Arithmetic Circuits 2

COE 202

Digital Logic Design

Dr. Muhamed Mudawar

King Fahd University of Petroleum and Minerals

Zero versus Sign Extension

- **Unsigned** Integers are **Zero-Extended**
- **Signed** Integers are **Sign-Extended**
- Given that X is a 4-bit **unsigned** integer ➔ Range = 0 to 15
- Given that Y is a 4-bit **signed** integer ➔ Range = -8 to +7
- If **unsigned** X = 4’b1101 (binary), then X = 13 (decimal)
- If **signed** Y = 4’b1101 (binary), then Y = -3 (decimal)
- If X is **zero-extended** from 4 to 6 bits then X = 6’b001101 = 13
- If Y is **sign-extended** from 4 to 6 bits then Y = 6’b111101 = -3

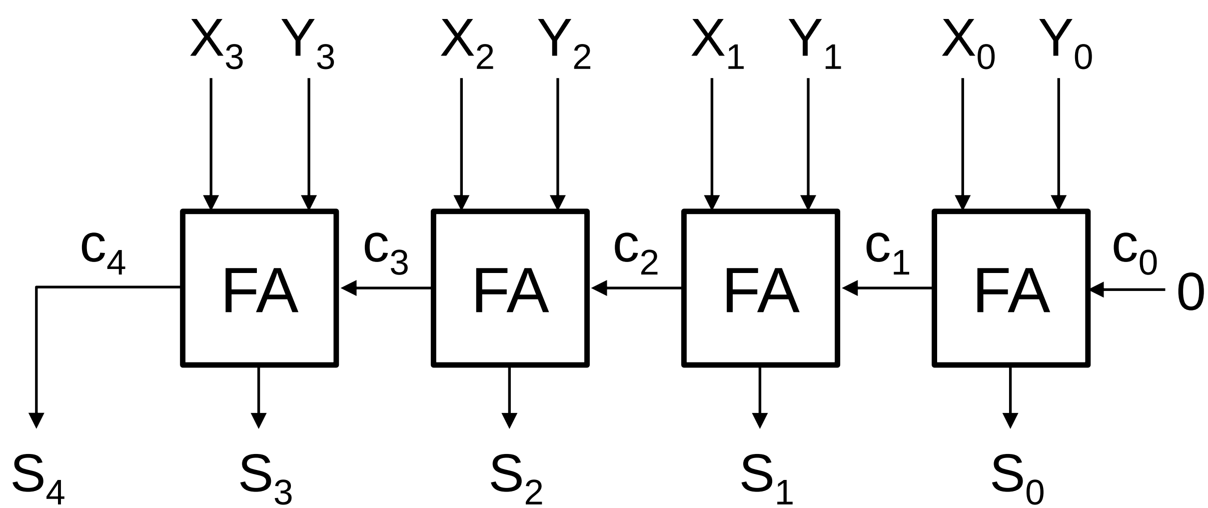
Unsigned Addition S = X + Y

- Design a circuit that computes: S = X + Y (**unsigned X and Y**)
- X[3:0] and Y[3:0] are 4-bit **unsigned** integers ➔ Range = 0 to 15

Solution:

- Maximum S = 15 + 15 = 30 ➔ unsigned S must be **5 bits**

Most-significant sum bit S_4 is the carry bit c_4



Signed Addition S = X + Y

❖ Design a circuit that computes: S = X + Y (**signed X and Y**)

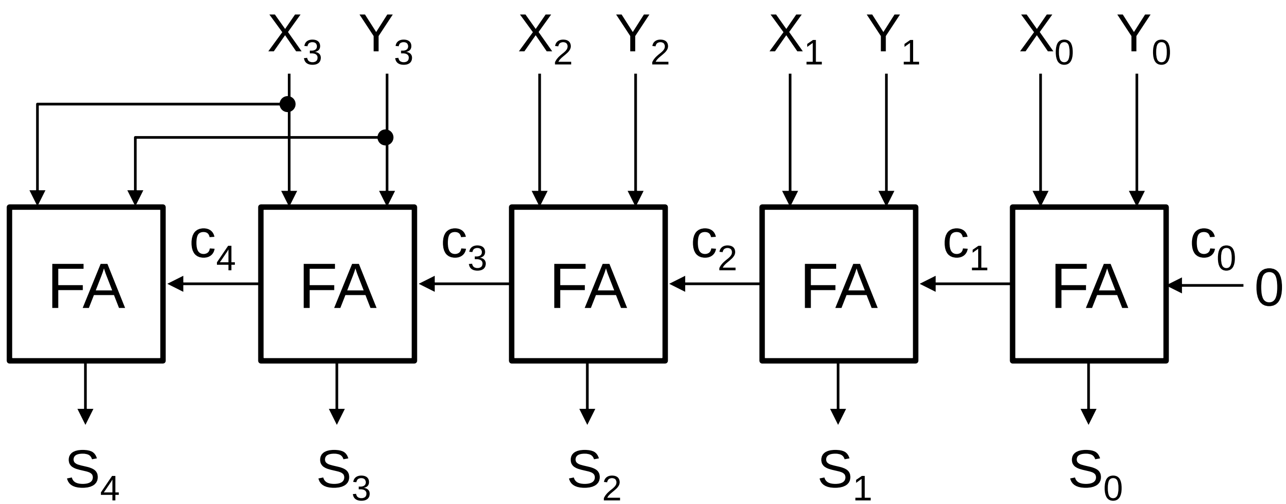
❖ X[3:0] and Y[3:0] are 4-bit **signed** integers ➔ Range = -8 to +7

