

# King Fahd University of Petroleum & Minerals Computer Engineering Dept

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COE 541 – Design and Analysis of  
Local Area Networks

Term 041

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# Revision – Fourier Transform

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- A “transformation” between the time domain and the frequency domain

**Time (t)**                      **Frequency (f)**  
**s(t)**                      **← →**                      **S(f)**

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt$$

Fourier Transform

$$s(t) = \int_{-\infty}^{\infty} S(f) e^{+j2\pi ft} df$$

Inverse Fourier Transform

# Revision – Fourier Transform (2)

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- **F.T. can be used to find the BANDWIDTH of a signal or system**
  - **Bandwidth - system: range of frequencies passed (perhaps scaled) by system**
  - **Bandwidth – signal: range of (+ve) frequencies contained in the signal**

# Revision – Fourier Transform (3)

- **Remember for periodic signals (i.e.  $s(t) = s(t+T)$  where  $T$  is the period) → Fourier Series expansion:**

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)]$$

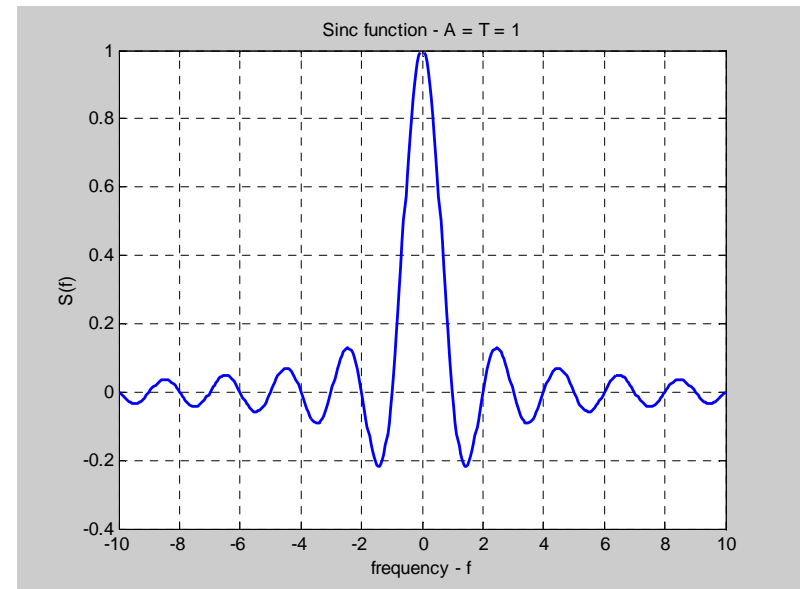
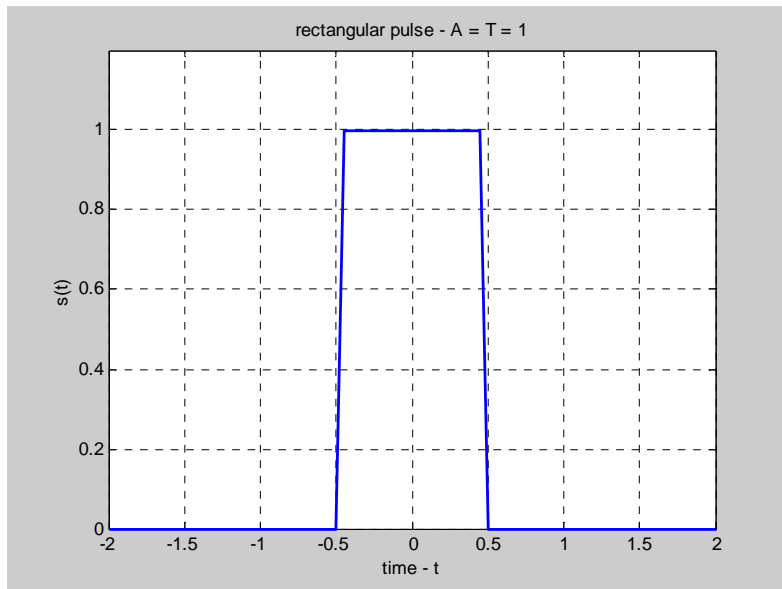
$$A_0 = \frac{2}{T} \int_0^T s(t) dt \quad B_n = \frac{2}{T} \int_0^T s(t) \sin(2\pi n f_0 t) dt$$

$$A_n = \frac{2}{T} \int_0^T s(t) \cos(2\pi n f_0 t) dt$$

$f_0$  is the fundamental frequency and is equal to  $1/T$

# Revision – Fourier Transform (4)

- **Famous pairs – rectangular pulse ( A = T = 1 )**



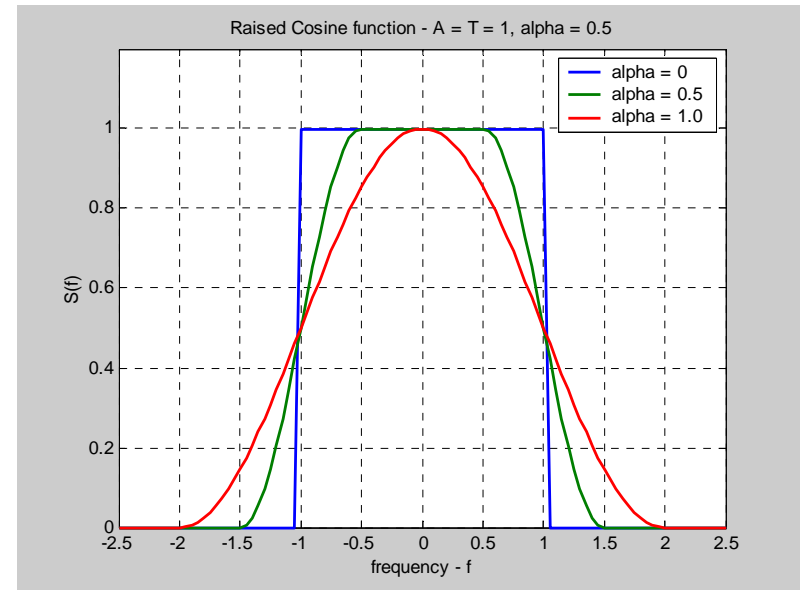
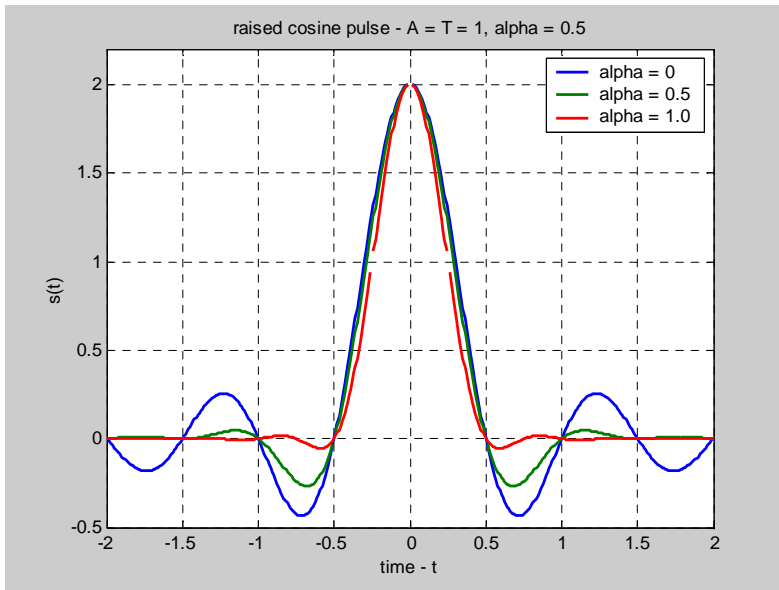
$$s(t) = \Pi(t / T)$$

$$S(f) = AT \frac{\sin(\pi f T)}{\pi f T}$$

$$\begin{aligned} S(f) &= AT \text{ for } f = 0 \\ &= 0 \text{ for } f = n/T; n = \pm 1, 2, \dots \end{aligned}$$

# Revision – Fourier Transform (5)

- **Famous pairs – Raised Cosine pulse ( A = T = 1), as a function of  $\alpha$**



$$s(t) = \frac{(2A)}{T} \frac{\cos(2\pi\alpha t)}{1 - (4\alpha t)^2} \frac{\sin(2\pi t / T)}{2\pi t / T}$$

$$S(f) = \begin{cases} A & |f| > \frac{1}{T} - \alpha \\ A \cos^2\left(\frac{\pi}{4\alpha}\left(|f| - \frac{1}{T} + \alpha\right)\right) & \frac{1}{T} - \alpha < |f| < \frac{1}{T} + \alpha \\ 0 & |f| > \frac{1}{T} + \alpha \end{cases}$$

# Revision – Fourier Transform (6)

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- **Raised Cosine Pulse:  $0 < \alpha < 1/T$**
- **Note that  $s(t) = 0$  for  $t = nT/2$  where  $n = +/- 1, 2, \dots$** 
  - **Very good for forming pulses**
  - **ZERO ISI for ideal situation**
- **BW for  $s(t) = 1/T + \alpha$** 
  - **Maximum =  $2 \times 1/T$  (for  $\alpha = 1/T$ )**
  - **Minimum =  $1/T$  (for  $\alpha = 0$ )**

# Revision – Fourier Transform (7)

- **Matlab code:**

```
clear all % clear all variables
```

```
A = 1;  
T = 1;  
alphas = [0 0.5 1];
```

```
for k = 1:length(alphas)  
    alpha = alphas(k);  
  
    t = -2:0.01:2; % define the time axis  
    s_t(k,:) = ((2*A)/T) * (cos(2*pi*alpha*t)./ ...  
        (1-(4*alpha*t).^2)) .* (sin(2*pi*t/T)./ ...  
        (2*pi*t/T)); % define s(t)  
  
    f = -2.5:0.05:2.5; % define the freq axis  
    S_f(k,:) = zeros(size(f));  
    i = find(abs(f) <= (1/T-alpha));  
    S_f(k,i) = A;  
    i = find((abs(f) <= (1/T+alpha)) & ...  
        (abs(f) > (1/T-alpha)));  
    S_f(k,i) = A*(cos(pi/(4*alpha)* ...  
        (abs(f(i))-1/T+alpha))).^2;% define S(f)  
end
```

```
figure(1);  
plot(t, s_t); % plot s(t)  
title('raised cosine pulse - A = T = 1');  
xlabel('time - t');  
ylabel('s(t)');  
legend('alpha = 0', 'alpha = 0.5', 'alpha = 1.0');  
axis([-2 2 -0.5 2.2]);  
grid
```

```
figure(2);  
plot(f, S_f); % plot S(f)  
title('Raised Cosine function - A = T = 1');  
xlabel('frequency - f');  
ylabel('S(f)');  
legend('alpha = 0', 'alpha = 0.5', 'alpha = 1.0');  
axis([-2.5 2.5 0 1.2]);  
grid
```



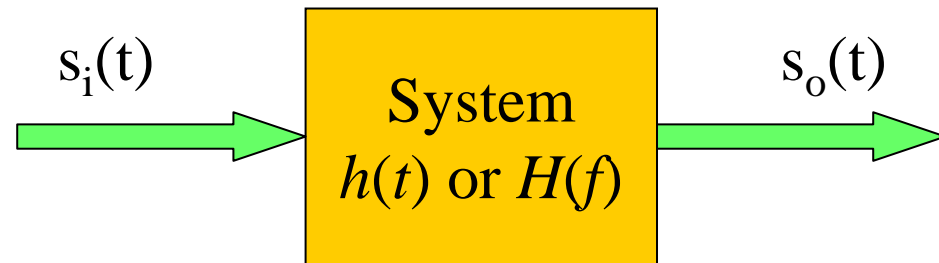
# Signals and Systems

- **For linear Systems:**
  - **$h(t)$  is the system's impulse response – i.e.  $s_o(t) = h(t)$  when  $s_i(t) = \delta(t)$**
  - **$S_i(t)$  is system input signal**
  - **$S_o(t)$  is system output signal**

$$s_o(t) = \int_{-\infty}^{\infty} s_i(\tau) h(t - \tau) d\tau$$

$$s_o(t) = s_i(t) * h(t)$$

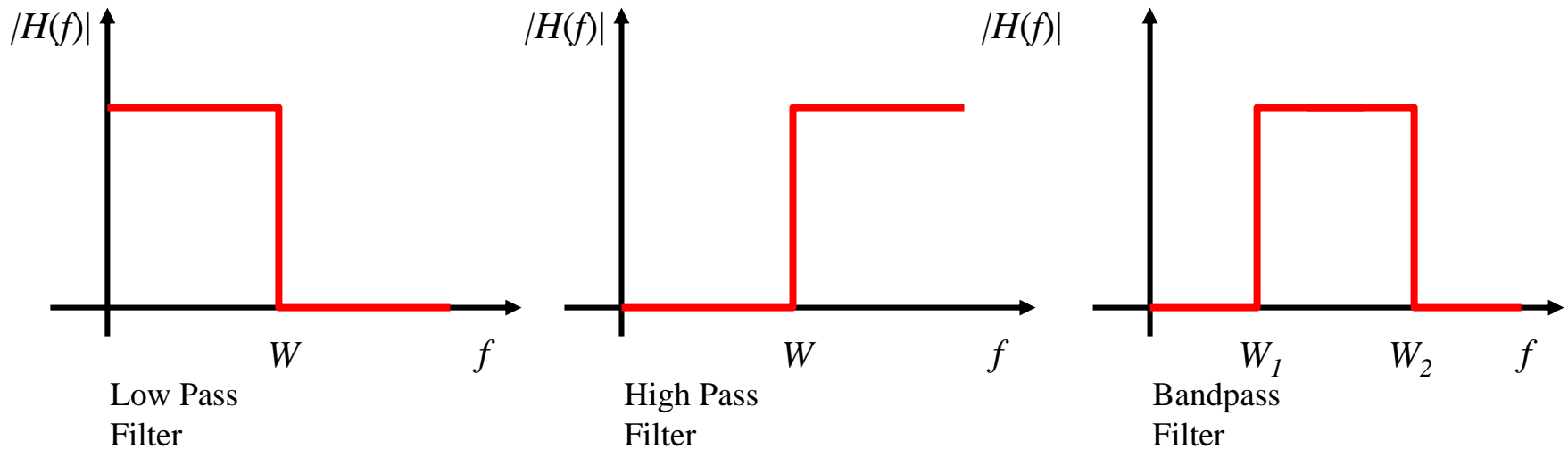
$$S_o(f) = S_i(f) H(f)$$



A good introduction into linear systems is found at  
[http://www.ece.utexas.edu/~bevans/courses/ee313/lectures/04\\_Convolution/lecture4.pdf](http://www.ece.utexas.edu/~bevans/courses/ee313/lectures/04_Convolution/lecture4.pdf)

