Solution to Quiz#2

Question 1: Simplify the following Boolean function using algebraic manipulation:

X'YZ + X'YZ' + XZ

Solution:

X'YZ + X'YZ' + XZ
= X'Y (Z + Z') + XZ
= X'Y + XZ

Question 2: Prove the identity of the following Boolean function using algebraic manipulation:

A'B'C'D' + A'B'CD' + AB'C'D' + AB'CD' = B'D'

Solution:

Left hand side = A'B'C'D' + A'B'CD' + AB'C'D' + AB'CD'
= B'D' (A'C' + A'C + AC' + AC)
= B'D' (A' (C' + C) + A (C' + C))
= B'D' (A' + A)
= B'D' = Right hand side

Solution to Homework#2

Question 2-1

a) $\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$

Verification of DeMorgan's Theorem

X	Y	Z	XYZ	$\overline{XYZ}$	$\overline{X}+\overline{Y}+\overline{Z}$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	0	0

b) $X + YZ = (X + Y) \cdot (X + Z)$

The Second Distributive Law

X	Y	Z	YZ	$X+YZ$	$X+Y$	$X+Z$	$(X+Y)(X+Z)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

c) $\bar{X}Y + \bar{Y}Z + X\bar{Z} = X\bar{Y} + Y\bar{Z} + \bar{X}Z$

X	Y	Z	$\bar{X}Y$	$\bar{Y}Z$	$X\bar{Z}$	$\bar{X}Y+\bar{Y}Z+X\bar{Z}$	$X\bar{Y}$	$Y\bar{Z}$	$\bar{X}Z$	$X\bar{Y}+Y\bar{Z}+\bar{X}Z$
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	0	1	0	1
0	1	1	1	0	0	1	0	0	1	1
1	0	0	0	0	1	1	1	0	0	1
1	0	1	0	1	0	1	1	0	0	1
1	1	0	0	0	1	1	0	1	0	1
1	1	1	0	0	0	0	0	0	0	0

Question 2-2

a) $\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$

$= (\bar{X}Y + \bar{X}\bar{Y}) + (\bar{X}Y + XY)$

$= \bar{X}(Y + \bar{Y}) + Y(X + \bar{X})$

$= \bar{X} + Y$

b) $\overline{A}B + \overline{B}\overline{C} + AB + \overline{B}C = 1$

$= (\overline{A}B + AB) + (\overline{B}\overline{C} + \overline{B}C)$

$= B(A + \overline{A}) + \overline{B}(C + \overline{C})$

$= \overline{B + \overline{B}}$

$= 1$

c) $Y + \overline{X}Z + X\overline{Y} = X + Y + Z$

$= Y + X\overline{Y} + \overline{X}Z$

$= (Y + X)(Y + \overline{Y}) + \overline{X}Z$

$= Y + X + \overline{X}Z$

$= Y + (X + \overline{X})(X + Z)$

$= X + Y + Z$

d) $\overline{X}\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X}\overline{Y} + XZ + Y\overline{Z}$

$= \overline{X}\overline{Y} + \overline{Y}Z(X + \overline{X}) + XZ + XY + Y\overline{Z}$

$= \overline{X}\overline{Y} + X\overline{Y}Z + \overline{X}\overline{Y}Z + XZ + XY + Y\overline{Z}$

$= \overline{X}\overline{Y}(1 + Z) + X\overline{Y}Z + XZ + XY + Y\overline{Z}$

$= \overline{X}\overline{Y} + XZ(1 + \overline{Y}) + XY + Y\overline{Z}$

$= \overline{X}\overline{Y} + XZ + XY(Z + \overline{Z}) + Y\overline{Z}$

$= \overline{X}\overline{Y} + XZ + XYZ + Y\overline{Z}(1 + X)$

$= \overline{X}\overline{Y} + XZ(1 + Y) + Y\overline{Z}$

$= \overline{X}\overline{Y} + XZ + Y\overline{Z}$

Question 2-7

a) $\overline{X}\overline{Y} + XYZ + \overline{X}Y = \overline{X} + XYZ = (\overline{X} + XY)(\overline{X} + Z) = (\overline{X} + X)(\overline{X} + Y)(\overline{X} + Z)$