Solution:

❖ Minimum S = (-8) + (-8) = -16, Maximum S = (+7) + (+7) = + 14

❖ Therefore, signed range of S = -16 to +14 ➔ S must be **5 bits**

X and Y are **sign-extended**
X_3 and Y_3 are replicated to produce S_4



Unsigned Subtraction S = X - Y

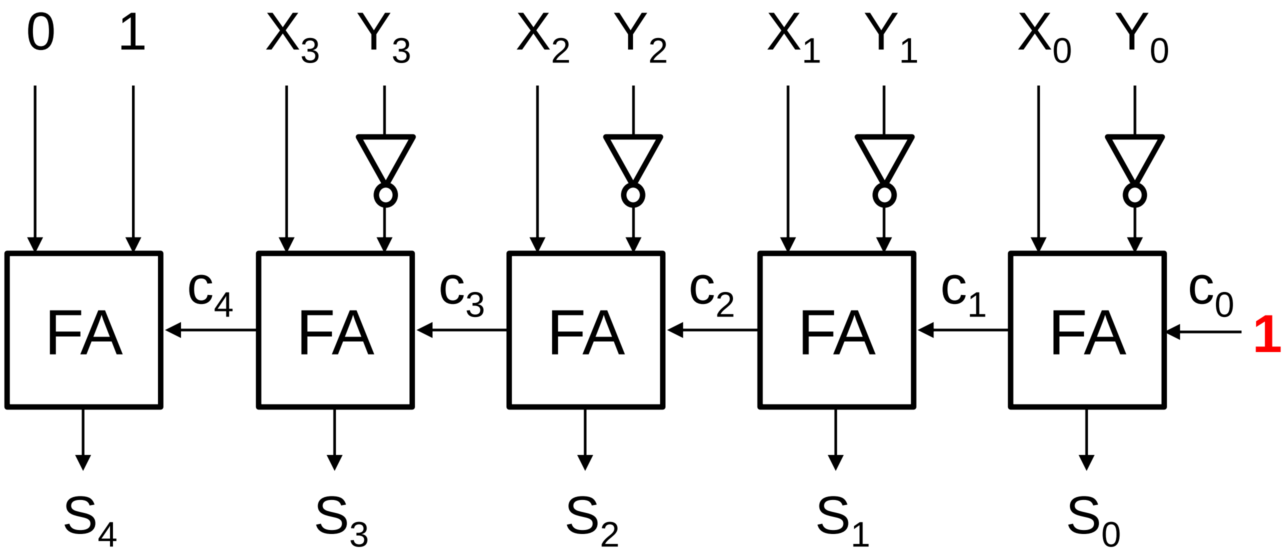
- ❖ Design a circuit that computes S = X – Y (**unsigned X and Y**)
- ❖ X[3:0] and Y[3:0] are 4-bit **unsigned** integers ➔ Range = 0 to 15

Solution: S = X – Y = 2's complement of Y = X + Y' + 1

- ❖ Minimum S = 0 – 15 = -15, Maximum S = 15 – 0 = +15
- ❖ S is **signed**, even though X are Y are **unsigned** ➔ S is **5 bits**