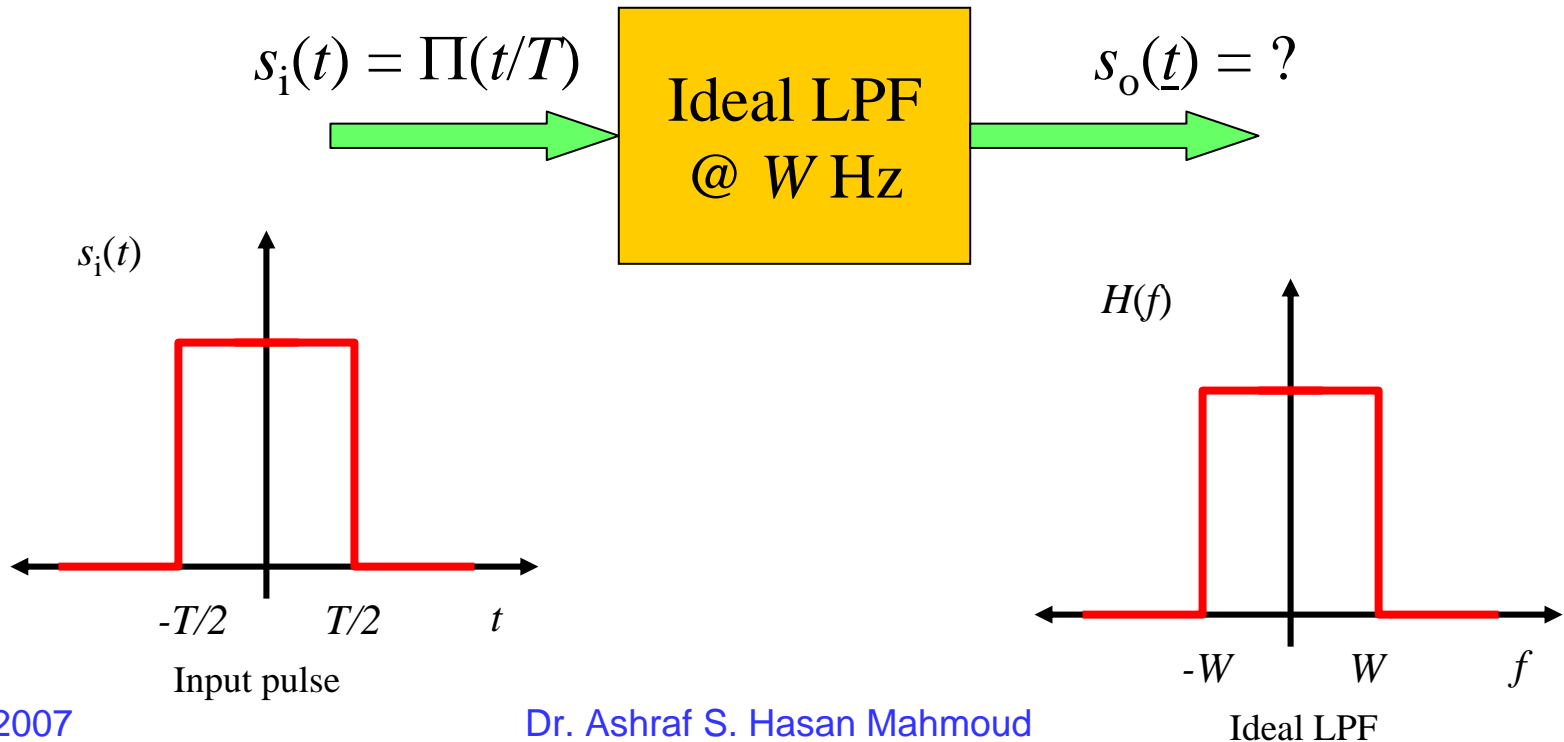
# Signals and Systems (2)

- **System bandwidth is determined by examining the Fourier transfer of the system function  $h(t)$ ,  $H(f)$**
- **Example (transmission) systems:**



# Signals and Systems - Example

- **Ideal Low Pass Filter – find the output signal for rectangular input pulse?**



# Signals and Systems - Example

- The input signal  $s_1(t)$  is given by:

$$s_1(t) = \begin{cases} A & |t| \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

- Where as its Fourier transform  $S_1(f)$  is given by (note that  $s_1(t)$  contains all frequencies from 0 till  $\infty$  - refer to Fourier transform of rectangular pulse):

$$S_1(f) = AT \frac{\sin(\pi fT)}{\pi fT} \quad \text{for all } f$$

- The Fourier transform of the system impulse response,  $H(f)$  is given by (note this transmission system limits frequencies to at most  $W$  Hz):

$$H(f) = \begin{cases} 1 & |f| \leq W \\ 0 & \text{otherwise} \end{cases}$$

# Signals and Systems - Example

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- Therefore the Fourier transform of the output signal is given by:
- $|S_o(f)| = |S_i(f)| \times |H(f)|$

$$\begin{aligned} & \sin(\pi fT) \\ &= AT \frac{\sin(\pi fT)}{\pi fT} \quad \text{for } |f| < W \\ &= 0 \quad \text{otherwise} \end{aligned}$$

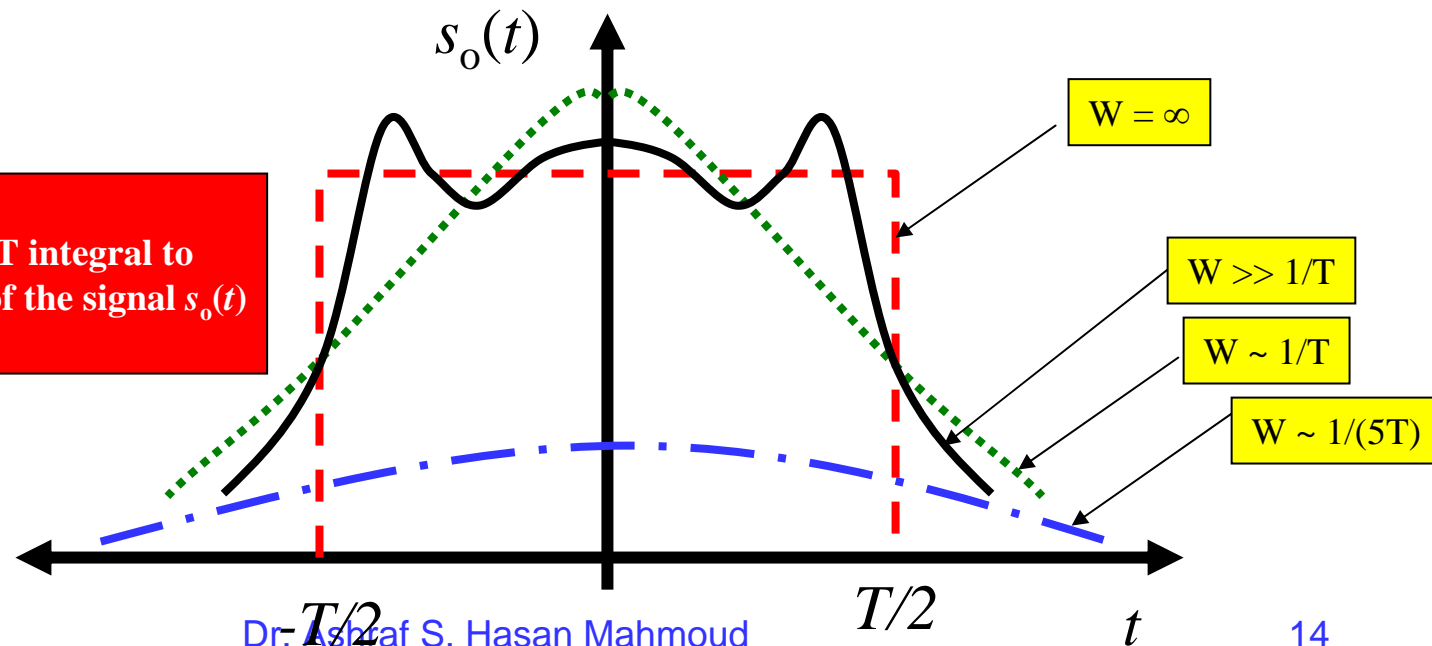
**(note the output signal has frequencies up to W Hz only)**

# Signals and Systems - Example

- To find the output signal  $s_o(t)$ , one has to use the inverse Fourier transform on  $S_o(f)$
- As the BW of the system is increased, the output signal approaches a rectangular pulse (copy of input)

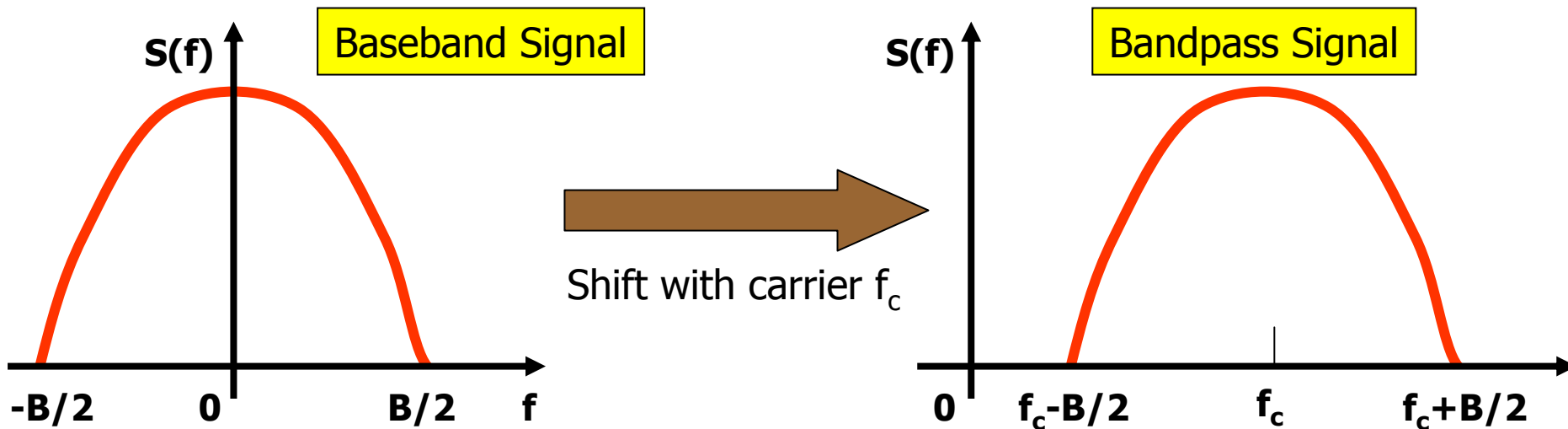
**Exercise:**

Try to apply the inverse F.T integral to obtain an analytical form of the signal  $s_o(t)$  as a function of  $W$  and  $T$ .



# Baseband vs. Bandband

- **Baseband Signal:**
  - Spectrum not centered around non zero frequency
  - May have a DC component
- **Bandpass Signal:**
  - Does not have a DC component
  - Finite bandwidth around or at  $f_c$



# Modulation

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- **Is used to shift the frequency content of a baseband signal**
  - **Basis for AM modulation**
  - **Basis for Frequency Division Multiplexing (FDM)**

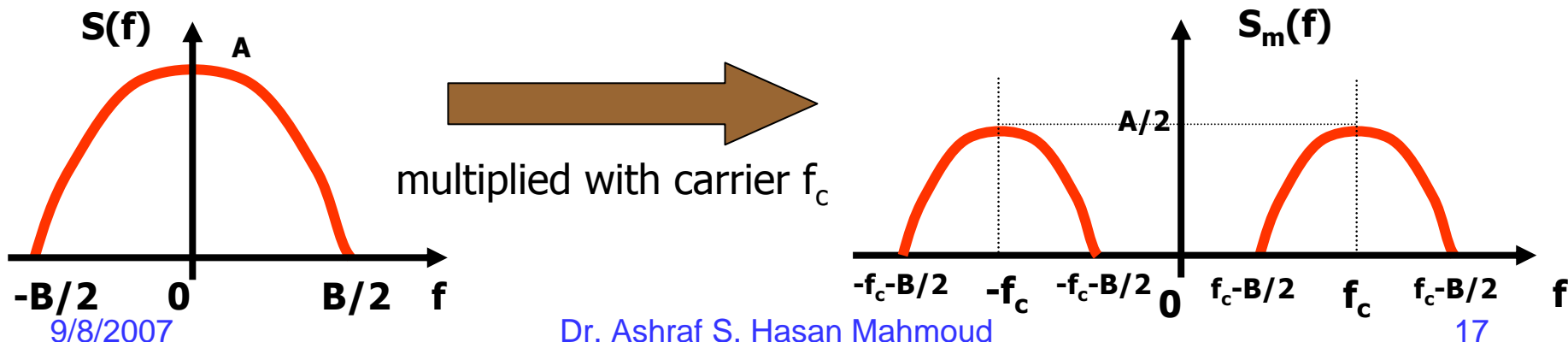


# Modulation

- Consider the signal  $s(t)$ ,  

$$s_m(t) = s(t) \times \cos(2\pi f_c t)$$
 The spectrum for  $s_m(t)$  is given by

$$S_m(f) = \frac{1}{2} \times \{S(f-f_c) + S(f+f_c)\}$$



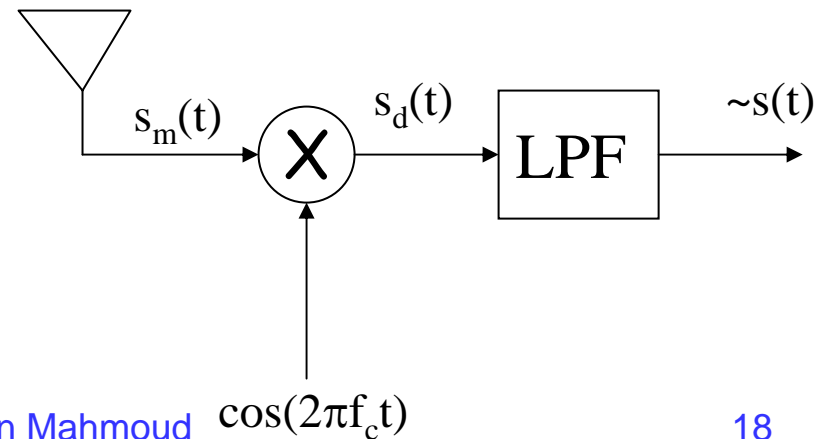
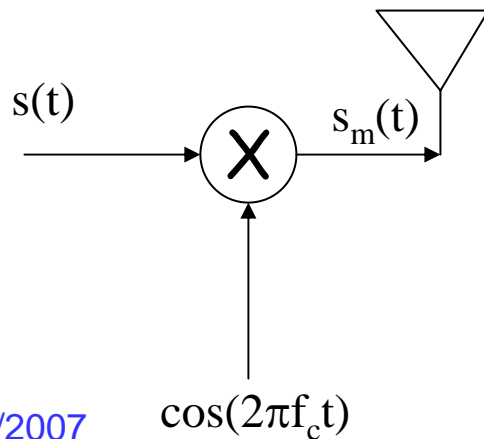
# Modulation – Txer/Rxer

- At the receiver side:

$$\begin{aligned}
 s_d(t) &= s_m(t) \times \cos(2\pi f_c t) \\
 &= s(t) \times \cos(2\pi f_c t) \times \cos(2\pi f_c t) \\
 &= \underbrace{\frac{1}{2} s(t)}_{\text{desired term}} + \underbrace{\frac{1}{2} s(t) \times \cos(2\pi 2f_c t)}_{\text{undesired term – signal centered around } 2f_c}
 \end{aligned}$$

desired term

undesired term – signal centered around  $2f_c$   
 filtered out using the LPF



# Nyquist Bandwidth

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- **For a noiseless channels of bandwidth  $B$ , the maximum attainable bit rate (or capacity) is given by**

$$C = 2B \log_2(M)$$

**Where  $M$  is the size of the signaling set**

# Shannon Capacity

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- **Capacity of a channel of bandwidth  $B$ , in the presence of noise is given by**

$$C = B \log_2(1 + \text{SNR})$$

**where SNR is the ratio of signal power to noise power – a measure of the signal quality**

# Example: Shannon Capacity

- Consider a GSM system with BW = 200 kHz. If SNR is equal to 15 dB, find the channel capacity?
- Solution:

$$\text{SNR} = 15 \text{ dB} = 10^{(15/10)} = 31.6$$

$$\begin{aligned} C &= 200 \times 10^3 \times \log_2(1+31.6) \\ &= 1005.6 \text{ kb/s} \end{aligned}$$

**Note GSM operates at 273 kb/s which is ~27% of maximum capacity at SNR = 30 dB.**

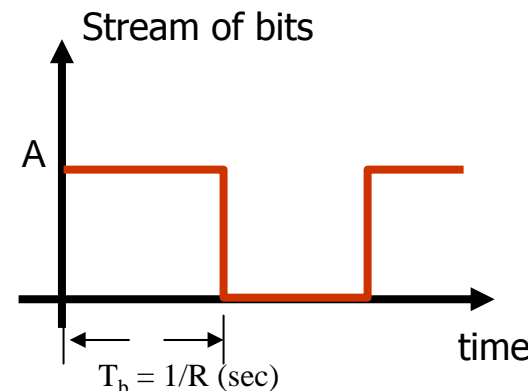
# E<sub>b</sub>/N<sub>0</sub> Expression

- An alternative representation of SNR
- Consider the bit stream shown in figure – for bit of rate R, then each bit duration is equal to  $T_b = 1/R$  seconds
- Energy of signal for the bit duration is equal to  $A^2 \times T_b$ , where its power is equal to bit energy /  $T_b$  or  $A^2$ .
- Noise power is equal to  $N_0 \times B$  (refer to thermal noise section)
- Hence, SNR is given by signal power / noise power or

$$SNR = \frac{\text{signal power}}{N_0 B} = \frac{E_b}{N_0} \times \frac{R}{B}$$

- One can also write

$$\left( \frac{E_b}{N_0} \right)_{dB} = \text{Signal Power (dBW)} - 10 \log R - 10 \log k - 10 \log T$$



