COMPUTER ENGINEERING DEPARTMENT

INTRODUCTION TO ROBOTICS **EXAM 1**
COE 484 **March 15, 2008**

GRADING

Problems	A	B	C	Total/10
Grades				

Name:____________________________-

I.D.:____________________________-

A - FRAME OF REFERENCES AND ROTATION MATRICES

Simplify the following matrix expression:

$$M = [(ROTZ^t(\theta_1)ROTZ(2\theta_1)]^t ROTZ(\theta_2)ROTY^t(\theta_3)ROTY(\theta_4)$$

Using the rules on rotation matrices we have

$$M = [ROTZ(-\theta_1)ROTZ(2\theta_1)]^t ROTZ(\theta_2)ROTY(-\theta_3)ROTY(\theta_4) =$$
$$[ROTZ(\theta_1)]^t ROTZ(\theta_2)ROTY(\theta_4 - \theta_3) =$$
$$ROTZ(-\theta_1)ROTZ(\theta_2)ROTY(\theta_4 - \theta_3) =$$
$$ROTZ(\theta_2 - \theta_1)ROTY(\theta_4 - \theta_3).$$

B - FIELD, ROBOT AND CAMERA GEOMETRY

The center of a soccer **Field** has a fixed frame of reference F_f which is defined by its origin O_f and its orthogonal unit vectors X_f, Y_f, and Z_f. A humanoid robot and its camera are known at any time by the following vector and rotation matrices:

- The robot is moving over the above field and its frame of reference F_h is defined by the position of its origin $O_fO_{h,f}$ and its rotation matrix M_f^h which are both regularly updated with respect to field frame F_f.
- The humanoid holds a Camera having a frame of reference F_c which is defined by the position of its origin $O_hO_{c,h}$ and its rotation matrix M_h^c, are both regularly updated with respect to the humanoid frame F_h.

Answer each of following questions:

1. Suppose the robot hand position is $A_{/R_h}$ with respect to the robot frame R_h. Evaluate $O_fA_{/R_f}$ which is the position of hand with respect to field frame R_f.

   The vector $O_fA_{/R_f}$ is the position of hand with respect to field frame R_f which is defined by $O_fA_{/R_f} = O_fO_{h,f} + M_f^h A_{/R_h}$.

2. Evaluate the overall Camera rotation matrix M_h^c with respect to field frame R_f.
   The overall Camera rotation matrix M_h^c with respect to field frame R_f is defined by $M_f^c = M_f^h M_h^c$.

3. Suppose the Camera detect a ball at a location $B_{/R_c}$ with respect to the Camera frame R_c. Evaluate $O_fB_{/R_f}$ which is the position of ball with respect to field frame R_f.

   The vector $O_fB_{/R_f}$ is the position of ball with respect to field frame R_f which is defined by $O_fB_{/R_f} = O_fO_{h,f} + M_f^h M_h^c O_h B_{/R_c}$.

4. The goal edge E is located at a position $O_fE_{/R_f}$ with respect to the Field frame R_f. What is the position of E in the Camera frame F_c, i.e. evaluate vector $O_cE_{/R_c}$.

   The vector $O_fE_{/R_f}$ is the position of goal edge E with respect to field frame R_f which is defined by $O_fE_{/R_f} = O_fO_{h,f} + M_f^h M_h^c O_c E_{/R_c}$. Therefore, $O_fE_{/R_f} O_fO_{h,f} = M_f^h M_h^c O_c E_{/R_c}$ and $O_cE_{/R_c} = [M_f^h M_h^c]^{-1}(O_fE_{/R_f} - O_fO_{h,f} = M_c^f (O_fE_{/R_f} - O_fO_{h,f})$.

B - HUMANOID WALKING MOTION

The KONDO KHR-1 humanoid robot has five joints in each leg:

Link-1 (Revolute (Z), Y(L1=0))
Link-2 (Revolute (X), Y(L2))
Link-3 (Revolute (X), Y(L3))
Link-4 (Revolute (X), Y(L4))
Link-5 (Revolute (Z), Y(L5))

The above notation is described as follows:

- The first link denoted as Link-1 (Revolute (Z), Y(L1)) indicates that frame F_1 is Revolute(Z) and its rotations is about its Z axis and Y(L1) means that this link extends along the Y axis.
- Each link refers to its frame of reference.
- The Z axis is forward in the horizontal plane, the Y axis in downward along the vertical axis, and the X axis is along the right side of the robot.
- The first link is attached to the robot body. The last link is attached to the robot foot.
- The origin of robot body frame F_0 coincide with the origin of link 1 frame F_1, i.e. $L1 = 0$ and $O_0O_{1,0} = (0, 0, 0)^t$.

Answer each of following questions:

1. Evaluate the expression giving the position of frame origin $O_0O_{2,0}$, $O_0O_{3,0}$, and $O_0O_{4,0}$ with respect to body frame F_0 as function of $L2$, $L3$, $L4$ and the joint variables θ_1, θ_2, θ_3, and θ_4.