X−Y = X+Y'+1

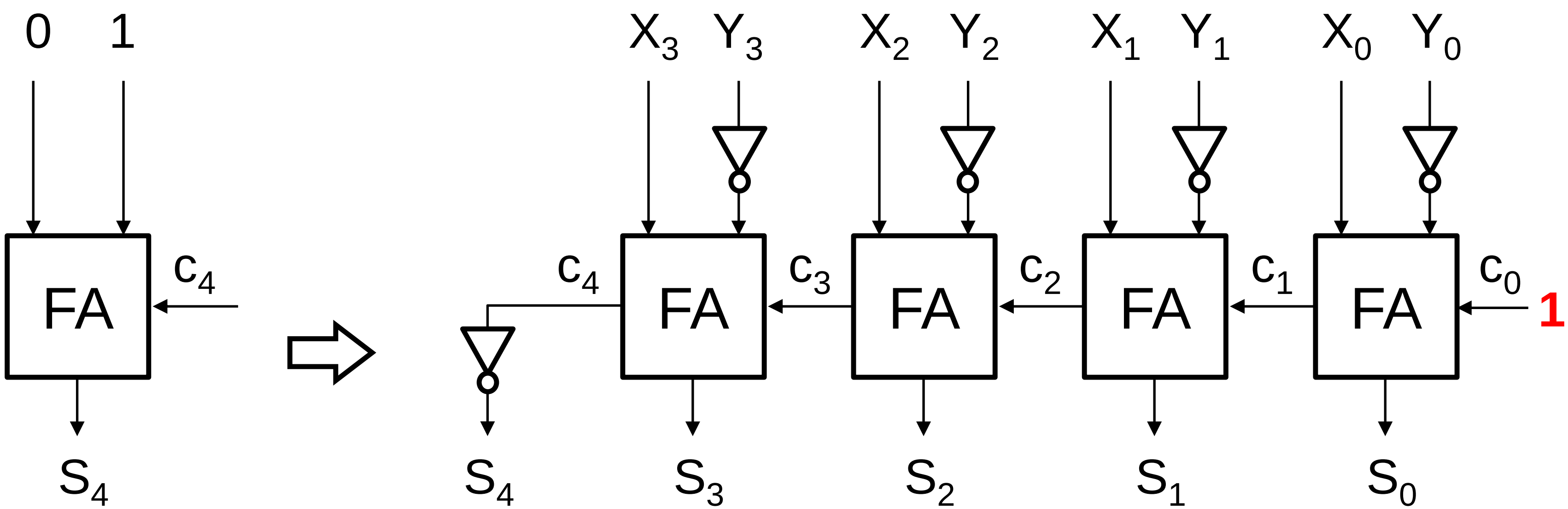
X and Y are **zero-extended**.



Unsigned Subtraction S = X - Y

❖ Most-significant bit: $S_4 = 0 + 0' + c_4 = 1 + c_4 = c_4'$

❖ Full Adder for S_4 can be replaced by an **inverter**



Signed Subtraction S = X - Y

❖ Design a circuit that computes S = X – Y (**signed X and Y**)

❖ X[3:0] and Y[3:0] are 4-bit **signed** integers ➔ Range = -8 to +7

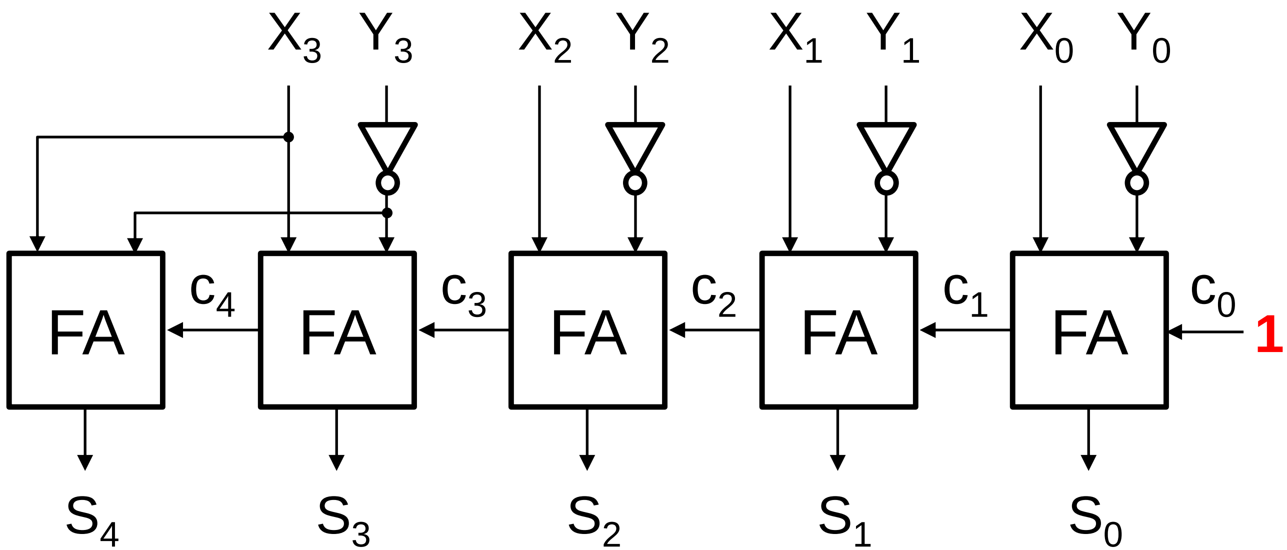
Solution: S = X – Y = X + Y' + 1

❖ Minimum S = -8 – (+7) = -15, Maximum S = +7 – (-8) = +15

❖ Signed range for S is -15 to +15 ➔ S is **5 bits**

X–Y = X+Y'+1

X and Y are **sign-extended**.