# Signal Elements or Pulses

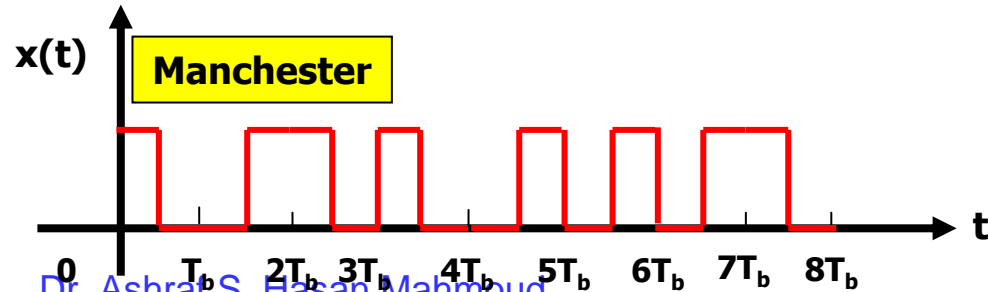
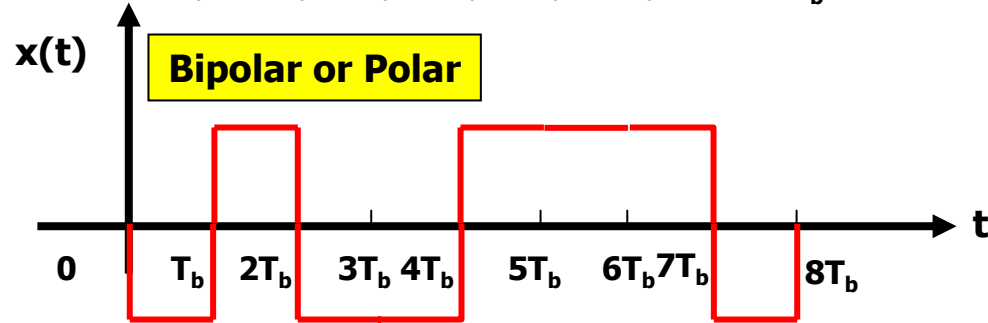
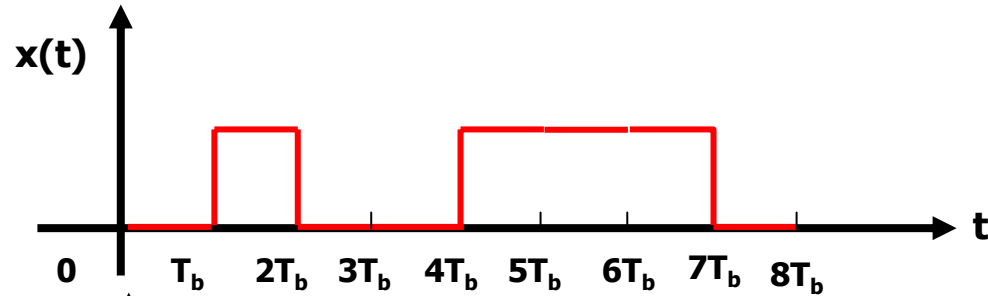
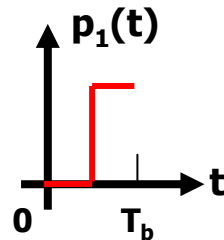
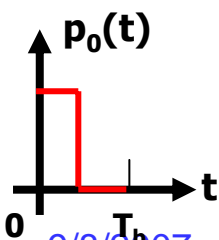
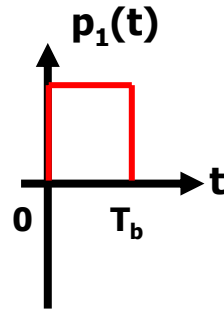
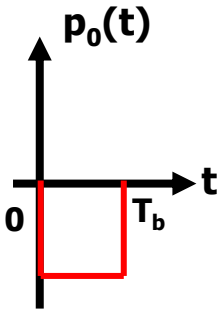
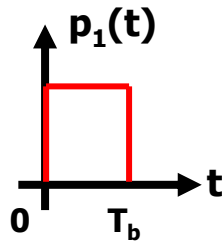
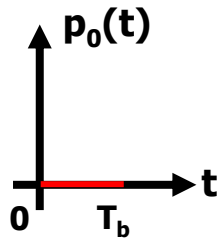
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- **Unit of transmission – repeated to form the overall signal**
- ***Shape* of pulse determines the bandwidth of the transmitted signal**
- **Digital data is mapped or encoded to the different pulses or units of transmission**
- **Baud/Modulation or Symbol Rate ( $R_s$ )**
  - **The bit rate  $R_b = R_s \log_2(M)$**
- **Please refer to earlier examples of pulses and the corresponding BW**

# Signal Elements or Pulses

## Definitions of Pulses

## Encoded Signal: 0 1 0 0 1 1 1 0



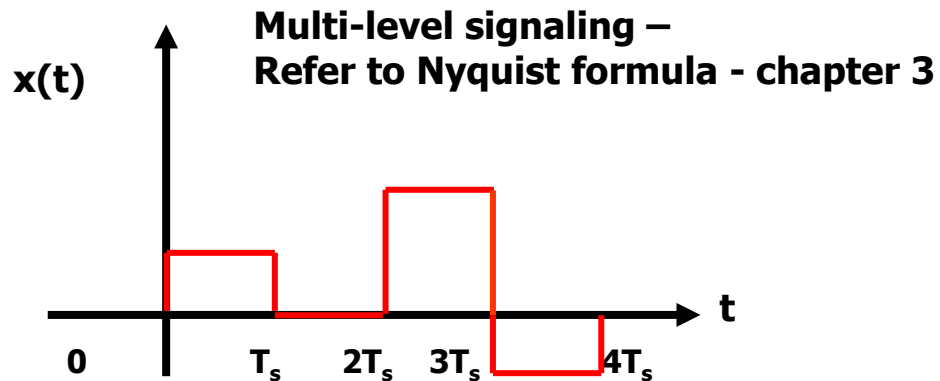
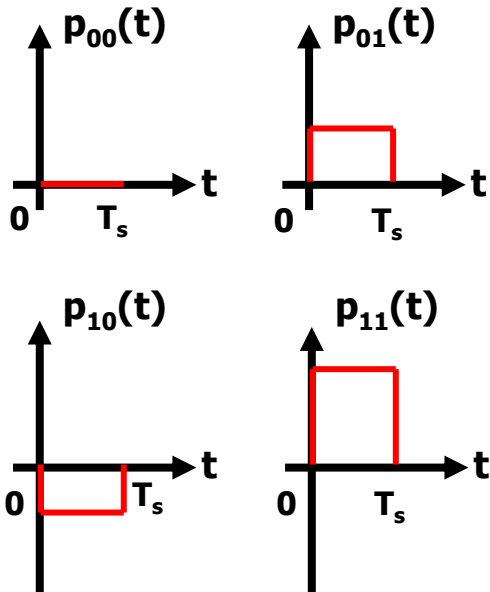
Examples of Digital Signaling



# Signal Elements or Pulses

## Pluses Definitions

## Encoded Signal: 0 1 0 0 1 1 1 0

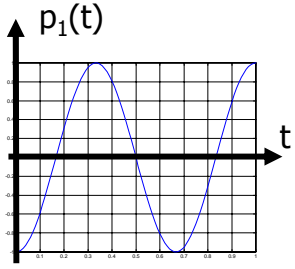
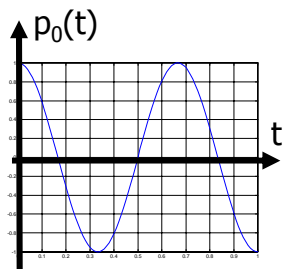
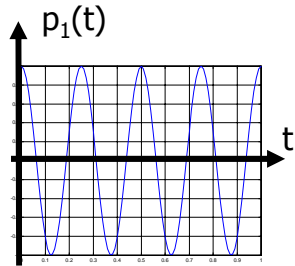
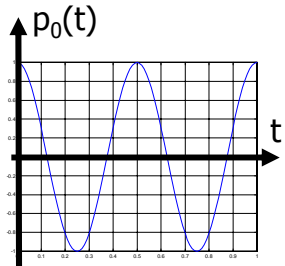
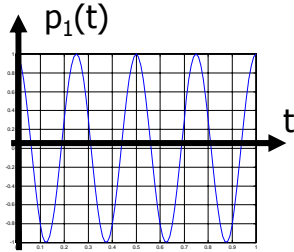
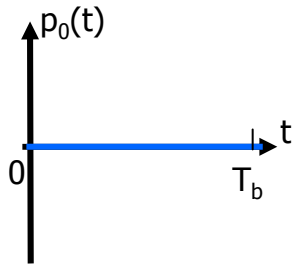


Example of Digital Signaling

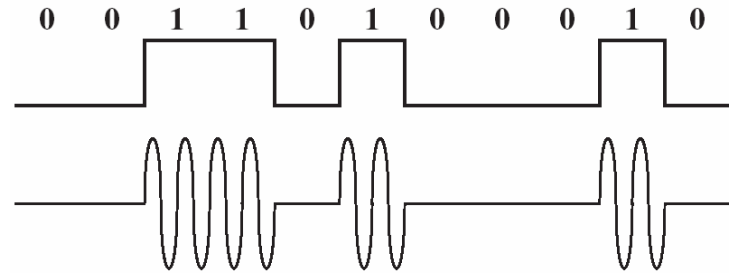
- Note that each symbol or pulse carries 2 bits
- Symbol duration is  $T_s = 2T_b$
- Bit rate  $R$  equal to  $1/T_b$
- Symbol rate or *baud rate*  $R_s$  equal to  $1/T_s \rightarrow R = 2R_s$
- In general to encode  $n$  bits per pulse, you need  $2^n$  pulses

# Signal Elements or Pulses

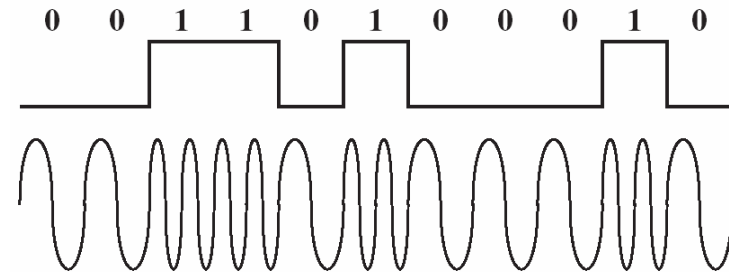
## Definitions of Pulses



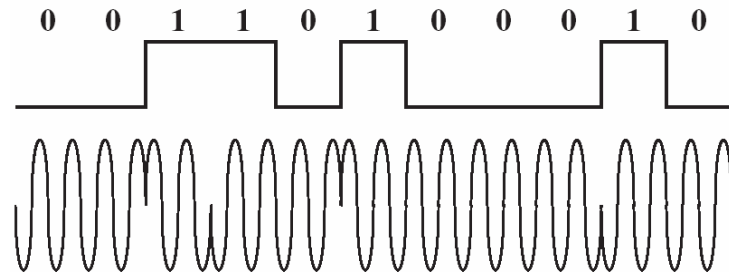
## Encoded Signal:



