EE 200- Digital Logic Circuit Design

1.4 Octal & Hexadecimal Numbers
1.5 Complements

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- Can you name number systems other than binary or decimal?

Objectives

- General number-base conversion.
- Decimal ↔ Octal conversion.
- Binary ↔ Octal ↔ Hex Shortcut.
- Complements

- **What? Why More Number Systems?**
- Humans understand decimal.
- Digital electronics (computers) understand binary.
- Since computers have 32, 64, and even 128 bit busses, displaying numbers in binary is cumbersome.
- Data on a 32 bit data bus would look like the following:

0110 1001 0111 0001 0011 0100 1100 1010

- Data on a 32 bit data bus would look like the following:

  0110 1001 0111 0001 0011 0100 1100 1010

- Hexadecimal (base 16) and octal (base 8) number systems are used to represent binary data in a more compact form.
- Next we'll explore the process for converting numbers between the decimal number system and the hexadecimal & octal number systems.

- **Counting . . . 2, 8, 10, 16**

Decimal	Binary	Octal	Hexadecimal
0	00000	0	0
1	00001	1	1
2	00010	2	2
3	00011	3	3
4	00100	4	4
5	00101	5	5
6	00110	6	6
7	00111	7	7
8	01000	10	8
9	01001	11	9
10	01010	12	A
11	01011	13	B
12	01100	14	C
13	01101	15	D
14	01110	16	E
15	01111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13

- Conversion Process Decimal → Base_N

Base_{10} DECIMAL → Successive Division → Base_N ANY BASE

a) Divide the decimal number by **N**; the remainder is the LSB of the **ANY BASE** Number.

b) If the quotient is zero, the conversion is complete. Otherwise repeat step (a) using the quotient as the decimal number. The new remainder is the next most significant bit of the **ANY BASE** number.

- Conversion Process $Base_N$ →Decimal

a) Multiply each bit of the **ANY BASE** number by its corresponding bit-weighting factor (i.e., Bit-0→**N**^{0}; Bit-1→**N**^{1}; Bit-2→**N**^{2}; etc).

b) Sum up all of the products in step (a) to get the decimal number.

• Decimal ↔ Octal Conversion

The Process: Successive Division

1. Divide the decimal number by **8**; the remainder is the LSB of the **octal** number.
2. If the quotient is zero, the conversion is complete. Otherwise repeat step (1) using the quotation as the decimal number. The new remainder is the next most significant bit of the **octal** number.

- **Decimal ↔ Octal Conversion**

<u>Example:</u> Convert the decimal number 94_{10} into its octal equivalent.

$$8\,\overline{)\,94}\;\; \text{quotient } 11 \qquad r = 6 \;\leftarrow \text{LSB}$$

$$8\,\overline{)\,11}\;\; \text{quotient } 1 \qquad r = 3$$

$$8\,\overline{)\,1}\;\; \text{quotient } 0 \qquad r = 1 \;\leftarrow \text{MSB}$$

$$\therefore\;\; 94_{10} = 136_8$$

- **Octal ↔ Decimal Process**

The Process: Weighted Multiplication:

1. Multiply each bit of the Octal Number by its corresponding bit-weighting factor (i.e., Bit-0→8^0=1; Bit-1→8^1=8; Bit-2→8^2=64; etc.).
2. Sum up all of the products in step (1) to get the decimal number.