$= (\overline{X} + Y)(\overline{X} + Z) = \overline{X} + YZ$

b) $X + Y(Z + \overline{X + Z}) = X + Y(Z + \overline{X}\overline{Z}) = X + Y(Z + \overline{X})(Z + \overline{Z}) = X + YZ + \overline{X}Y$

$= (X + \overline{X})(X + Y) + YZ = X + Y + YZ = X + Y$

c) $\overline{W}X(\overline{Z} + \overline{Y}Z) + X(W + \overline{W}YZ) = \overline{W}X\overline{Z} + \overline{W}X\overline{Y}Z + WX + \overline{W}XYZ$

$= \overline{W}X\overline{Z} + \overline{W}XZ + WX = \overline{W}X + WX = X$

d) $(AB + \overline{A}\overline{B})(\overline{C}\overline{D} + CD) + \overline{AC} = AB\overline{C}\overline{D} + ABCD + \overline{A}\overline{B}CD + \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A} + \overline{C}$

$= ABCD + \overline{A} + \overline{C} = \overline{A} + \overline{C} + A(BCD) = \overline{A} + \overline{C} + C(BD) = \overline{A} + \overline{C} + BD$

Question 2-9

a) $\overline{F} = (\overline{A} + B)(A + \overline{B})$

b) $\overline{F} = ((V + \overline{W})\overline{X} + \overline{Y})Z$

c) $\overline{F} = [\overline{W} + \overline{X} + (Y + \overline{Z})(\overline{Y} + Z)][W + X + Y\overline{Z} + \overline{Y}Z]$

d) $\overline{F} = \overline{A}B\overline{C} + (A + B)\overline{C} + \overline{A}(B + C)$

Question 2-10

a) Sum of Minterms: $\overline{X}YZ + X\overline{Y}Z + XY\overline{Z} + XYZ$

Product of Maxterms: $(X + Y + Z)(X + Y + \overline{Z})(X + \overline{Y} + Z)(\overline{X} + Y + Z)$

b) Sum of Minterms: $\overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}BC + ABC$

Product of Maxterms: $(A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$

c) Sum of Minterms: $\overline{W}\overline{X}Y\overline{Z} + \overline{W}XY\overline{Z} + W\overline{X}Y\overline{Z} + WX\overline{Y}\overline{Z} + WX\overline{Y}Z + WXY\overline{Z} + WXYZ$

Product of Maxterms: $(W + X + Y + Z)(W + X + Y + \overline{Z})(W + X + \overline{Y} + \overline{Z})$
$(W + \overline{X} + Y + Z)(W + \overline{X} + Y + \overline{Z})(W + \overline{X} + \overline{Y} + \overline{Z})$
$(\overline{W} + X + Y + Z)(\overline{W} + X + Y + \overline{Z})(\overline{W} + X + \overline{Y} + \overline{Z})$

Question 2-12

a) $(AB + C)(B + \overline{C}D) = AB + AB\overline{C}D + BC = AB + BC$ s.o.p.

$= B(A + C)$ p.o.s.

b) $\overline{X} + X(X + \overline{Y})(Y + \overline{Z}) = (\overline{X} + X)(\overline{X} + (X + \overline{Y})(Y + \overline{Z}))$

$= (\overline{X} + X + \overline{Y})(\overline{X} + Y + \overline{Z})$ p.o.s.

$= (1 + \overline{Y})(\overline{X} + Y + \overline{Z}) = \overline{X} + Y + \overline{Z}$ s.o.p.

c) $(A + B\overline{C} + CD)(\overline{B} + EF) = (A + B + C)(A + B + D)(A + \overline{C} + D)(\overline{B} + EF)$

$= (A + B + C)(A + B + D)(A + \overline{C} + D)(\overline{B} + E)(\overline{B} + F)$ p.o.s.

$(A + B\overline{C} + CD)(\overline{B} + EF) = A(\overline{B} + EF) + B\overline{C}(\overline{B} + EF) + CD(\overline{B} + EF)$

$= A\overline{B} + AEF + B\overline{C}EF + \overline{B}CD + CDEF$ s.o.p.