Amplitude-shift keying



Frequency-shift keying



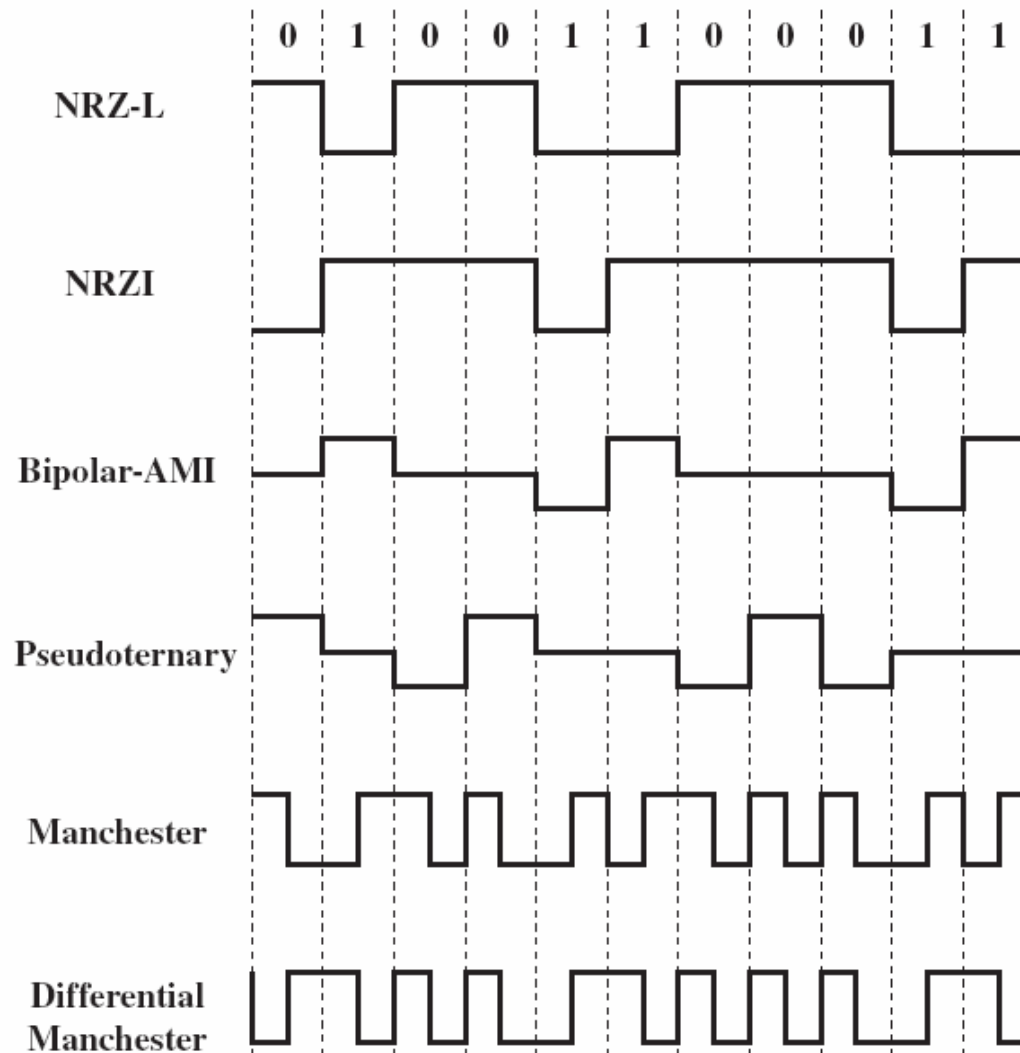
Phase-shift keying

Example of Analog Signaling

# Digital Signal Encoding Formats

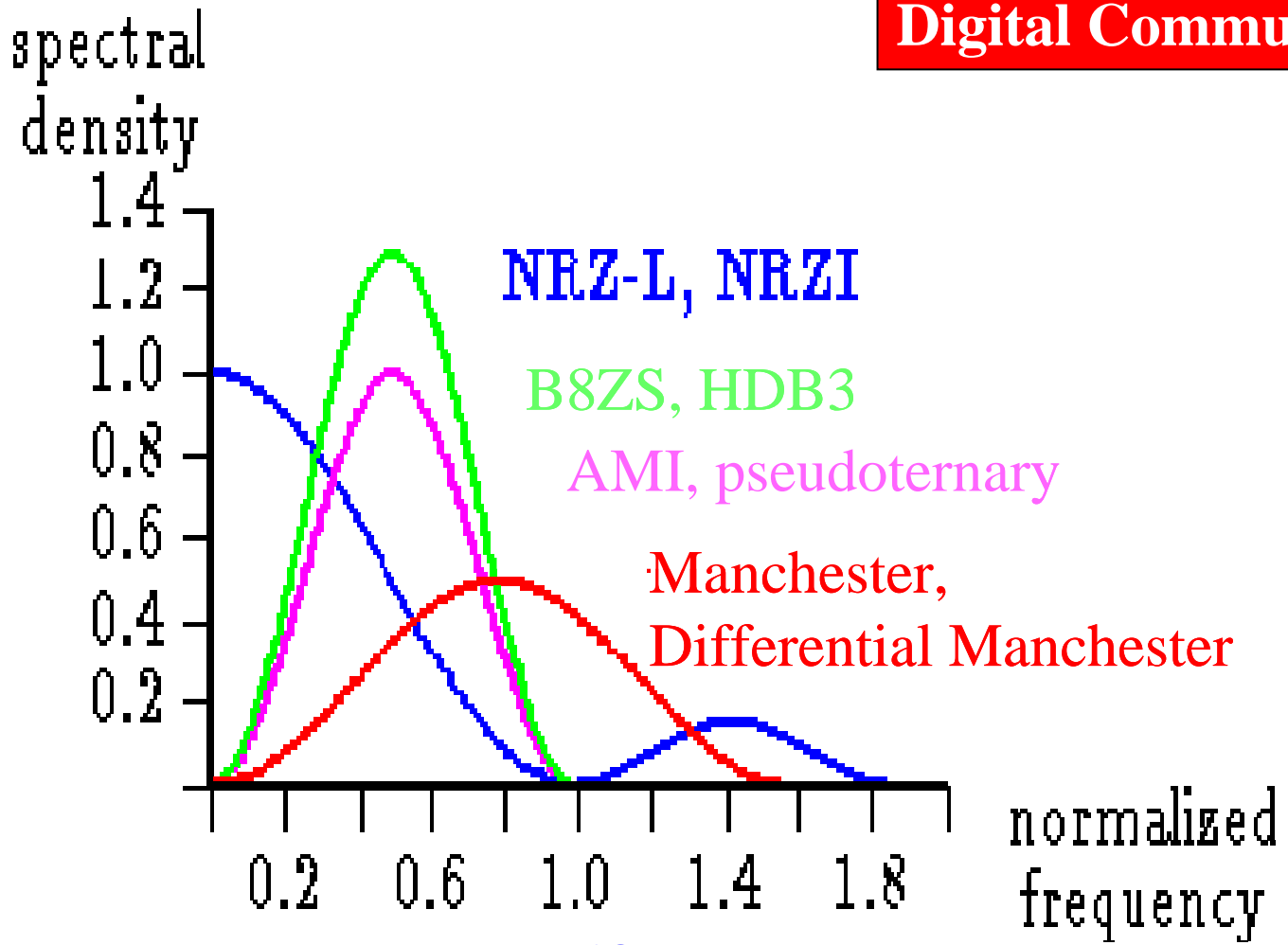
- **Nonreturn to Zero-Level (NRZ-L)**
  - 0 = high level
  - 1 = low level
- **Nonreturn to Zero Inverted (NRZI)**
  - 0 = no transition at beginning of interval
  - 1 = transition at beginning of interval
- **Bipolar-AMI**
  - 0 = no line signal
  - 1 = +ve or -ve level; alternating successive ones
- **Pseudoternary**
  - 0 = +ve or -ve level; alternating for successive ones
  - 1 = no line signal
- **Doubinary**
  - 0 = no line signal
  - 1 = +ve or -ve level; depending on number of separating 0s (even – same polarity, odd – opposite polarity)
- **Manchester**
  - 0 = transition from high to low in middle of interval
  - 1 = transition from low to high in middle of interval
- **Differential Manchester: Always transition in middle of interval**
  - 0 = transition at beginning of interval
  - 1 = no transition at beginning of interval

# Digital Signal Encoding Formats



# Spectrum Characteristics of Digital Encoding Schemes

**Digital Communications**



# Asynchronous Data Transmission

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- **Digital Info:**
  - **Bits**
  - **Characters**
  - **Packets**
  - **Messages or files**
- **Serial vs. Parallel character**

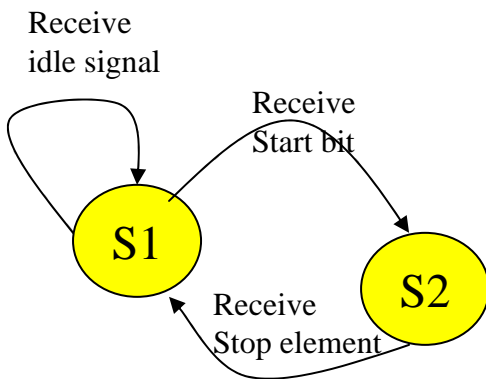
# Asynchronous Transmission

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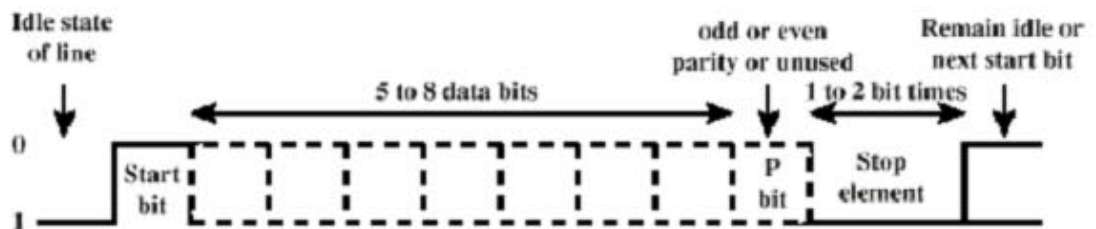
- Exploits: Rx-er can remain for short period in synch with Tx-er
- Used for short stream of bits – data transmitted one character (5 ~ 8 bits) at a time
- Synchronization is needed to be maintained for the length of short transmission
- Character is delimited (start & end) by known signal elements: start bit – stop element
- Rx-er re-synchs with the arrival of new character

# Asynchronous Transmission

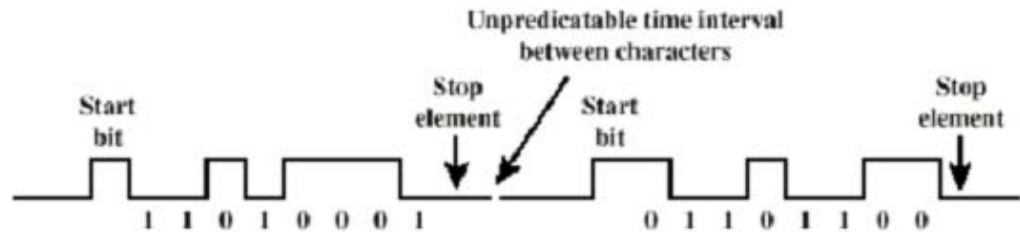
- Simple / Cheap
- Efficiency: transmit 1 start bit + 8 bit of data + 2 stop bits → Efficiency =  $8/11 = 72\%$  (or overhead =  $3/11 = 28\%$ )
- Good for data with large gaps (e.g. keyboard, etc)



S1: receiver in idle state  
S2: receiver is receiving character



(a) Character format



(b) 8-bit asynchronous character stream



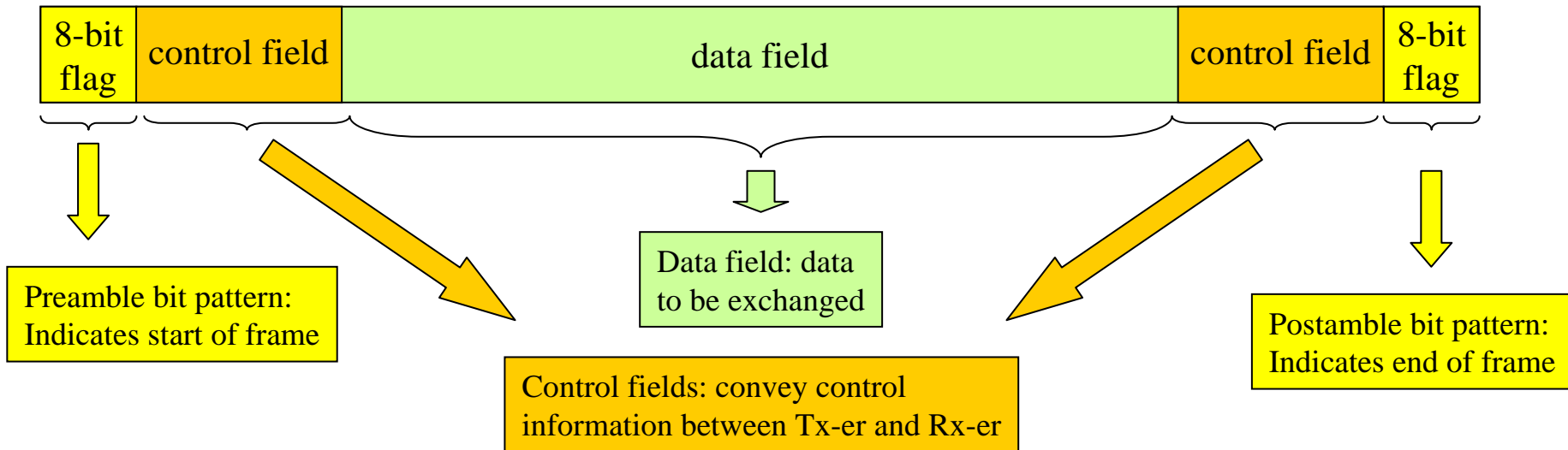
# Synchronous Data Transmission

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- **Two ends remain in sync for significant period of time**
- **Use of SYNC or PREAMBLE characters**
- **Noise + Data = may create another SYNC character → frame split**
  - **Solution – use two SYNC characters**
- **Rx-er must buffer incoming frames and search for SYNC character(s)**

# Synchronous Frame Format

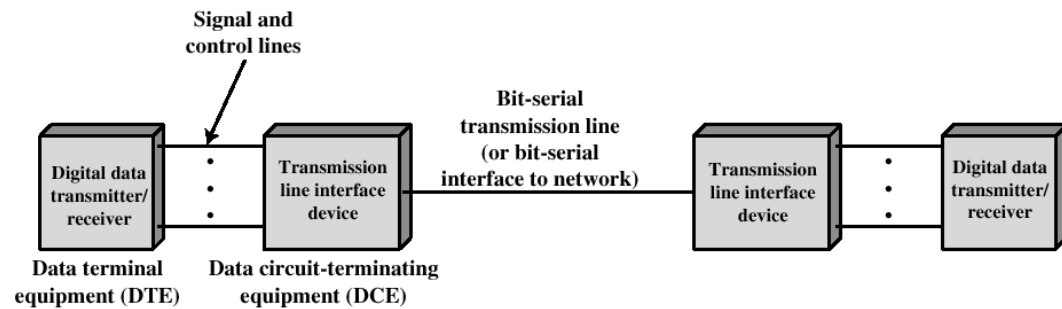
- Typical Frame Structure



- For large data blocks, synchronous transmission is far more efficient than asynchronous:
  - E.g. HDLC frame (to be discussed in Chapter 7): 48 bits are used for control, preamble, and postamble – if 1000 bits are used for data → efficiency = 99.4% (or overhead = 0.6%)

# Interfacing

- Data Terminal Equipment (DTE): terminals or computers
- Data Circuit Equipment (DCE): modem
- Two DCEs exchanging data on behalf of DTEs must use exact same protocol



(a) Generic interface to transmission medium



(b) Typical configuration

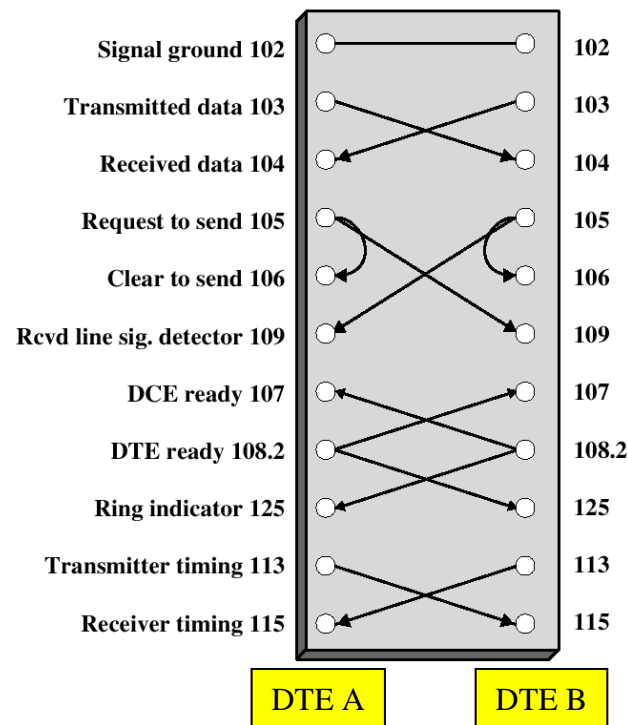
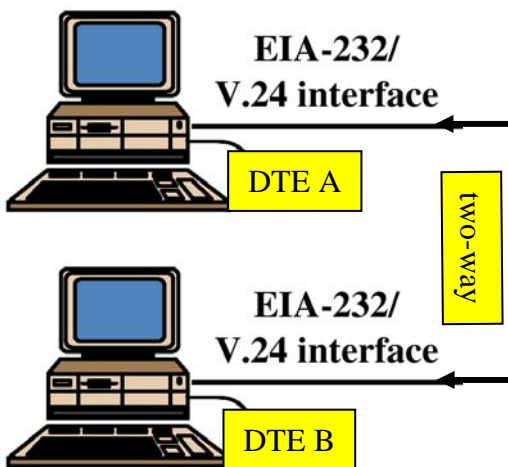
# DTE-DCE Interface Definition

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- Mechanical: physical specification of connection – type, dimensions, location of pins, etc
- Electrical: voltage levels and timing signals used
- Functional: specify functions that are performed for circuits – rx circuit, tx circuit, etc.
- Procedural: specification of sequence of event for transmitting data based on functional specification
- Two examples:
  - V.24/EIA-232-F, and
  - ISDN physical interface

# V.24/EIA-232-F - Procedural Specification - Examples

- Example: Two terminals connected back-to-back through the V.24 interface BUT with no DCEs
- This is referred to as the NULL modem connection
- For short distance connections



# Error Control

---

- Error Detection
  - Parity Checks
  - Cyclic Redundancy Check (CRC)
- For a channel of bit error rate (or BER) of  $P$ , the probability of  $m$  bits in error in a block of  $n$  bits ( $m \leq n$ ) is given by

$$\binom{n}{m} p^m (1-p)^{n-m}$$

or

$$\frac{n!}{(n-m)!m!} p^m (1-p)^{n-m}$$

The above simple formula assumes iid error probability across all bits in block – How realistic is that?

## Error Control (2)

---

- The probability the frame or block is correct is given by

$$(1 - p)^n$$

Therefore the probability, the frame is in error (one or more bits in error) is given by

$$1 - (1 - p)^n$$

The above quantity is referred to as FER

# Error Control - Example

---

- Consider a channel with BER =  $10^{-3}$ , for a block (packet of  $n = 100$  bits), the probability of having one bit in error is equal to

$$100 \times P \times (1-P)^{99} = 9 \times 10^{-2}$$

While the probability of having 4 bits in error is equal to

$$(100 \text{ choose } 4) \times P^4 \times (1-P)^{96} = 3.6 \times 10^{-6}$$

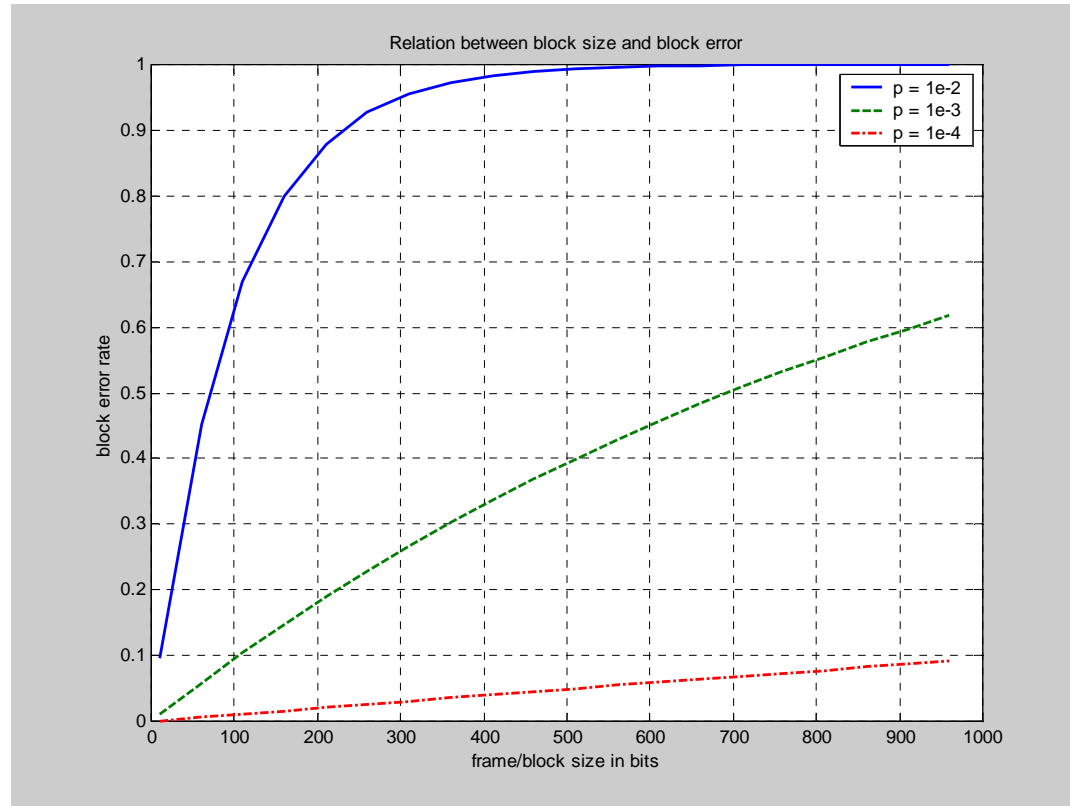
The probability that the frame is correct is equal to

$$(1-P)^{100} = 0.905 \quad \rightarrow \text{i.e. } \sim 10\% \text{ of the time the frame is in error!!}$$