$S = 2*X + Y$ (Unsigned X and Y)

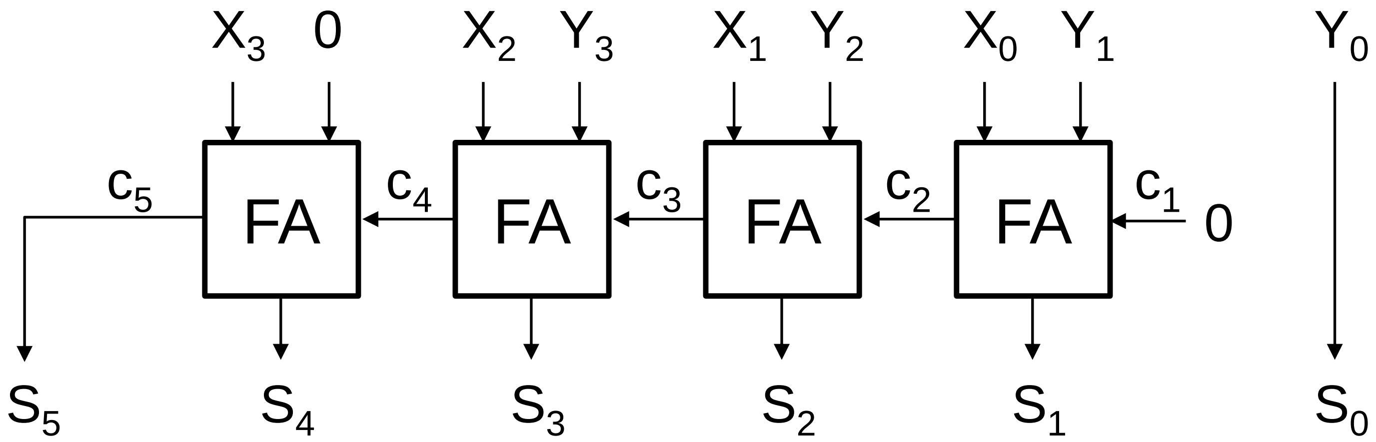
❖ Design a circuit that computes S = 2*X + Y (**unsigned X and Y**)

❖ X[3:0] and Y[3:0] are 4-bit **unsigned** integers ➔ range = 0 to 15

Solution:

❖ **2*X + Y = X << 1 + Y (Shift-Left X by 1 bit)**

❖ Maximum value of S = 2*15 + 15 = 45 ➔ S is **6 bits** = S[5:0]



S = 2*X + Y (Signed X and Y)

❖ Design a circuit that computes S = 2*X + Y using Full Adders

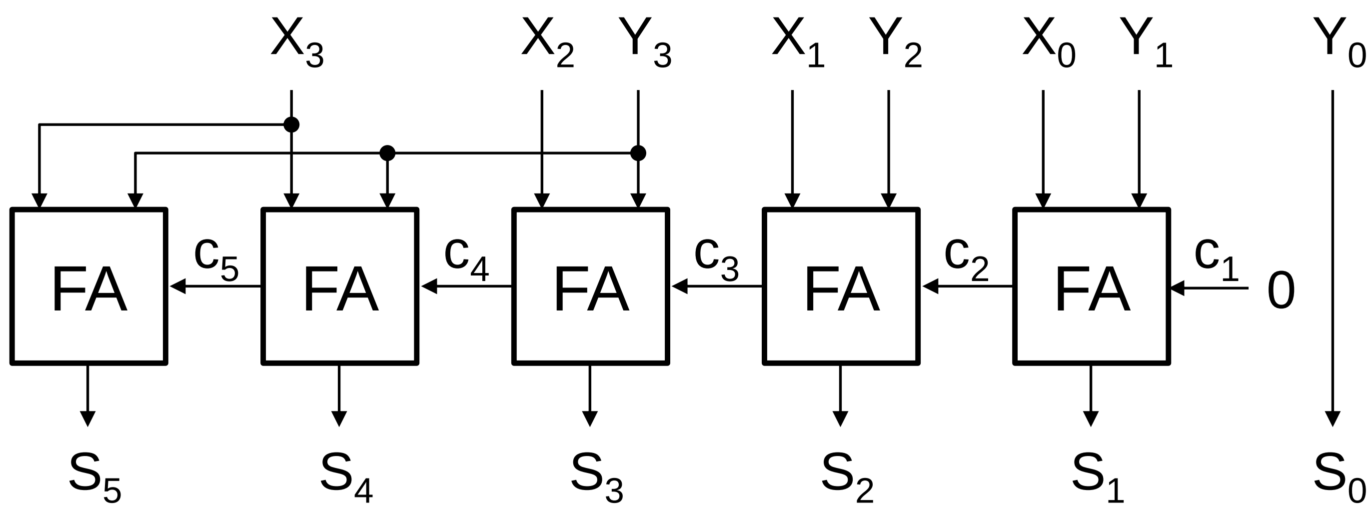
❖ X[3:0] and Y[3:0] are 4-bit **signed** integers ➔ range = -8 to +7