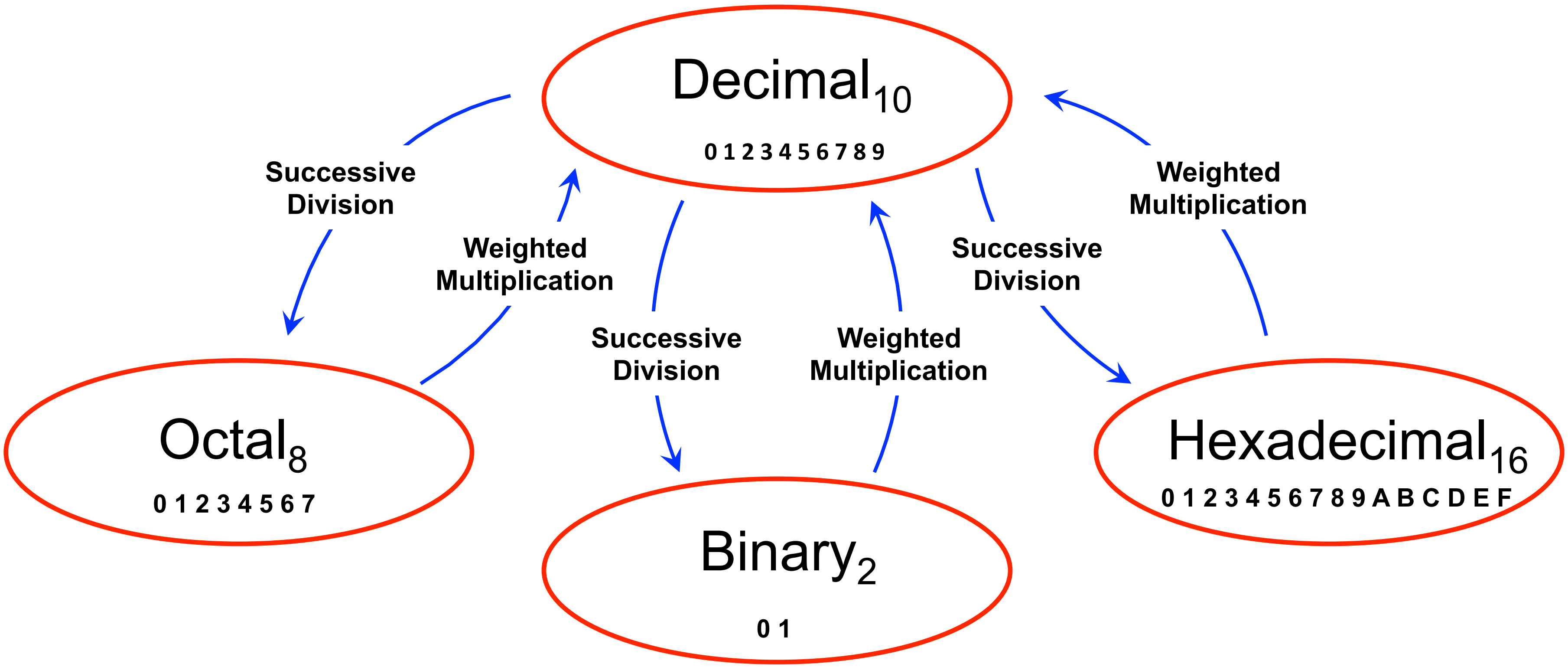
- **Decimal ↔ Octal Conversion**

<u>Example:</u> Convert the octal number 136_8 into its decimal equivalent.

1		3		6		
8^2		8^1		8^0		
64		8		1		
64	+	24	+	6	=	94_{10}

$$\therefore \; 136_8 = 94_{10}$$

- **Converting To and From Decimal**



- **Binary ↔ Octal ↔ Hex Shortcut**

Because binary, octal, and hex number systems are all powers of two (which is the reason we use them) there is a relationship that we can exploit to make conversion easier.

$$1\,0\,1\,1\,0\,1\,0_2 = 132_8 = 5A_H$$

$$\underbrace{0\,0\,1}_{1}\;\underbrace{0\,1\,1}_{3}\;\underbrace{0\,1\,0}_{2}\,{}_2 = \underbrace{1}_{0\,0\,1}\;\underbrace{3}_{0\,1\,1}\;\underbrace{2}_{0\,1\,0}\,{}_8$$

$$\underbrace{0\,1\,0\,1}_{5}\;\underbrace{1\,0\,1\,0}_{A}\,{}_2 = \underbrace{5}_{0\,1\,0\,1}\;\underbrace{A}_{1\,0\,1\,0}\,{}_{16}$$

Signed Magnitude:

The leading digit set it, "0" is positive and "1" is negatave.

One's Complement:

flip bits for *-ve* numbers

Two's Complement:

one's complement + 1

OR toggle bits after the *1st* 1 from the *LSB*

Signed Magnitude:

12 = 00001100

-12 = 10001100

One's Complement:

12 = 00001100

-12 = 11110011

Two's Complement:

12 = 00001100

-12 = 11110100

- **Number's Complements, a basic concepts**
  - Complements are used to simplify the subtraction operation and for logical manipulations.
  - For each base (r), there are two complements:
    - r's complement, also called radix/base complement,
    - (r-1)'s complement.

- **Decimal number complements:**
- 9’s complement of the decimal number
  - subtract each digit from 9
  - **example:** 9’s complement of 134795 is
    865204

- **Decimal number complements:**
- 10’s complement of the decimal number
  - 9’s complement + 1
  - **example:** 10’s complement of 134795 is its 9’s complement + 1

9’s	865204
10’s	865205

- **Binary number complements:**
- 1's complement of the binary number
  - Flip each digit (each 0 to 1 and vice versa)
  - **example:** 1's complement of 110100101 is 001011010

- **Binary number complements:**
- 2’s complement of the binary number
  - 1’s complement + 1
  - **example:** 2’s complement of 0010110010 is its 1’s complement + 1

1’s	1101001101
2’s	~~0~~1101001110

Summary

- General number-base conversion.
- Decimal ↔ Octal conversion.
- Binary ↔ Octal ↔ Hex Shortcut.
- Complements

- **Next Lecture**
  - Complements
  - Subtraction using complements.