# Error Control – Example (2)

- Relation between block size ( $n$ ) and frame error rate (FER)



# Simple Parity Check

---

- Add one extra bit for each character such that:
  - Even Parity: no of 1s even
  - Odd Parity: no of 1s odd
- Simple
- Can not detect even no of errors in character
- Adding one extra bit to a group of n bits →  
Excess redundancy =  $1/(n+1)$

# VRC/LRC Parity Check

- Extension of simple parity: Vertical Redundancy Check (VRC) and Longitudinal Redundancy Check (LRC)

Original data to send									
Char 1	1	0	0	1	1	0	0	0	1
Char 2	0	1	1	1	0	1	0	1	1
Char 3	1	1	0	0	1	1	0	0	0
Char 4	1	0	0	0	1	0	0	0	0
Char 5	0	1	0	0	1	1	1	0	0
Checking char	1	1	1	0	0	1	1	1	0

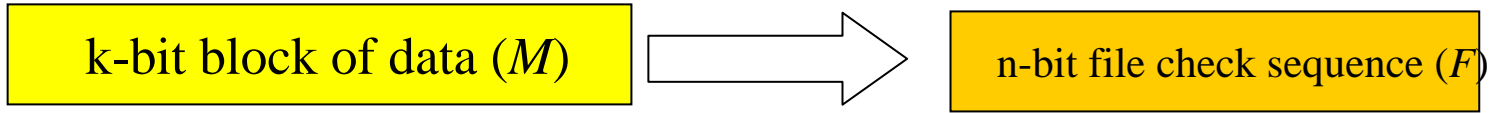
Parity check

# VRC/LRC Parity Check (2)

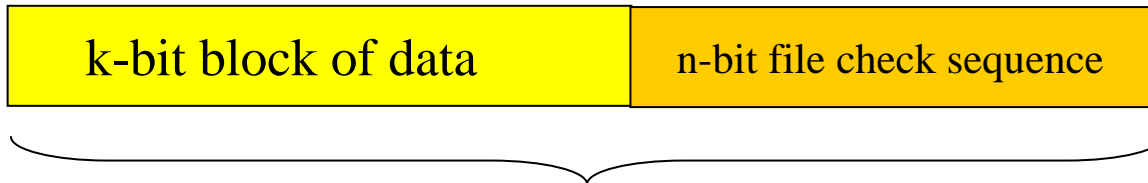
---

- Can detect all odd errors – same as the simple parity check
- Can detect any combination of even error in characters that DO NOT result in even number of errors in a column
- Excess Redundancy:  $14/(40+14) = 0.26$
- There could be undetected errors – How?

# Cyclic Redundancy Check (CRC)



Processing: compute FCS (for some given an  $n+1$  bit polynomial  $P$ )



$k+n$  bit frame to be transmitted =  $T$

- Modulo 2 arithmetic is used to generate the FCS:
  - $0 \pm 0 = 0; 1 \pm 0 = 1; 0 \pm 1 = 1; 1 \pm 1 = 0$
  - $1 \times 0 = 0; 0 \times 1 = 0; 1 \times 1 = 1$

# CRC – Mapping Binary Bits into Polynomials

---

- Consider the following k-bit word or frame and its polynomial equivalent:

$$b_{k-1} b_{k-2} \dots b_2 b_1 b_0 \rightarrow b_{k-1}x^{k-1} + b_{k-2}x^{k-2} + \dots + b_1x^1 + b_0$$

where  $b_i$  ( $k-1 \leq i \leq 0$ ) is either 1 or 0

# CRC – Mapping Binary Bits into Polynomials - Examples

---

- Example1: an 8 bit word  $M = 11011001$  is represented as  $M(x) = x^7 + x^6 + x^4 + x^3 + 1$

- Example2: What is  $x^4M(x)$  equal to?

$x^4M(x) = x^4(x^7 + x^6 + x^4 + x^3 + 1) = x^{11} + x^{10} + x^8 + x^7 + x^4$ , the equivalent bit pattern is 110110010000 (i.e. four zeros added to the right of the original M pattern)

- Example3: What is  $x^4M(x) + (x^3 + x + 1)$ ?

$x^4M(x) + (x^3 + x + 1) = x^{11} + x^{10} + x^8 + x^7 + x^4 + x^3 + x + 1$ , the equivalent bit pattern is 110110011011 (i.e. pattern 1011 =  $x^3 + x + 1$  added to the right of the original M pattern)

# CRC Calculation

- $T = (k+n)$ -bit frame to be tx-ed,  $n < k$
- $M = k$ -bit message, the first  $k$  bits of frame  $T$
- $F = n$ -bit FCS, the last  $n$  bits of frame  $T$
- $P =$  pattern of  $n+1$  bits (a predetermined divisor)

$T = (n+k)$ -bit frame



$P = (n+1)$  bit divisor

Note:

- $T(x)$  is the polynomial (of  $k+n-1^{\text{st}}$  degree or less) representation of frame  $T$
- $M(x)$  is the polynomial (of  $k-1^{\text{st}}$  degree or less) representation of message  $M$
- $F(x)$  is the polynomial (of  $n-1^{\text{st}}$  degree or less) representation of FCS
- $P(x)$  is the polynomial (of  $n^{\text{th}}$  degree or less) representation of the divisor  $P$
- $T(x) = X^n M(x) + F(x)$  – refer to example 3 on previous slide



# CRC Calculation (2)

---

- Design: frame T such that it divides the pattern P with no remainder?
- Solution: Since the first component of T, M, is the data part, it is required to find F (or the FCS) such that T divides P with no remainder

Using the polynomial equivalent:

$$T(x) = X^n M(x) + F(x)$$

One can show that  $F(x) = \text{remainder of } x^n M(x) / P(x)$

i.e if  $x^n M(x) / P(x)$  is equal to  $Q(x) + R(x)/P(x)$ , then  $F(X)$  is set to be equal to  $R(X)$ .

Note that:

Polynomial of degree k+n

----- = polynomial of degree k + remainder polynomial of degree n or less

Polynomial of degree n

# CRC Calculation - Procedure

---

- 1. Shift pattern M n bits to the left**
- 2. Divide the new pattern  $2^nM$  by the pattern P**
- 3. The remainder of the division R (n bits) is set to be the FCS**
- 4. The desired frame T is  $2^nM$  plus the FCS bits**

**Note:**

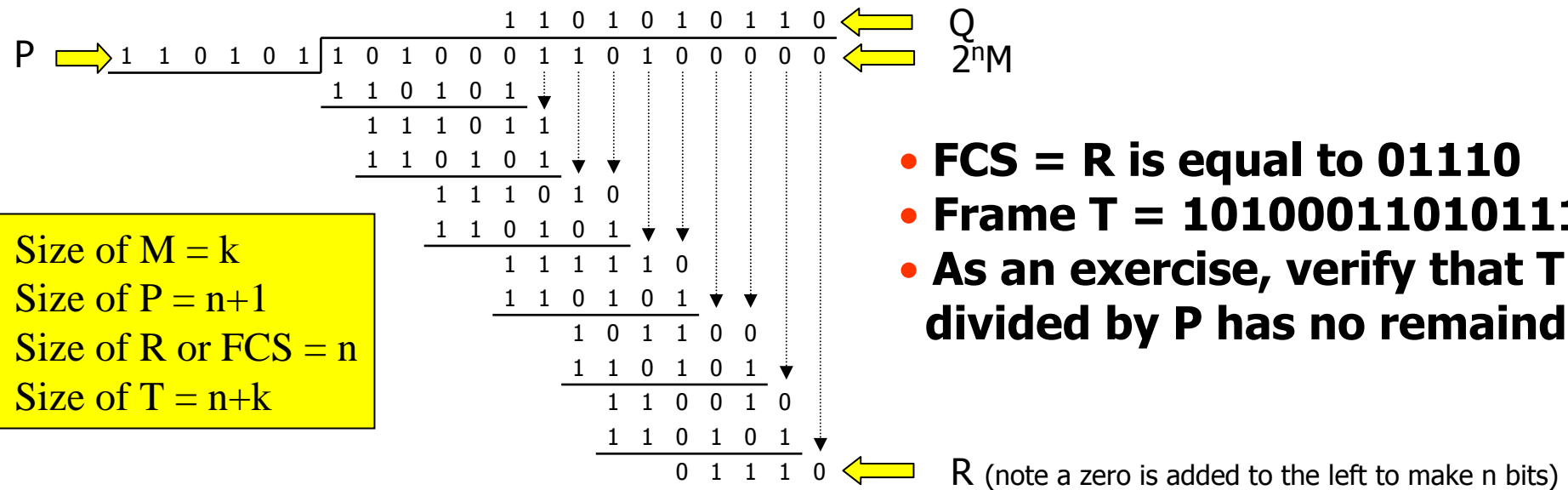
$2^nM$  is the pattern resulting from shifting the pattern M n bits to the left. In other words, the polynomial equivalent of the pattern  $2^nM$  is  $x^nM(x)$

# CRC Calculation - Example

- **Message M = 1010001101 (10 bits) → k = 10**  
**Pattern P = 110101 (6 bits – note 0<sup>th</sup> and n<sup>th</sup> bits are 1s)**  
**→ n + 1 = 6 → n = 5**

**Find the frame T to be transmitted?**

- **Solution:**



# CRC Calculation – The previous example BUT using Polynomials

- **Message M = 1010001101 (10 bits) → k = 10**
- **$M(x) = x^9 + x^7 + x^3 + x^2 + 1 \rightarrow x^5M(x) = x^{14} + x^{12} + x^8 + x^7 + x^5$**
- **Pattern P = 110101 (6 bits – note 0<sup>th</sup> and n<sup>th</sup> bits are 1s) → n + 1 = 6 → n = 5**
- **$P(x) = x^5 + x^4 + x^2 + 1$**
- **Find the frame T to be transmitted?**
- **Solution:**

$x^5$	$+x^4$	$+x^2$	$+1$	$x^9$	$+x^8$	$+x^6$	$+x^4$	$+x^2$	$+x$	
$x^{14}$	$+x^{12}$	$+x^8$	$+x^7$	$+x^5$						
$x^{14}$	$+x^{13}$	$+x^{11}$	$+x^9$							
$x^{13}$	$+x^{12}$	$+x^{11}$	$+x^9$	$+x^8$	$+x^7$	$+x^5$				
$x^{13}$	$+x^{12}$	$+x^{10}$	$+x^8$							
				$x^{11}$	$+x^{10}$	$+x^9$	$+x^7$	$+x^5$		
				$x^{11}$	$+x^{10}$	$+x^8$	$+x^6$			
				$x^9$	$+x^8$	$+x^7$	$+x^6$	$+x^5$		
				$x^9$	$+x^8$	$+x^6$	$+x^4$			
						$+x^7$	$+x^5$	$+x^4$		
				$+x^7$	$+x^6$	$+x^4$	$+x^2$			
				$x^6$	$+x^5$	$+x^2$				
				$x^6$	$+x^5$	$+x^3$	$+x$			
				$+x^3$	$+x^2$	$+x$				

- **FCS = R(x) =  $x^3 + x^2 + x$**   
(or  $0x^4 + x^3 + x^2 + x$ )
- **→ R is equal to 01110**
- **Frame T = 101000110101110**
- **As an exercise, verify that T divided by P has no remainder**

# CRC Calculation – The previous example BUT using Polynomials – cont'd

---

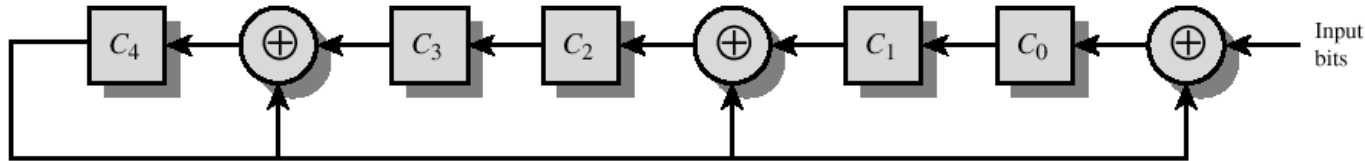
- **Message M = 1010001101 (10 bits)**
- $M(x) = x^9 + x^7 + x^3 + x^2 + 1$
- $x^5M(x) = x^{14} + x^{12} + x^8 + x^7 + x^5$
- **Pattern P = 110101**
- $P(x) = x^5 + x^4 + x^2 + 1$
- $R(x) = x^3 + x^2 + x$
- $Q(x) = x^9 + x^8 + x^6 + x^4 + x^2 + x$
- $T(x) = x^5M(x) + R(x)$   
 $= x^{14} + x^{12} + x^8 + x^7 + x^5 + x^3 + x^2 + x$ , or  
**T = 101000110101110**
- **Exercise: Verify that  $Q(x)P(x) + R(x) = x^5M(x)$**

# CRC – Receiver Procedure

---

- **Tx-er transmits frame T**
- **Channel introduces error pattern E**
- **Rx-er receives frame  $T_r = T \oplus E$**  (note that if  $E = 000..000$ , then  $T_r$  is equal to  $T$ , i.e. error free transmission)
- **$T_r$  is divided by  $P$ , Remainder of division is  $R$**
- **if  $R$  is ZERO, Rx-er assumes no errors in frame; else Rx-er assumes erroneous frame**
- **If an error occurs and  $T_r$  is still divisible by  $P \rightarrow$  UNDETECTABLE error** (this means the  $E$  is also divisible by  $P$ )

# CRC - Transmitter Circuit



□ = 1-bit shift register    ⊕ = Exclusive-OR circuit

(a) Shift-register implementation

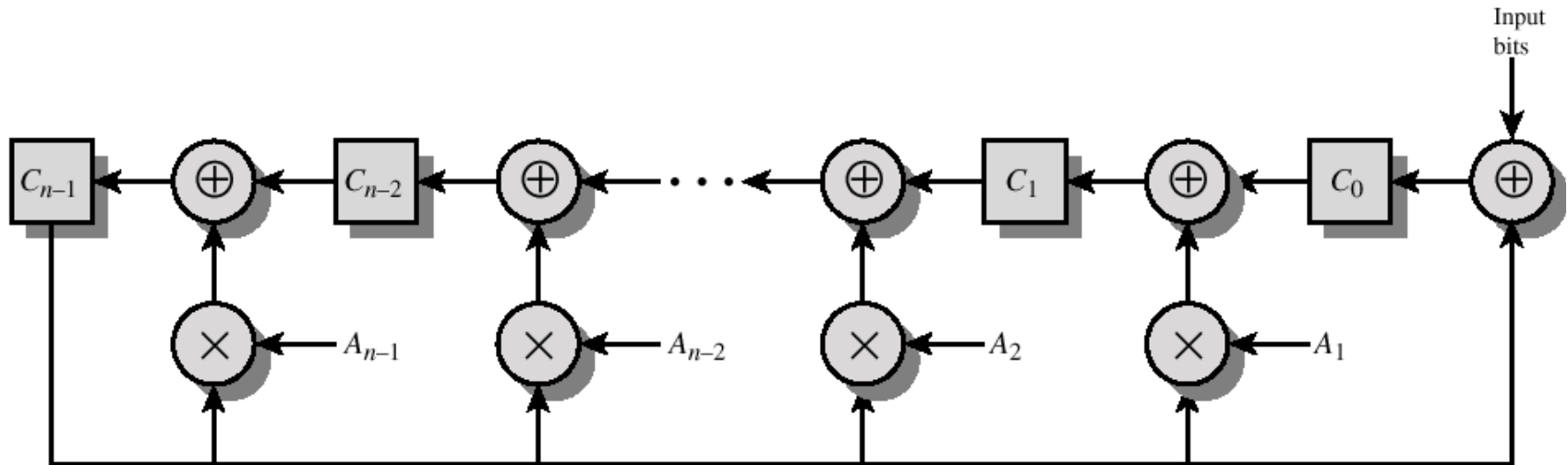
Shift register circuit for dividing by  
 $P = X^5 + X^4 + X^2 + 1$

