsing Method

- Consider the following grammar *G3* …

  0: $S' \rightarrow S$ **$** 1: $S \rightarrow$ **id** 2: $S \rightarrow V$ **:=** E 3: $V \rightarrow$ **id** 4: $E \rightarrow V$ 5: $E \rightarrow$ **n**

- The initial state consists of 4 items as shown below
  - When **id** is shifted in state 0, we obtain 2 items: $S \rightarrow$ **id** • and $V \rightarrow$ **id** •
- FOLLOW(*S*) = {**$**} and FOLLOW(*V*) = {**:=** , **$**}
- **Reduce-reduce conflict** in state 1 when lookahead token is **$**
  - Therefore, grammar *G3* is **not** SLR(1)
  - The reduce-reduce conflict is caused by the weakness of SLR(1) method
  - $V \rightarrow$ **id** should be reduced only when lookahead token is **:=** (but not **$**)



General LR(1) Parsing – Items and States

- Even more powerful than SLR(1) is the LR(1) parsing method
- LR(1) generalizes LR(0) by including a lookahead token in items
- An LR(1) item consists of …
  - **Grammar production rule**
  - **Right-hand position represented by the dot**, and
  - **Lookahead token**

  $A \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_n \ , \ l$ where l is a **lookahead token**
- The • represents how much of the right-hand side has been seen
  - $X_1 \ldots X_i$ appear on top of the stack
  - $X_{i+1} \ldots X_n$ are expected to appear
- The lookahead token l is expected **after** $X_1 \ldots X_n$ appear on stack
- **An LR(1) state is a set of LR(1) items**

LR(1) Parser Generation – Initial State

- Consider again grammar *G3* …

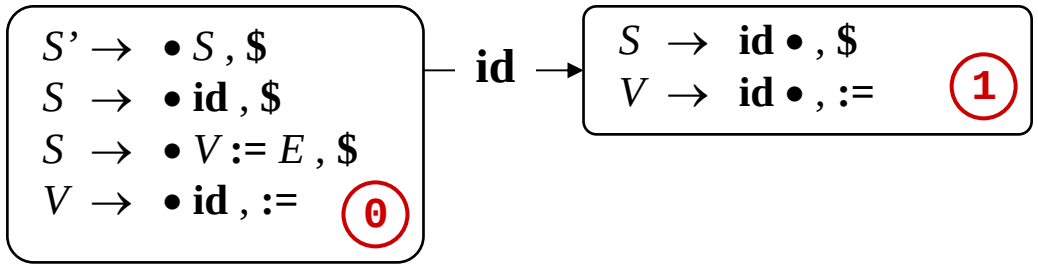
  0: $S' \rightarrow S$ **$** 1: $S \rightarrow$ **id** 2: $S \rightarrow V$ **:=** E 3: $V \rightarrow$ **id** 4: $E \rightarrow V$ 5: $E \rightarrow$ **n**

- The initial state contains the LR(1) item: $S' \rightarrow \bullet S$, **$**
  - $S' \rightarrow \bullet S$, **$** means that S is expected and to be followed by **$**
- The closure of ($S' \rightarrow \bullet S$, **$**) produces the initial state items
  - Since the dot appears before S, an S is expected
  - There are two productions of S: $S \rightarrow$ **id** and $S \rightarrow V$ **:=** E
  - The LR(1) items ($S \rightarrow \bullet$ **id** , **$**) and ($S \rightarrow \bullet V$ **:=** E , **$**) are obtained
    - The lookahead token is **$** (end-of-file token)
  - Since the $\bullet$ appears before V in ($S \rightarrow \bullet V$ **:=** E , **$**), a V is expected
  - The LR(1) item ($V \rightarrow \bullet$ **id** , **:=**) is obtained
    - The lookahead token is **:=** because it appears after V in ($S \rightarrow \bullet V$ **:=** E , **$**)