Solution:

❖ Range of X and Y is -8 to +7 ➔ Minimum S = 2*(-8) + (-8) = -24

❖ Maximum S = 2*(+7) + 7 = +21 ➔ S is **6 bits** = S[5:0]

X and Y are **sign-extended**. Sign bits X_3 and Y_3 are **replicated**.



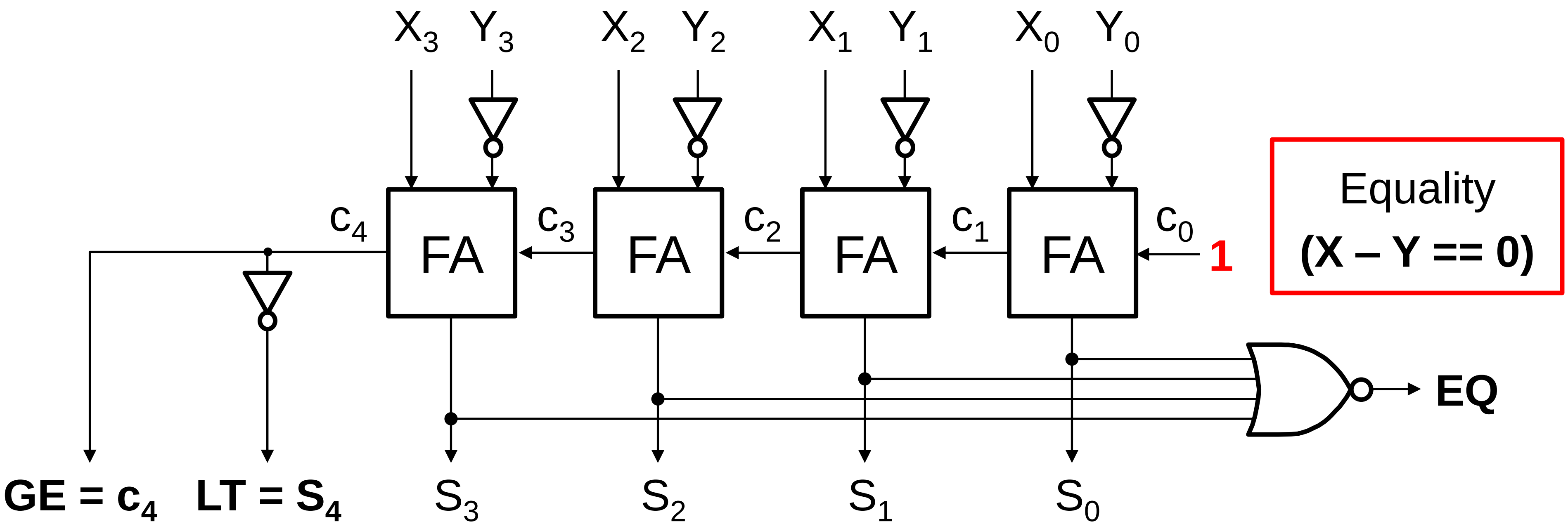
Unsigned Less Than: LT = X < Y

❖ Design a circuit that computes **unsigned LT** (**unsigned X and Y**)

Solution:

