Computer Engineering Department
ROBOTICS
LIST OF HOMEWORKS AND THEIR SOLUTION
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Homework No 1 (Due on March 12, 2008)

Bipedal Humanoid Geometric Motion

1. Simplify the following matrix expression:

   $(ROTZ(\theta_1)ROTZ^t(2\theta_1)[ROTY(\theta_2)ROTZ^t(\theta_3)]^{-1}ROTY(\theta_4))^t$

   Using the rules on rotation matrices we have

   $[ROTZ(-\theta_1)ROTZ(\theta_3)ROTY(-\theta_2)ROTY(\theta_4)]^t =$
   $[ROTZ(\theta_3-\theta_1)ROTY(\theta_4-\theta_2)]^t =$
   $ROTY(\theta_2-\theta_4)ROTZ(\theta_1-\theta_3)$**.**

2. Consider the KONDO KHR-1 which has the following five joints in each leg:

   Link-1 (Revolute (Z), Y(L1))
   Link-2 (Revolute (X), Y(L2))
   Link-3 (Revolute (X), Y(L3))
   Link-3 (Revolute (X), Y(L3))
   Link-3 (Revolute (z), Y(L3))

   where Revolute(Z) and Y(L1) means that this dof is revolute and its rotations is about its Z axis and the link itself is along the Y axis. Each link refers to its frame of reference. The Z axis is forward in the horizontal plane, the Y axis in downward along the vertical axis, and the X axis is along the right side of the robot. The first link is attached to the robot body. The last link is attached to the robot foot.

   (a) Give the general two equations (rotation and position) for iteratively finding the forward geometric model(FGM).

   (b) Apply the FGM iterative equations to the above robot arm to determine the expression of the position and orientation of each link $O_iO_{i+1,0}$ for i in $[1,..,5]$ as function of the joint parameters such as $\theta_1,...,\theta_5$. The model is to give the position and orientation of the foot frame which the fifth frame R_5 and the coordinate of its origin.

(c) Indicate a method to obtain the Inverse geometric model (IGM) for the KONDO roboto leg, i.e. given the frame R_5 orientation matrix and its origin evaluate the joint solution $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$.

(d) In order to let the KONDO walking each leg must follow a specific trajectory. Answer the following: (1) Determine the control parameters for the walking of the Kondo humanoid and explain how each parameter is to be controlled, and (2) Find how R_5 should be moved to provide the robot some walking mechanism.

(e) Generate the trajectory of the joint solution when the robot is walking for a few steps. Plot the solution and attach the graphics with your comments.

SOLUTION:

(a) The General two equations (rotation and position) for iteratively finding the forward geometric model are:

$$M_0^i = M_0^{i-1}.M_{i-1}^i$$

$$O_0O_{i,0} = O_0O_{i-1,0} + M_0^i.O_{i-1}O_{i,i}$$

(b) The geometric model for the 3 DOF arm is as follows. This robot arm is observed relative to a fixed frame of reference R_0.

The geometric model for the 5 DOF Kondo Leg is as follows. The leg dofs are observed relative to a fixed frame of reference R_0.

First step: The link $L_1 = 0$ is defined along the y_0 axis and the first degree of freedom θ_1 is revolute, the coordinates of point $O_{1,0}$ becomes:

$$M_0^1 = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$O_0O_{1,0} = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = (0,0,0)^t$$

Second step: The link L_3 is defined along the Y axis and θ_2 is revolute, the coordinates of point $O_0O_{2,0}$ becomes:

$$O_0O_{2,0} = O_0O_{1,0} + M_0^1.O_1O_{2,1} = M_0^1.O_1O_{2,1}$$

$$O_0O_{2,0} = M_0^1.O_1O_{2,1} = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L2 \\ 0 \end{bmatrix} = \begin{bmatrix} -S1L2 \\ C1L2 \\ 0 \end{bmatrix}$$

$$M_0^2 = M_0^1.M_1^2 = M_0^2 = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C2 & -S2 \\ 0 & S2 & C2 \end{bmatrix} =$$

$$\begin{bmatrix} C1 & -S1C2 & S1S2 \\ S1 & C1C2 & -C1S2 \\ 0 & S2 & C2 \end{bmatrix}$$