Shift Action

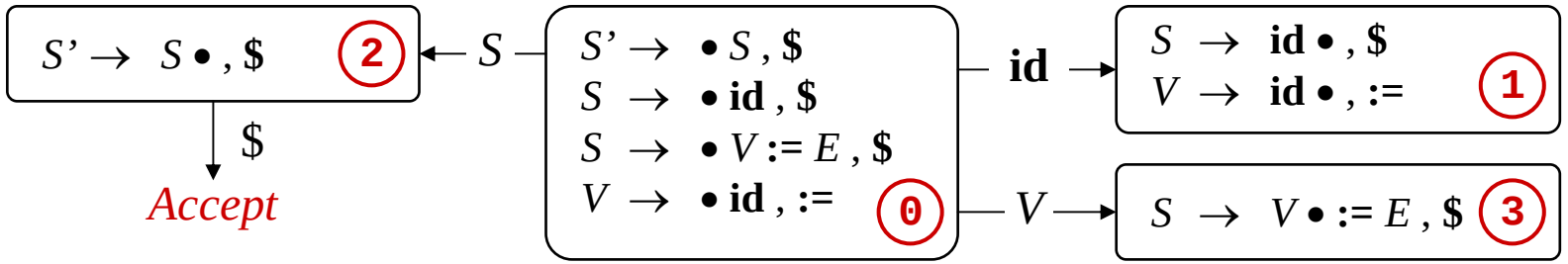
- The initial state (state 0) consists of 4 items
- In state 0, we can shift an **id**
  - The token **id** can be shifted in two items
  - When shifting **id**, we shift the dot past the **id**
  - We obtain ($S \rightarrow$ **id** $\bullet$, **$**) and ($V \rightarrow$ **id** $\bullet$, **:=**)
  - The two LR(1) items form a new state (state 1)
  - The two items are **reduce items**
  - No additional item can be added to state 1

$S' \rightarrow \bullet S$, **$**
$S \rightarrow \bullet$ **id** , **$**
$S \rightarrow \bullet V$ **:=** E , **$**
$V \rightarrow \bullet$ **id** , **:=**
0