❖ If **(X < Y)** then **(X – Y) < 0**, If **(X == Y)** then **(X – Y == 0)**

❖ Do **unsigned subtraction**, **LT = S_4 =** **sign-bit** of the result

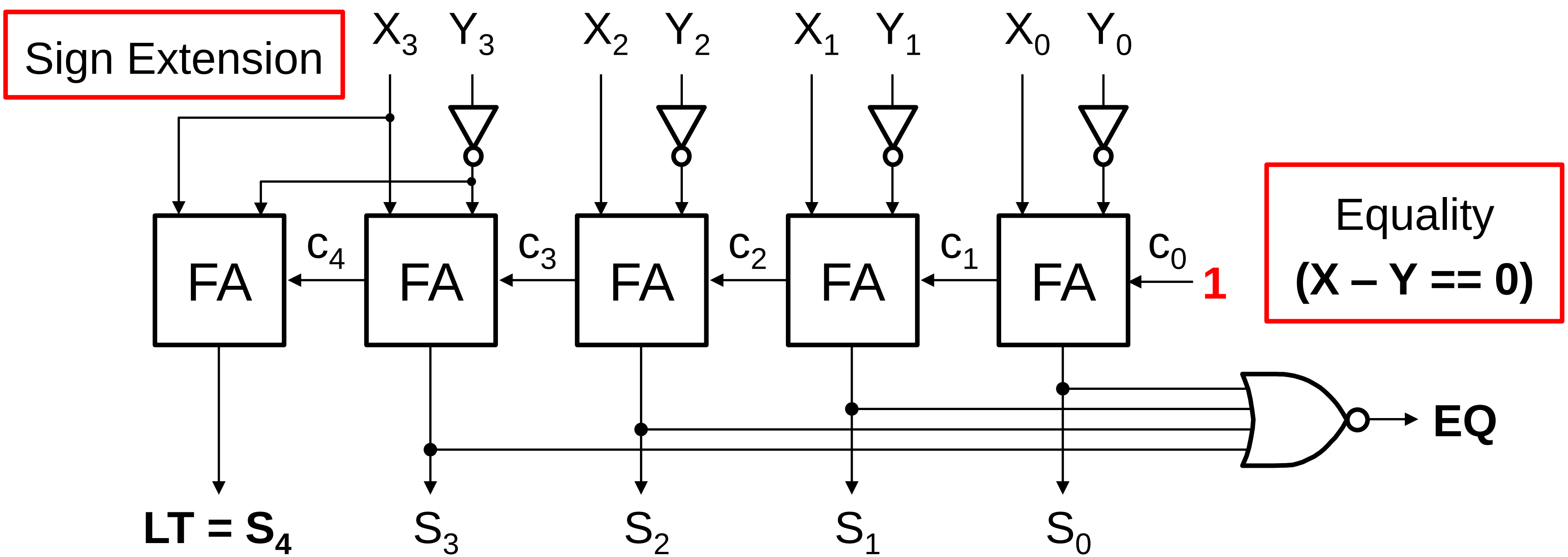


Signed Less Than: LT = X < Y

- Design a circuit that computes **signed LT** (**Signed X and Y**)

Solution:

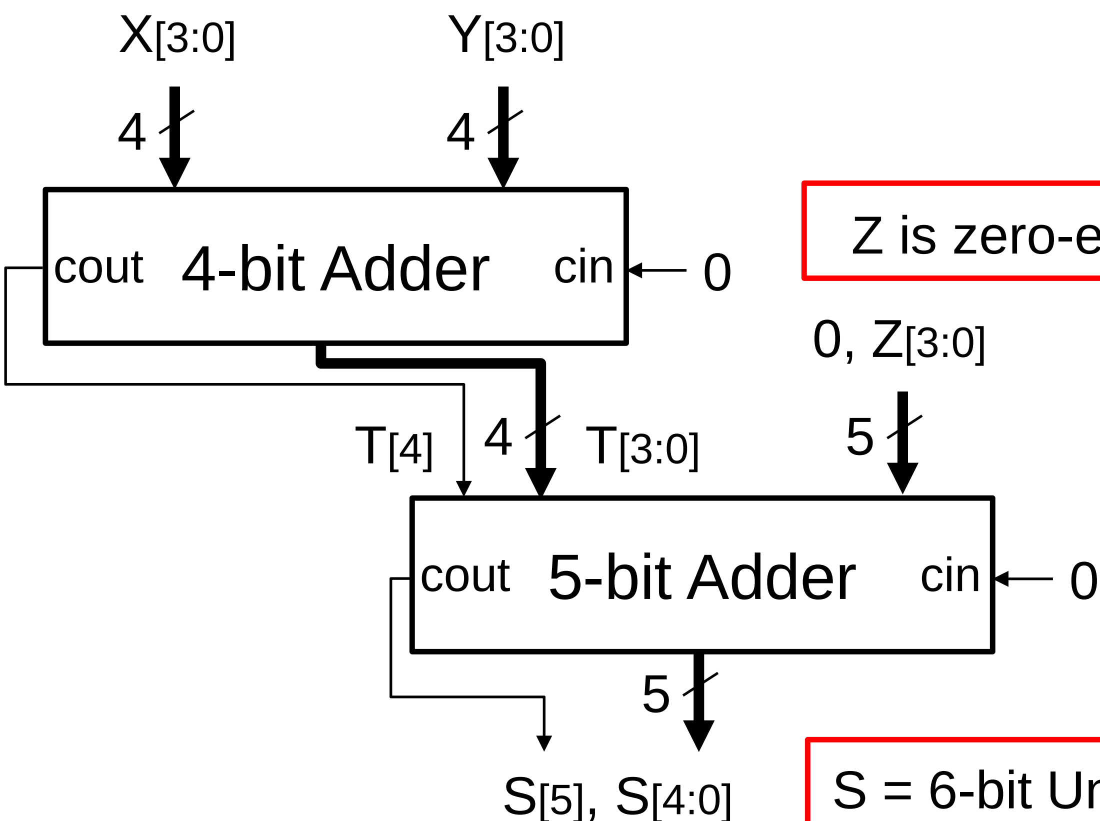
- If **(X < Y)** then **(X – Y) < 0**, If **(X == Y)** then **(X – Y == 0)**
- Do **signed subtraction**, **LT = S_4** = **sign-bit** of the result