Third step: The link L_3 is defined along the Y_0 axis and θ_3 is revolute, the coordinates of point $O_0O_{3,0}$ becomes:

$$O_0O_{3,0} = O_0O_{2,0} + M_0^2.O_2O_{3,2}$$

$$M_0^2.O_2O_{3,2} = \begin{bmatrix} C1 & -S1C2 & S1S2 \\ S1 & C1C2 & -C1C2 \\ 0 & S2 & C2 \end{bmatrix} \begin{bmatrix} 0 \\ L3 \\ 0 \end{bmatrix} = \begin{bmatrix} -S1C2L2 \\ C1C2L2 \\ S2L2 \end{bmatrix}$$

$$O_0O_{3,0} = \begin{bmatrix} -S1L2 \\ C1L2 \\ 0 \end{bmatrix} + \begin{bmatrix} -S1C2L3 \\ C1C2L3 \\ S2L3 \end{bmatrix} = \begin{bmatrix} -S1L2 - S1C2L3 \\ C1L2 + C1C2L3 \\ S2L3 \end{bmatrix}$$

$$M_0^3 = M_0^2.M_2^3 = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C2 & -S2 \\ 0 & S2 & C2 \end{bmatrix} =$$

$$\begin{bmatrix} C1 & -S1C23 & S1S23 \\ S1 & C1C23 & -C1S23 \\ 0 & S23 & C23 \end{bmatrix}$$

Fourth step: The link L_4 is defined along the Y axis and θ_4 is revolute, the coordinates of point $O_0O_{4,0}$ becomes:

$$O_0O_{4,0} = O_0O_{3,0} + M_0^3.O_3O_{4,3}$$

$$O_0O_{4,0} = \begin{bmatrix} -S1(L2 + C2L3 + C23L4) \\ C1(L2 + C2L3 + C1C23L4) \\ S2L3 + S23L4 \end{bmatrix}$$

$$M_0^4 = M_0^3.M_3^4 = \begin{bmatrix} C1 & -S1C234 & S1S234 \\ S1 & C1C234 & -C1S234 \\ 0 & S234 & C234 \end{bmatrix}$$

Fifth step: The link L_5 is defined along the Y axis and θ_5 is revolute, the coordinates of point $O_0O_{5,0}$ becomes:

$$O_0O_{5,0} = O_0O_{4,0} + M_0^4.O_4O_{5,4}$$

$$O_0O_{5,0} = \begin{bmatrix} -S1(L2 + C2L3 + C23L4 + C234L5) \\ C1(L2 + C2L3 + C23L4 + C234L5) \\ S2L3 + S23L4 + S234L5 \end{bmatrix}$$

$$M_0^5 = M_0^4.M_4^5 = \begin{bmatrix} C1 & -S1C234 & S1S234 \\ S1 & C1C234 & -C1C234 \\ 0 & S234 & C234 \end{bmatrix} \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C1C5 - S1C234S5 & -C1S5 - S1C234C5 & S1S234 \\ S1C5 + C1C2S5 & S1S5 + C1C234C5 & -C1S234 \\ S234S5 & S234C5 & C234 \end{bmatrix}$$

(c) **The Inverse geometric model (IGM) for the KONDO roboto leg consists of finding the expression of solution of the joint vector $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$ given the frame R_5 orientation matrix and its origin $O0O5,0$. It consists of solving a non-linear system equation like:**

$$O_0O_{5,0} = (X, Y, Z)^t = \begin{bmatrix} -S1(L2 + C2L3 + C23L4 + C234L5) \\ C1(L2 + C2L3 + C23L4 + C234L5) \\ S2L3 + S23L4 + S234L5 \end{bmatrix}$$

We notice that $X^2 + Y^2 = (L2 + C2L3 + C23L4 + C234L5)^2$, thus we can evaluate $S1 = -X/sqrt(X^2+Y^2)$ and $C1 = -Y/sqrt(X^2+Y^2)$ which allows finding θ_1 as function of X, Y, and Z. Once the θ_1 is found we can move on and reduce the system equation by eliminating the known expressions and find the other 4 DOFs.

(d) **The control parameters for the walking of the Kondo humanoid are θ_1 which control the inclined angle of the body so that the Center of Mass is always projected on a posed foot. Angle θ_2 is useful to control the direction of the extension $E = O_2O_4$ which allows moving a leg by polling it from the floor moving forward one step by increasing θ_2.The angle θ_3 controls the extension $E = O_2O_4$. The angle θ_4 and θ_5 allow controlling the pose of the foot on the field. The above parameters allow controlling the robot walking mechanism.**

(e) **Please see the instructor notes**
http://faculty.kfupm.edu.sa/COE/mayez/ps-coe484/core/Picture%20016.jpg
for the generation of the trajectory of the joint solution during a walk of a few steps. An intuitive solution plot is shown.