	$C_4$	$C_3$	$C_2$	$C_1$	$C_0$	$C_4 \oplus C_3$	$C_4 \oplus C_1$	$C_4 \oplus \text{input}$	input	
Initial	0	0	0	0	0	0	0	1	1	} Message to be sent
Step1	0	0	0	0	1	0	0	0	0	
Step2	0	0	0	1	0	0	1	1	1	
Step3	0	0	1	0	1	0	0	0	0	
Step4	0	1	0	1	0	1	1	0	0	
Step5	1	0	1	0	0	1	1	1	0	
Step6	1	1	1	0	1	0	1	0	1	
Step7	0	1	1	1	0	1	1	1	1	
Step8	1	1	1	0	1	0	1	1	0	
Step9	0	1	1	1	1	1	1	1	1	
Step10	1	1	1	1	1	0	0	1	0	} Five zeros added
Step11	0	1	0	1	1	1	1	0	0	
Step12	1	0	1	1	0	1	0	1	0	
Step13	1	1	0	0	1	0	1	1	0	
Step14	0	0	1	1	1	0	1	0	0	
Step15	0	1	1	1	0	1	1	0	—	

(b) Example with input of 1010001101

# CRC – Receiver Circuit

- **Tx-er transmits frame T**

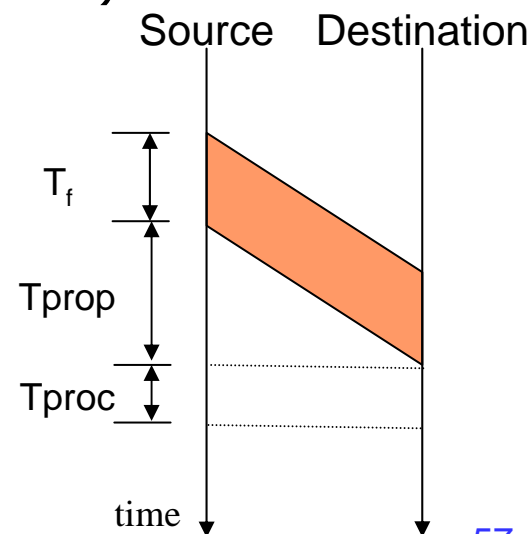


**Figure 7.7** General CRC Architecture to Implement Divisor  
 $1 + A_1X + A_2X^2 + \dots + A_{n-1}X^{n-1} + X^n$

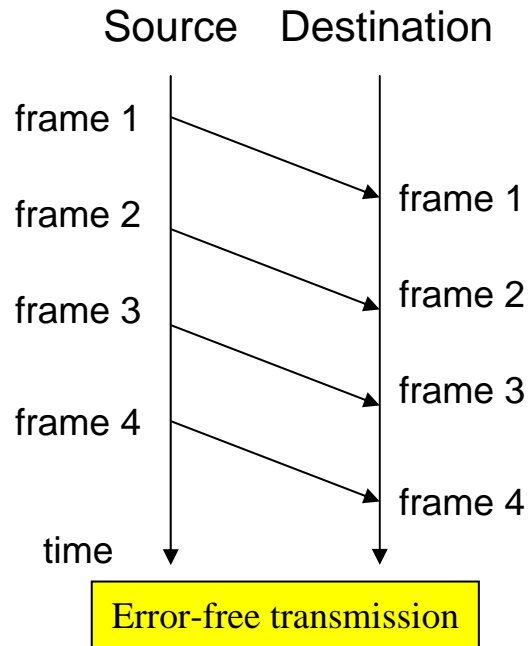


# Flow Control

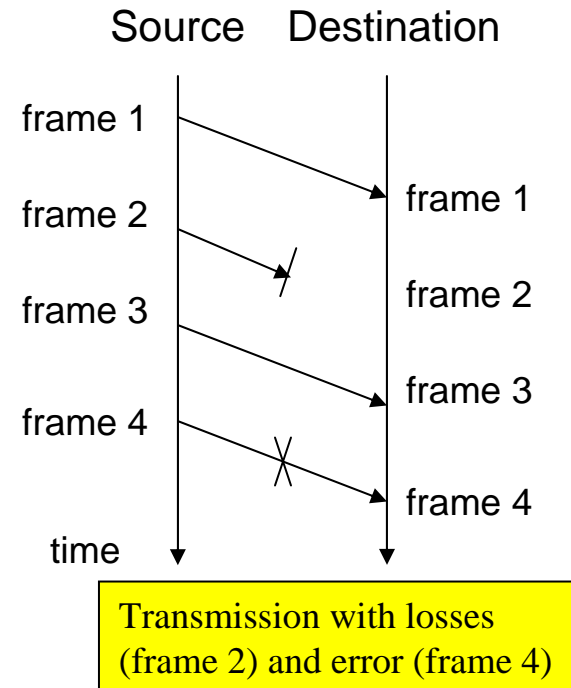
- A scheme to ensure that transmitter does not overwhelm receiver with data
- Transmission of one frame:
  - $T_f$ : time to transmit frame
  - $T_{prop}$ : time for signal to propagate
  - $T_{proc}$ : time for destination to process received frame – small delay (usually ignored if not specified)
- $T_{proc}$  may be ignored if not specified



# Flow Control (2)



## Models of Frame Transmission



- The destination has a limited buffer space. How will the source know that destination is ready to receive the next frame?
- In case of errors or lost frame, the source need to retransmit frames – i.e. a copy of transmitted frames must be kept. How will the source know when to discard copies of old frames?
- Etc.

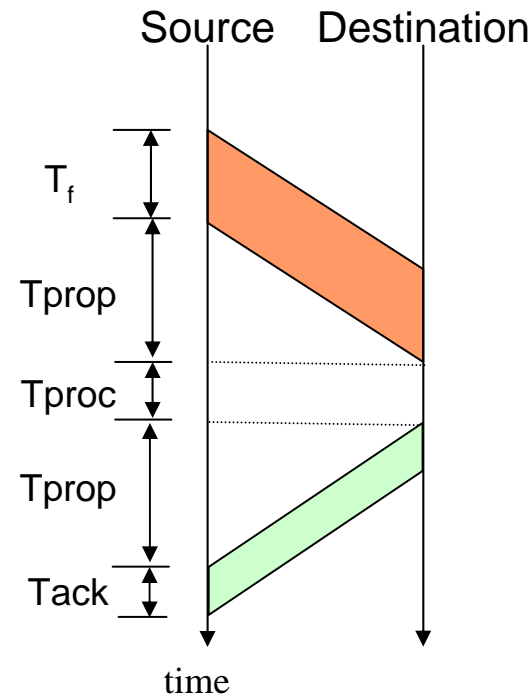
# Stop-and-Wait Protocol

---

- Protocol:
  - Source transmits a frame
  - After the destination receives frame, it sends ACK
  - Source, upon the receipt of ACK, can now send the next frame
- Destination can stop source by withholding the ACK
- Simple
- Animation for [Stop-and-Wait](#)
- NOTE: ONLY one frame can be in transit at any time

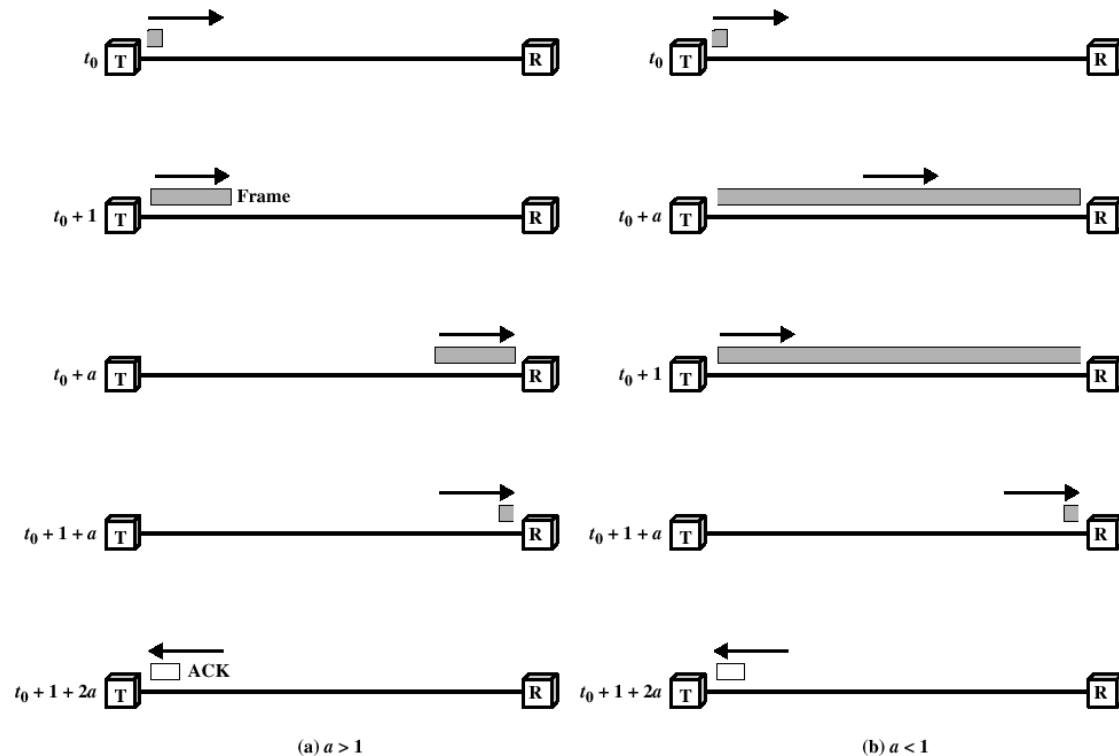
# Stop-and-Wait Protocol: Efficiency

- After every frame, source must wait till acknowledgment → Hence link propagation time is significant
- Total time to for one frame:  
$$T_{total} = T_f + 2T_{prop} + T_{proc} + T_{ack}$$
if we ignore  $T_{proc}$  and  $T_{ack}$  (usually very small)  
$$T_{total} = T_f + 2T_{prop}$$
- Link utilization,  $U$  is equal to  
$$U = T_f / (T_{total}), \text{ or}$$
$$= 1 / (1 + 2(T_{prop}/T_f)) = 1 / (1 + 2a)$$
where  $a = T_{prop}/T_f = \text{length of link in bits}$
- If  $a < 1$  (i.e.  $T_f > T_{prop}$  – when 1<sup>st</sup> transmitted bit reaches destination, source will still be transmitting →  $U$  is close 100%
- If  $a > 1$  (i.e.  $T_f < T_{prop}$  – frame transmission is completed before 1<sup>st</sup> bit reaches destination →  $U$  is low
- See figure 7.2



# Stop-and-Wait Protocol: Efficiency (2)

- Remember:  $a = T_{prop}/T_f =$  length of link in bits
- If  $a < 1$  (i.e.  $T_f > T_{prop}$  – when 1<sup>st</sup> transmitted bit reaches destination, source will still be transmitting  $\rightarrow U$  is close 100%)
- If  $a > 1$  (i.e.  $T_f < T_{prop}$  – frame transmission is completed before 1<sup>st</sup> bit reaches destination  $\rightarrow U$  is low)
- Stop-and-Wait is efficient for links where  $a \ll 1$  (long frames compared to propagation time)



# Sliding Window Protocol

---

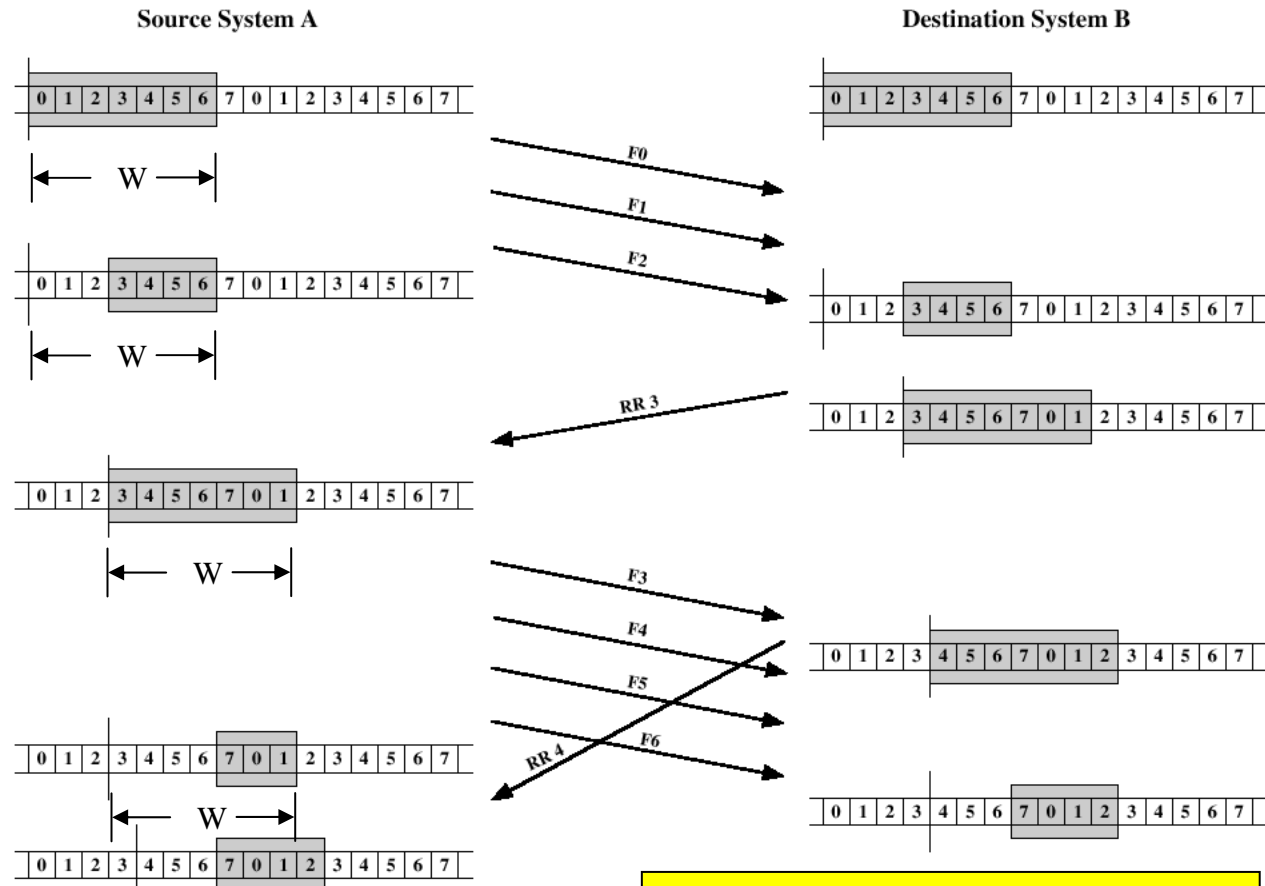
- Stop-and-Wait can be very inefficient when  $a > 1$
- Protocol:
  - Assumes full duplex line
  - Source A and Destination B have buffers each of size  $W$  frames
  - For  $k$ -bit sequence numbers:
    - Frames are numbered:  $0, 1, 2, \dots, 2^k-1, 0, 1, \dots$  (modulo  $2^k$ )
    - ACKs (RRs) are numbered:  $0, 1, 2, \dots, 2^k-1, 0, 1, \dots$  (modulo  $2^k$ )
  - A is allowed to transmit up to  $W$  frames without waiting for an ACK
  - B can receive up to  $W$  consecutive frames
  - ACK  $J$  (or RR  $J$ ), where  $0 \leq J < 2^k$ , sent by B means B has received frames up to frame  $J-1$  and is ready to receive frame  $J$
  - B can also send RNR  $J$ : B has received all frames up to  $J-1$  and is not ready to receive any more
- Window size,  $W$  can be less or equal to  $2^k-1$