Design a Circuit for Unsigned S = X + Y + Z

❖ X, Y, and Z are 4-bit **unsigned** integers ➔ Range = 0 to 15

Solution: Maximum S = 15 + 15 + 15 = 45 ➔ S must be **6 bits**



Z is zero-extended

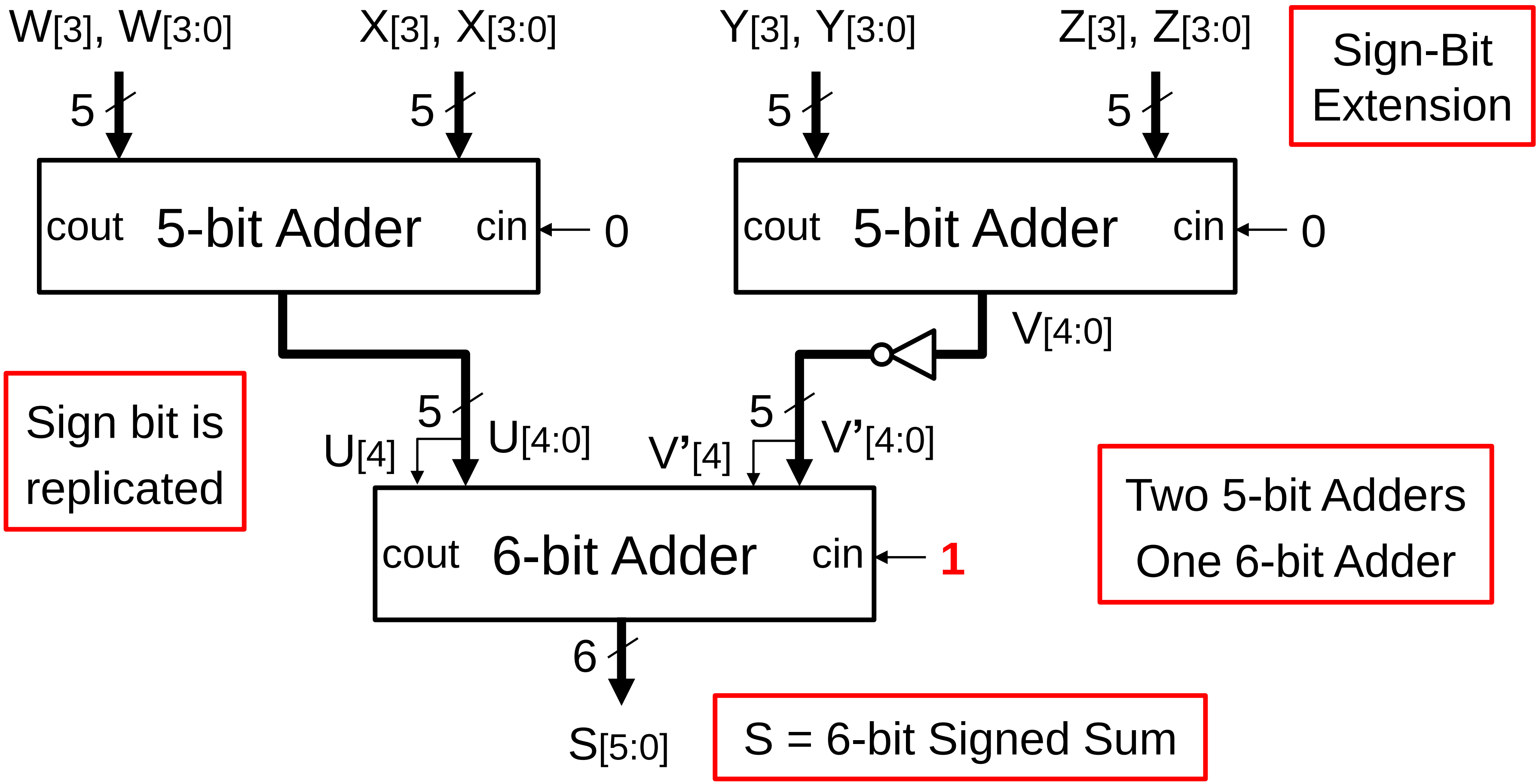
One 4-bit Adder
One 5-bit Adder

S = 6-bit Unsigned Sum

Design a Circuit for Signed S = W + X – Y – Z