   The geometric model for the 5 DOF Kondo Leg is as follows. This leg dofs are observed relative to a fixed frame of reference R_0.
   First step: The link $L_1 = 0$ is defined along the Z_0 axis and the first degree of freedom θ_1 is revolute, the coordinates of point $O_{1,0}$ becomes:

$$O_0O_{1,0} = (0, 0, 0)^t$$

$$M_0^1 = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   Second step: The link L_3 is defined along the Y axis and θ_2 is revolute, the coordinates of point $O_0O_{2,0}$ becomes:

$$O_0O_{2,0} = O_0O_{1,0} + M_0^1.O_1O_{2,1} = M_0^1.O_1O_{2,1}$$

$$O_0O_{2,0} = M_0^1.O_1O_{2,1} = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L2 \\ 0 \end{bmatrix} = \begin{bmatrix} -S1L2 \\ C1L2 \\ 0 \end{bmatrix}$$

$$M_0^2 = M_0^1.M_1^2 = M_0^2 = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C2 & -S2 \\ 0 & S2 & C2 \end{bmatrix} =$$

$$\begin{bmatrix} C1 & -S1C2 & S1S2 \\ S1 & C1C2 & -C1C2 \\ 0 & S2 & C2 \end{bmatrix}$$

Third step: The link L_3 is defined along the Y_0 axis and θ_3 is revolute, the coordinates of point $O_0O_{3,0}$ becomes:

$$O_0O_{3,0} = O_0O_{2,0} + M_0^2.O_2O_{3,2}$$

$$M_0^2.O_2O_{3,2} = \begin{bmatrix} C1 & -S1C2 & S1S2 \\ S1 & C1C2 & -C1C2 \\ 0 & S2 & C2 \end{bmatrix} \begin{bmatrix} 0 \\ L3 \\ 0 \end{bmatrix} = \begin{bmatrix} -S1C2L2 \\ C1C2L2 \\ S2L2 \end{bmatrix}$$

$$O_0O_{3,0} = \begin{bmatrix} -S1L2 \\ C1L2 \\ 0 \end{bmatrix} + \begin{bmatrix} -S1C2L3 \\ C1C2L3 \\ S2L3 \end{bmatrix} = \begin{bmatrix} -S1L2 - S1C2L3 \\ C1L2 + C1C2L3 \\ S2L3 \end{bmatrix}$$

$$M_0^3 = M_0^1.M_1^2M_2^3 = M_0^2.M_2^3 = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C2 & -S2 \\ 0 & S2 & C2 \end{bmatrix} =$$

$$\begin{bmatrix} C1 & -S1C23 & S1S23 \\ S1 & C1C23 & -C1C23 \\ 0 & S23 & C23 \end{bmatrix}$$

Fourth step: The link L_4 is defined along the Y axis and θ_4 is revolute, the coordinates of point $O_0O_{4,0}$ becomes:

$$O_0O_{4,0} = O_0O_{3,0} + M_0^3.O_3O_{4,3}$$

$$O_0O_{4,0} = \begin{bmatrix} -S1(L2 + C2L3 + C23L4) \\ C1(L2 + C2L3 + C1C23L4) \\ S2L3 + S23L4 \end{bmatrix}$$

$$M_0^4 = M_0^1.M_1^2M_2^3M_3^4 = M_0^3.M_3^4 = \begin{bmatrix} C1 & -S1C234 & S1S234 \\ S1 & C1C234 & -C1C234 \\ 0 & S234 & C234 \end{bmatrix}$$

Fifth step: The link L_5 is defined along the Y axis and θ_5 is revolute, the coordinates of point $O_0O_{5,0}$ becomes:

$$O_0O_{5,0} = O_0O_{4,0} + M_0^4.O_4O_{5,4}$$

$$O_0O_{5,0} = \begin{bmatrix} -S1(L2 + C2L3 + C23L4 + C234L5) \\ C1(L2 + C2L3 + C1C23L4 + C234L5) \\ S2L3 + S23L4 + S234L5 \end{bmatrix}$$

$$M_0^5 = M_0^4.M_4^5 = \begin{bmatrix} C1 & -S1C234 & S1S234 \\ S1 & C1C234 & -C1C234 \\ 0 & S234 & C234 \end{bmatrix} \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C1C5 - S1C2S5 & -C1S5 - S1C2C5 & S1S2 \\ S1C5 + C1C2S5 & S1S5 + C1C2C5 & -C1S2 \\ S2S5 & S2C5 & C2 \end{bmatrix}$$

2. Evaluate the length of the leg extension going from O_2 to O_4 by: (1) finding the expression of vector $O_2O_{4,1}$, and (2) evaluate the magnitude of vector $O_2O_{4,1}$, (3) show that the only variable that affect the above magnitude is θ_3.

   The length of the leg extension going from O_2 to O_4 can be evaluated by finding the expression of vector $O_2O_{4,1}$, and the magnitude of vector $O_2O_{4,1}$. Using the analysis of the previous question, we have

$$O_0O_{5,0} = O_0O_{4,0} + M_0^4.O_4O_{5,4}$$

$$O_2O_{4,1} = \begin{bmatrix} 0 \\ C2L3 + C23L4 \\ S2L3 + S23L4 \end{bmatrix}$$

   This allows finding the leg $E^2 = L_2^2 + L_3^2 + 2L_2L_3C3$ which indicates that the only variable that affect its magnitude is θ_3.

3. How one can control the motion of the above robot as closely as possible to human walking.

   The two leg control is based on moving the center of mass (CM) of the humanoid body from one leg to another. The walking is based on the following two simultaneous steps: (1) initially CM is on the posed leg and start moving toward the destination motion of the second leg, and (2) the other leg is in the air and moving forward for one walking step. Some synchronization is needed to make sure that at the time that CM will reach the destination position which projects on the other leg motion destination at the pose. This represent a cyclic forward walking scheme.