# Sliding Window Protocol (2)

- Example of Sliding-Window-Protocol:  $k = 3$  bits,  $W = 7$

## Observations:

- A may tx  $W = 7$  frames (F0, F1, ..., F6)
- After F0, F1, & F2 are tx-ed, window is shrunk (i.e. can not transmit except F3, F4, ..., F6)
- When B sends RR3, A knows F0, F1 & F2 have been received and B is ready to receive F3
- Window is advanced to cover 7 frames (starting with F3 up to F1)
- A sends F3, F4, F5, & F6
- B responds with RR4 when F3 is received – A advances the window by one position to include F2



$W =$  distance between first unacknowledged frame and last frame that can be sent

# Sliding Window Protocol - Piggybacking

---

- When using sliding window protocol in full duplex connections:
  - Node A maintains its own transmit window
  - Node B maintains its own transmit window
  - A frame contains: data field + ACK field
  - There is a sequence number for the data field, and a sequence number for the ACK field



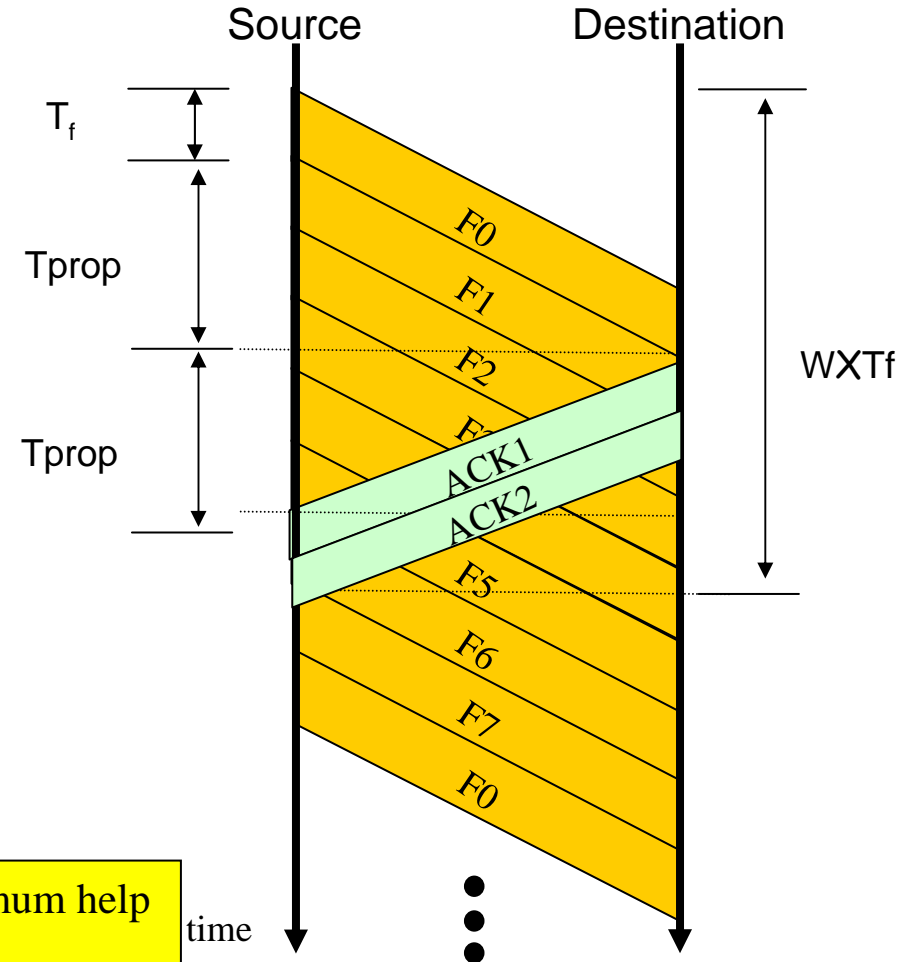
# Sliding Window Protocol - Efficiency

---

- Again we can distinguish two cases:
- Case 1:  $W \geq 2a + 1$
- Case 2:  $W < 2a + 1$

# Sliding Window Protocol - Efficiency - Case 1

- Assume  $k=3$ ,  $W = 7$   
(ignoring Tack)
- Source can continuously keep transmitting!!
  - Because the ACK can arrive to source before the window is completed
- Utilization = 100%



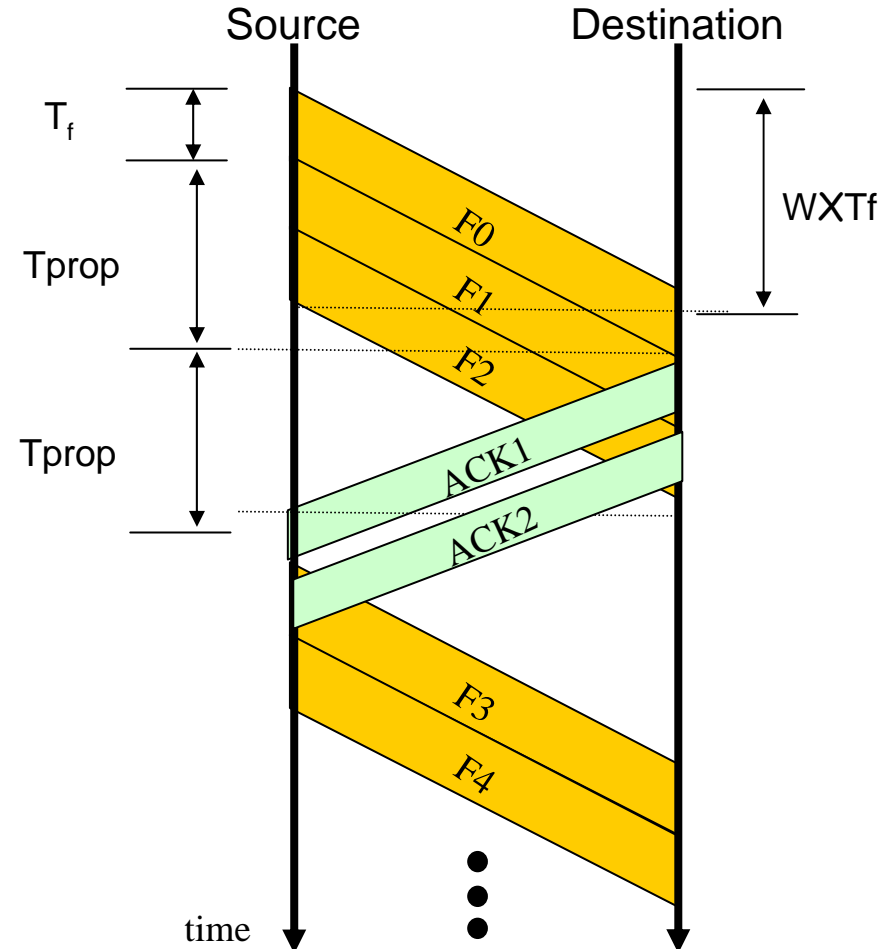
Sending ACK0 as soon as F0 is received is the maximum help the destination can do to increase utilization

# Sliding Window Protocol - Efficiency - Case 2

- Assume  $k = 3$ ,  $W = 3$  (ignoring Tack)
- Source can NOT continuously keep transmitting!!
  - Because the ACK can NOT arrive to source before the window is completed

$$\text{Utilization} = \frac{W \times T_f}{T_f + 2 \times T_{prop}}$$

$$= \frac{W}{1 + 2a}$$



# Sliding Window Protocol - Efficiency

---

- Refer to Appendix A
- When window size is  $W$  (for error free), link utilization,  $U$ , is given by

$$U = \begin{cases} 1 & W \geq (2a + 1) \\ \frac{W}{2a + 1} & W < (2a + 1) \end{cases}$$

where  $a = T_{prop}/T_f$  or length of link in bits

- Sliding window protocol can achieve 100% utilization if  $W \geq (2a + 1)$

# Sliding Window Protocol

---

- Animation for Sliding Window protocol
- Sliding Window Protocol Simulation  
(<http://www.cs.stir.ac.uk/~kjt/software/comms/jasper/SWP3.html>)

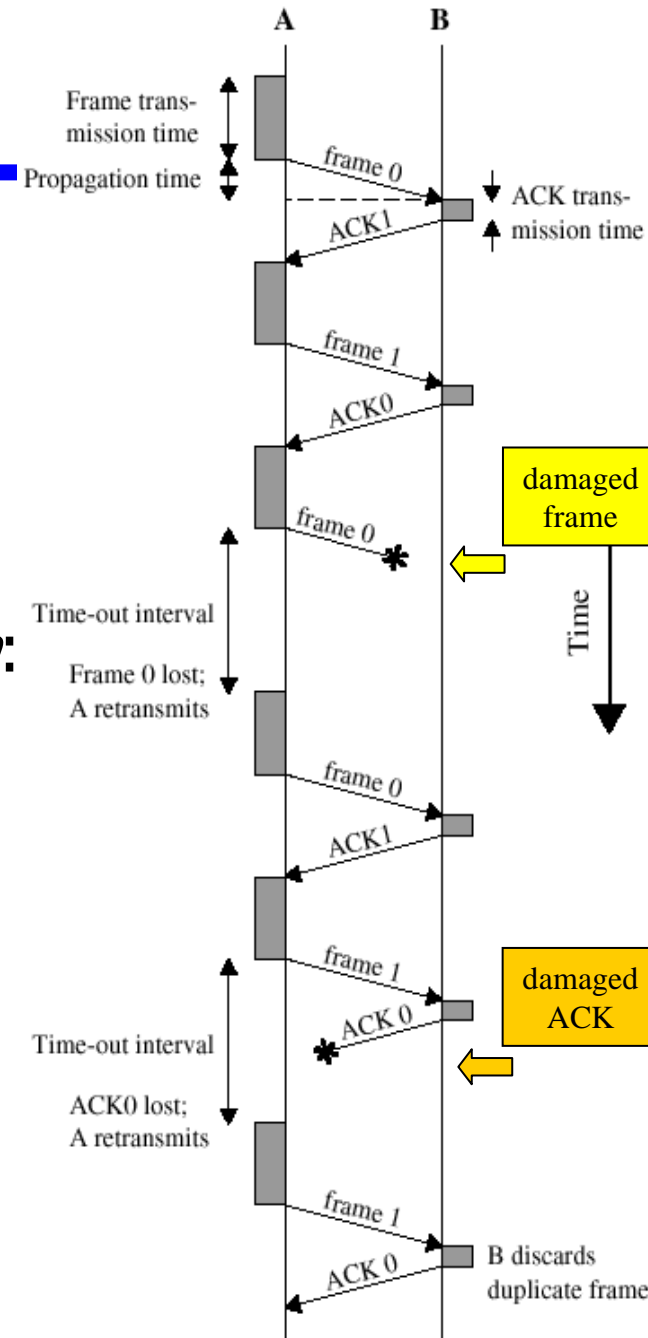
# Automatic Repeat Request - ARQ

---

- Types of Errors:
  - Lost frame
  - Damaged frame
- Error control Techniques:
  - Error detection – discussed previously
  - +ve ACK
  - Retransmission after timeout
  - -ve ACK and retransmission
- ARQ Procedures: convert an unreliable data link into a reliable one.
  - Stop-and-wait
  - Go-back-N
  - Selective-reject

# Stop-and-Wait ARQ

- Based on the stop-and-wait control flow procedure
- Two types of errors:
  1. Frame lost or damaged – Solution: timeout timer
  2. Damaged or lost ACK – The timeout timer solves this problem



# Sliding Window Protocol

---

- Stop-and-Wait can be very inefficient when  $a > 1$
- Protocol:
  - Assumes full duplex line
  - Source A and Destination B have buffers each of size  $W$  frames
  - For  $k$ -bit sequence numbers:
    - Frames are numbered:  $0, 1, 2, \dots, 2^k-1, 0, 1, \dots$  (modulo  $2^k$ )
    - ACKs (RRs) are numbered:  $0, 1, 2, \dots, 2^k-1, 0, 1, \dots$  (modulo  $2^k$ )
  - A is allowed to transmit up to  $W$  frames without waiting for an ACK
  - B can receive up to  $W$  consecutive frames
  - ACK  $J$  (or RR  $J$ ), where  $0 \leq J < 2^k$ , sent by B means B has received frames up to frame  $J-1$  and is ready to receive frame  $J$
  - B can also send RNR  $J$ : B has received all frames up to  $J-1$  and is not ready to receive any more
- Window size,  $W$  can be less or equal to  $2^k-1$



# Go-Back-N ARQ

---

- Based on the sliding-window flow control procedure
- If the  $i^{\text{th}}$  frame is lost or deemed lost (i.e.  $i+1^{\text{st}}$  is received before the  $i^{\text{th}}$  frame), the  $i^{\text{th}}$  frame and all subsequent frames are retransmitted

# Selective-Reject ARQ

---

- In contrast to Go-Back-N, the only frames retransmitted are those that receive –ve ACK (called SREJ) or those that time out
- More efficient:
  - Rx-er must have large enough buffer to save *post-SREJ* frames
  - Buffer manipulation – re-insertion of out-of-order frames

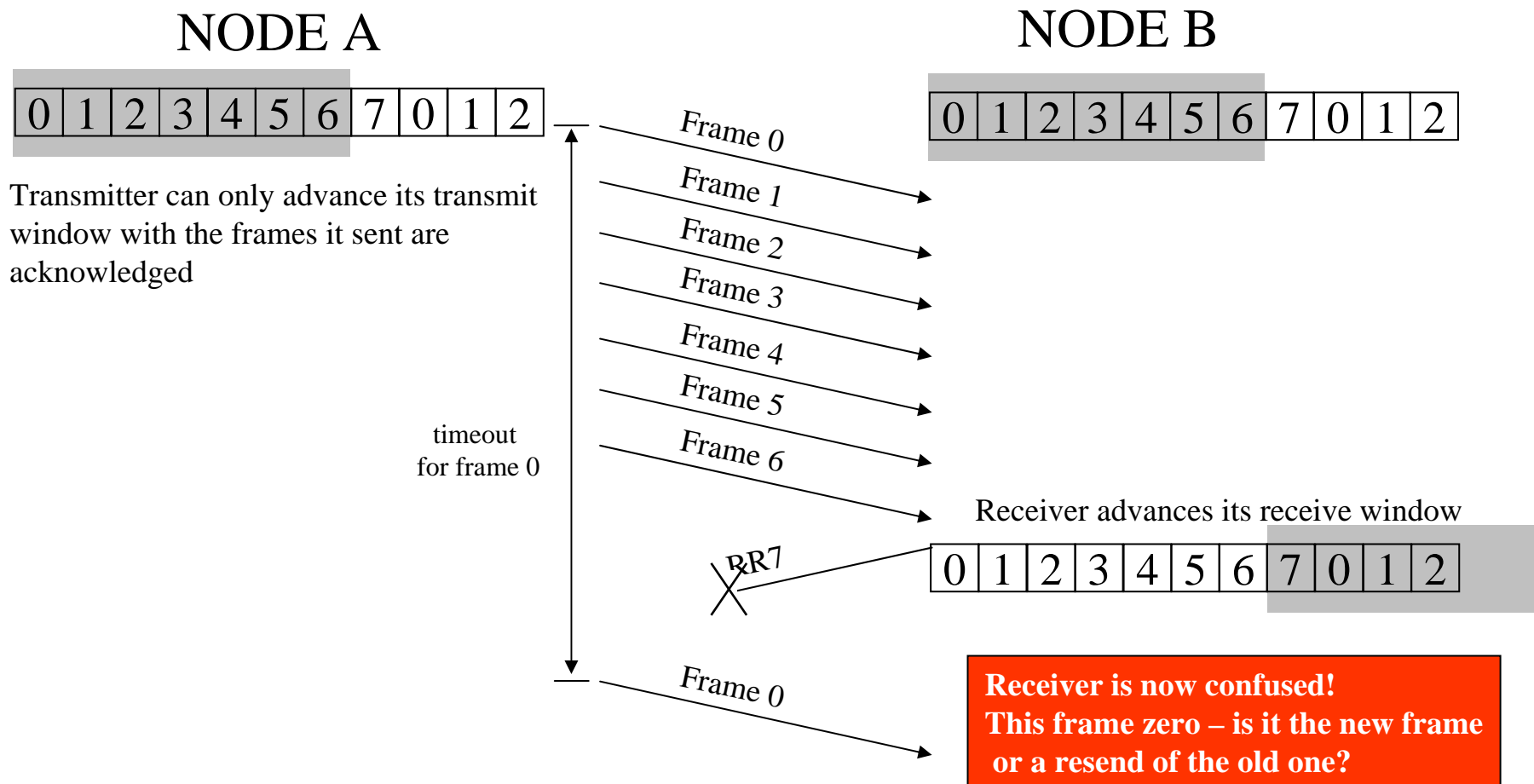
# Window Size for Selective-Reject ARQ – Why?