❖ W, X, Y, and Z are 4-bit **signed** integers ➔ Range = -8 to +7

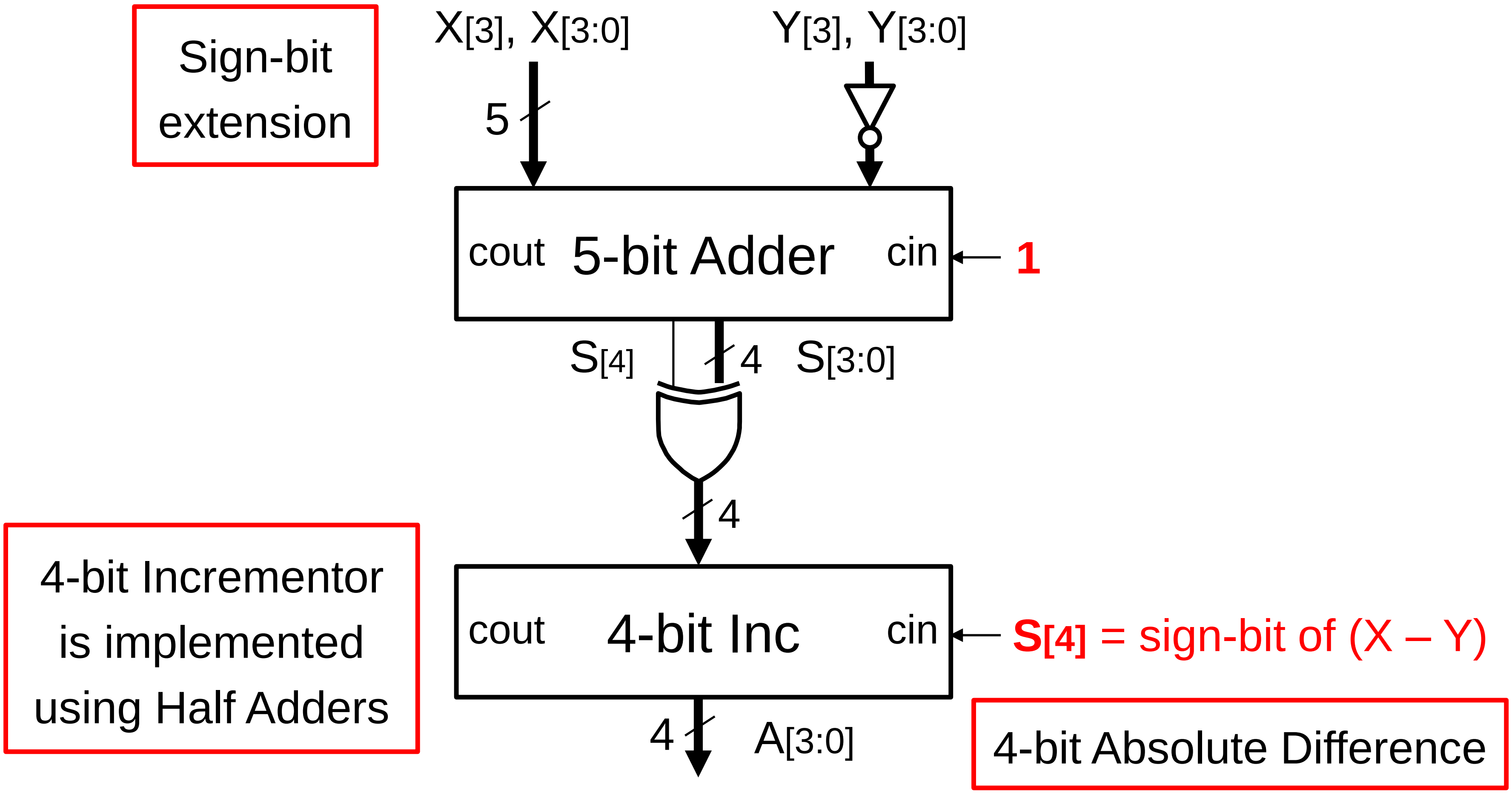
Solution: S = W + X – Y – Z = (W+X) – (Y+Z) ➔ 6 bits are used



Absolute Difference |X – Y| of Signed X, Y

❖ Design a circuit that computes A = |X – Y| (absolute difference)

Solution: Maximum A = |X – Y| = |-8 – +7| = 15 ➔ 4 bits are used



Incrementor Circuit