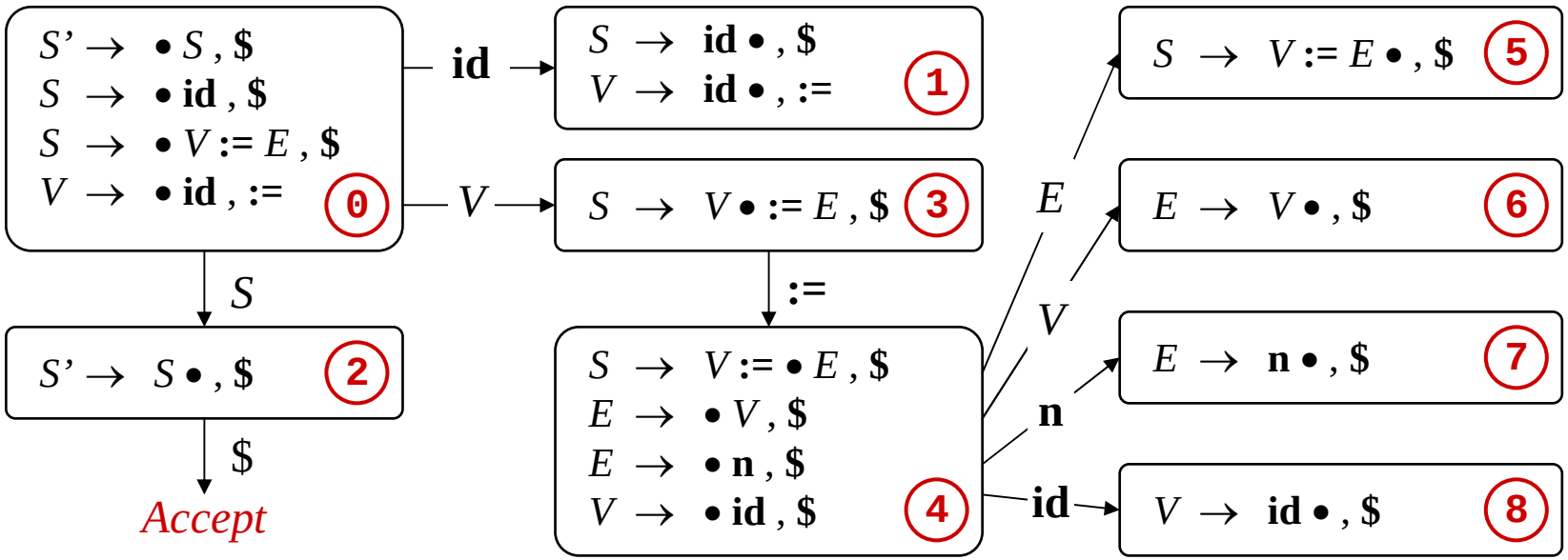
Reduce and Goto Actions

- In state 1, • appears at end of ($S \rightarrow$ **id** • , **$**) and ($V \rightarrow$ **id** • , **:=**)
  - This means that **id** appears on top of stack and can be reduced
  - Two productions can be reduced: $S \rightarrow$ **id** and $V \rightarrow$ **id**
- The lookahead token eliminates the conflict of the reduce items
  - If lookahead token is **$** then **id** is reduced to S
  - If lookahead token is **:=** then **id** is reduced to V
- When in state 0 after a reduce action …
  - If S is pushed, we obtain item ($S' \rightarrow S$ • , **$**) and go to state 2
  - If V is pushed, we obtain item ($S \rightarrow V$ • **:=** E , **$**) and go to state 3



LR(1) State Diagram

- The LR(1) state diagram of grammar *G3* is shown below
- Grammar *G3*, which was not SLR(1), is now LR(1)
- The reduce-reduce conflict that existed in state 1 is now removed
- The lookahead token in LR(1) items eliminated the conflict



LR(1) Closure and Goto Functions

- To construct the LR(1) DFA, we need *closure* and *goto* functions
- *Closure*(*I*) returns the complete set of LR(1) items for a state
  - For items $(A \rightarrow \alpha \bullet B\,\gamma\ ,\ l) \in I$, add $(B \rightarrow \bullet\ \beta\ ,\ x)$ forall $x \in \text{FIRST}(\gamma l)$
  - $x \in \text{FIRST}(\gamma l)$ is what follows B in $(A \rightarrow \alpha \bullet B\,\gamma\ ,\ l)$
- *Goto*(*I*, *X*) determines next state for a given state *I* and symbol *X*

```
function closure(I)
   J := I
   forall item (A → α • B γ , l) in J do
      forall production B → β do
         forall x ∈ FIRST(γl) do
            if (B → • β , x) ∉ J then
               J := J ∪ {(B → • β , x)}
   return J
end function closure
```

```
function goto(I, X)
   J := ∅
   forall item (A → α • X γ , l) in I do
      J := J ∪ {(A → α X • γ , l)}
   return closure(J)
end function goto
```

LR(1) Grammars

- A grammar is LR(1) if the following two conditions are met …
  - If a state contains $(A \rightarrow \alpha \bullet x\, \gamma, a)$ and $(B \rightarrow \beta \bullet, b)$ then $b \neq x$
  - If a state contains $(A \rightarrow \alpha \bullet, a)$ and $(B \rightarrow \beta \bullet, b)$ then $a \neq b$
- Violation of first condition results in a **shift-reduce conflict**
- If a state contains $(A \rightarrow \alpha \bullet x\, \gamma, a)$ and $(B \rightarrow \beta \bullet, x)$ then …
  - It can shift x and can reduce $B \rightarrow \beta$ when lookahead token is x
- Violation of second condition results in **reduce-reduce conflict**
- If a state contains $(A \rightarrow \alpha \bullet, a)$ and $(B \rightarrow \beta \bullet, a)$ then …
  - It can reduce $A \rightarrow \alpha$ and $B \rightarrow \beta$ when lookahead token is a
- LR(1) grammars are a superset of SLR(1) grammars