---

- Window size: should be less or equal to half range of sequence numbers
  - For n-bit sequence numbers, Window size is  $\leq 2^{n-1}$  (remember sequence numbers range from 0,1, ...,  $2^n-1$ )
- Why? See next example

# Window Size for Selective-Reject ARQ – Why? (2)

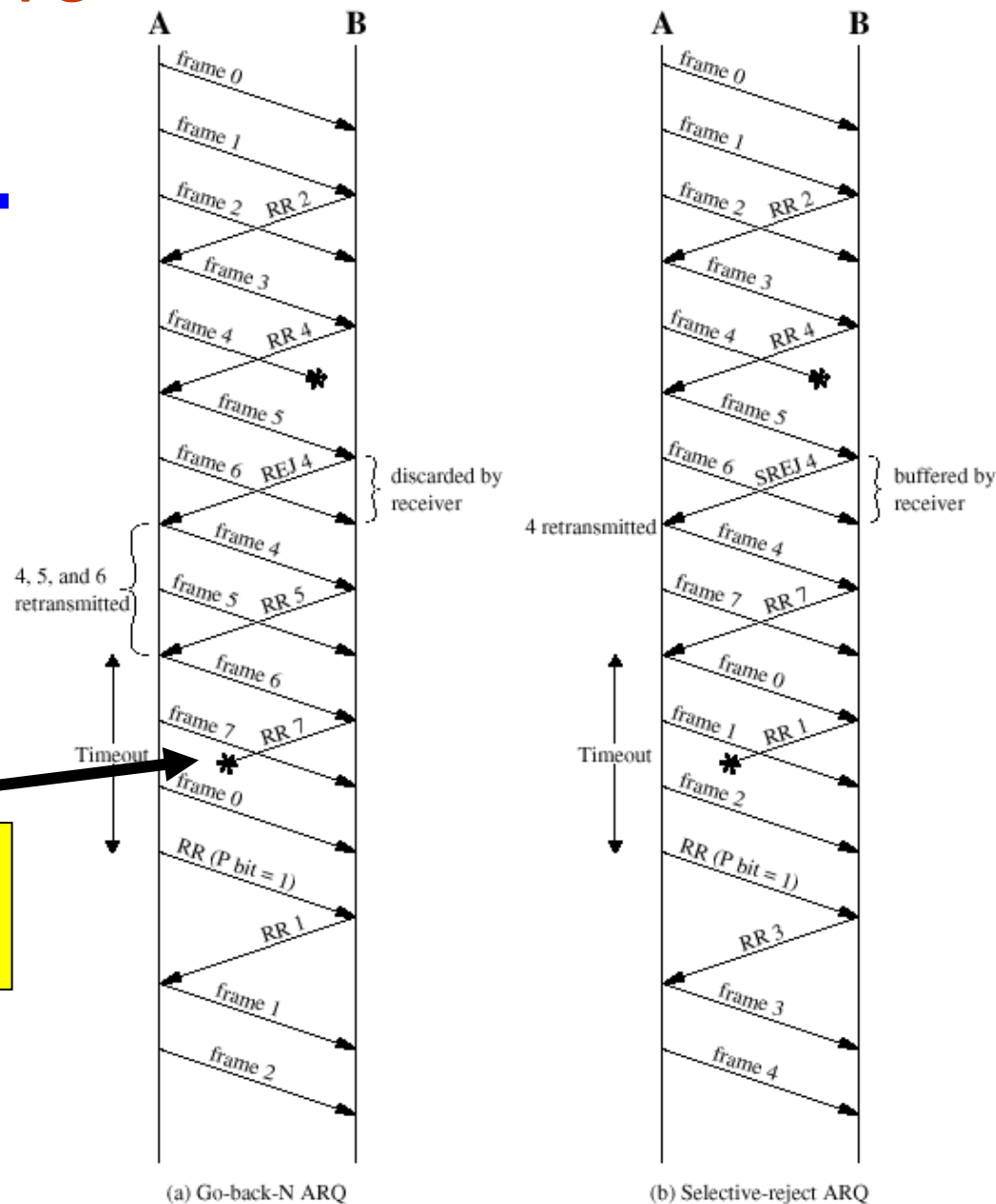
- Example: Consider 3-bit sequence number and window size of 7



# Go-Back-N/Selective- Reject ARQ Examples

- With Go-back-N frames 4,5 and 6 are retransmitted
- With Selective-Reject only frame 4 is retransmitted

Did this lost RR7 affect flow?  
How did the link recover?



(a) Go-back-N ARQ

(b) Selective-reject ARQ

# Switching

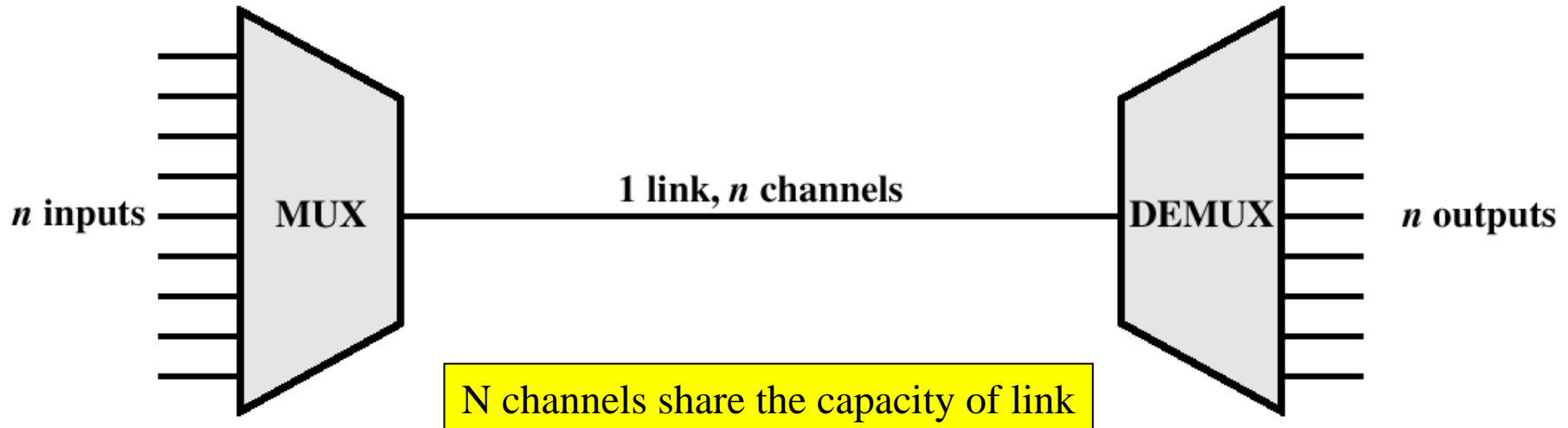
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- Circuit Switching
  - Call Setup
  - Data Exchange
  - Call Termination
- Store-and-forward (Packet Switching)
  - Virtual Circuit
  - Datagram

# What is MULTIPLEXING?

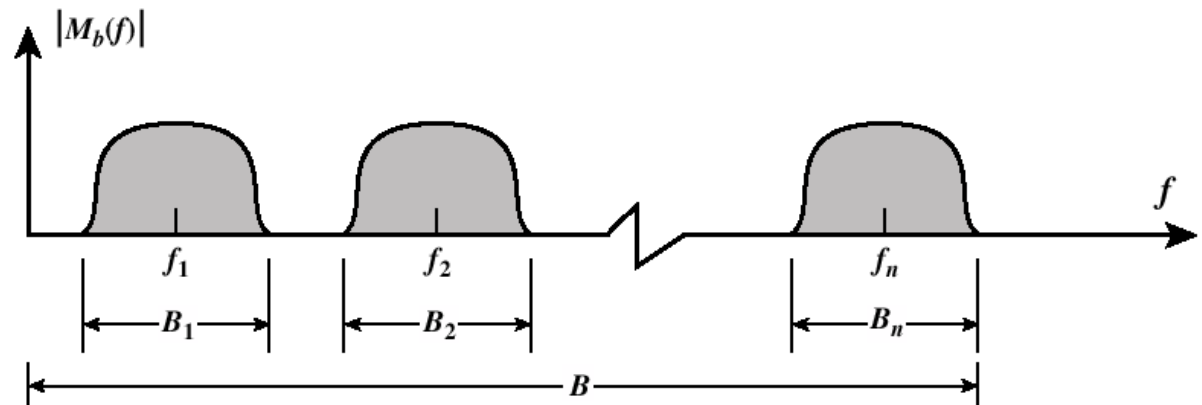
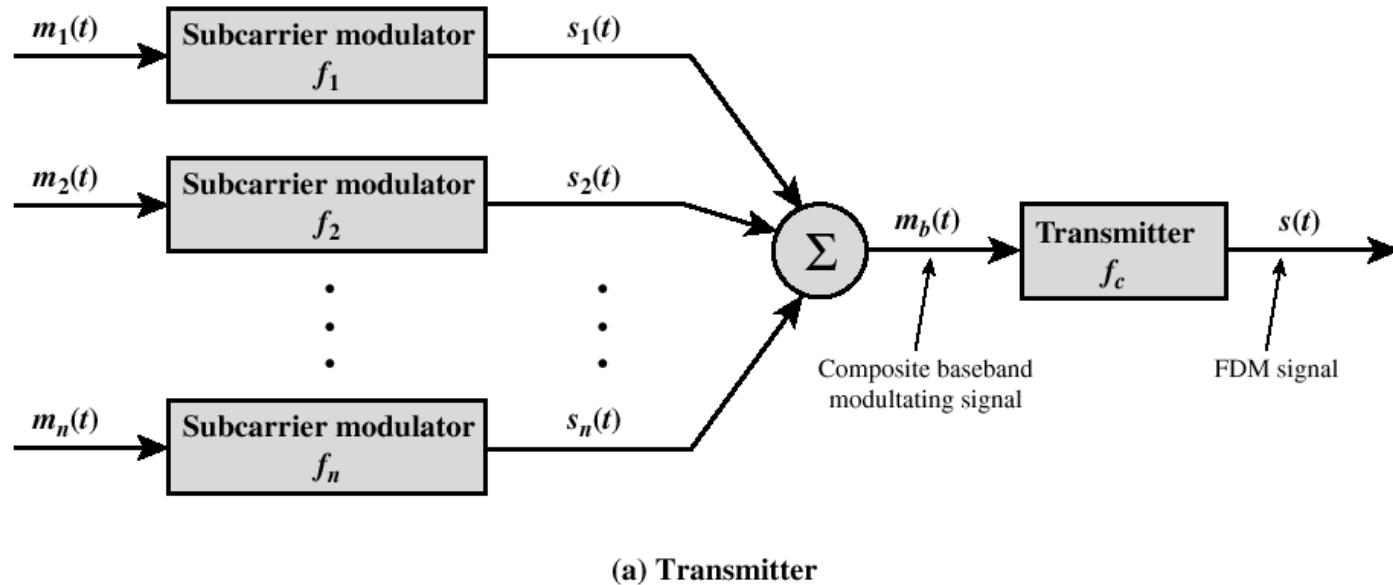
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- A generic term used where more than one application or connection share the capacity of one link
- Why?
  - To achieve better utilization of resources



# Frequency-Division Multiplexing - Transmitter

- $m_i(t)$ : analog or digital information
- Modulated with subcarrier  $f_i \rightarrow s_i(t)$
- $m_b(t)$  composite baseband modulating signal
- $m_b(t)$  modulated by  $f_c \rightarrow$  The overall FDM signal  $s(t)$

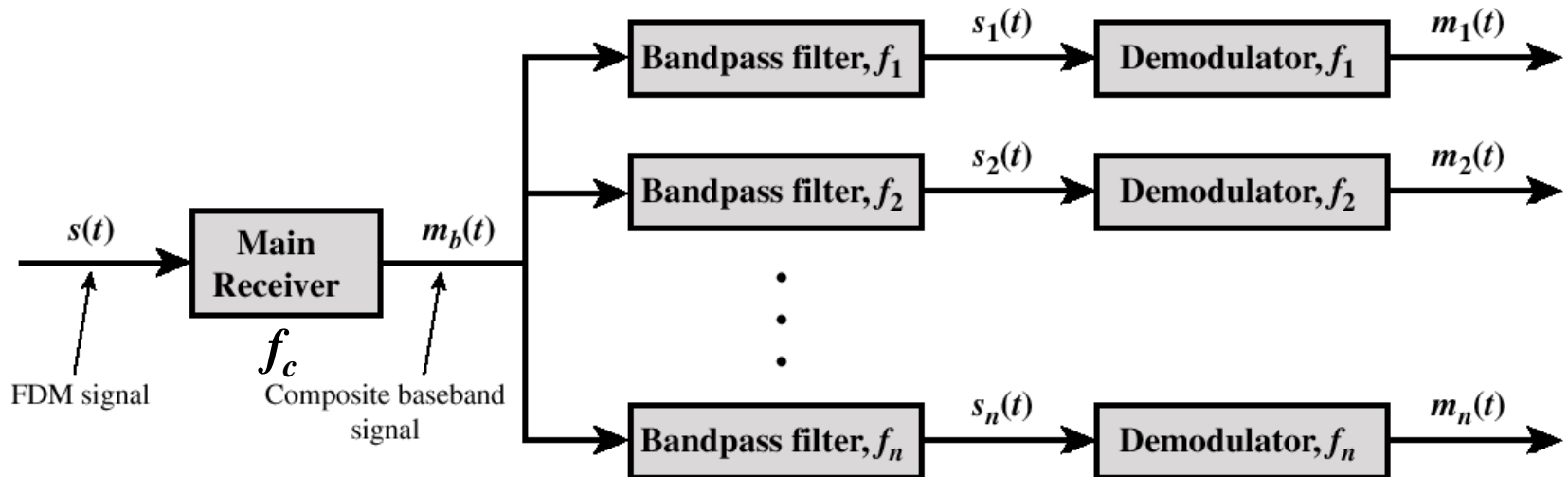




# Frequency-Division Multiplexing

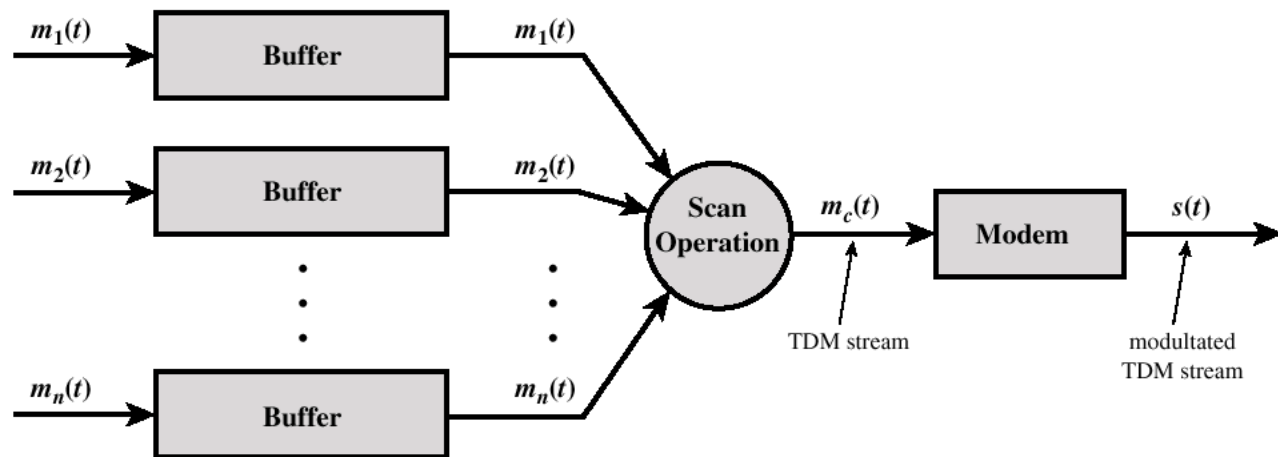
## - Receiver

- $m_b(t)$  is retrieved by demodulating the FDM signal  $s(t)$  using carrier  $f_c$
- $m_b(t)$  is passed through a parallel bank of bandpass filters – centered around  $f_i$
- The output of the  $i^{\text{th}}$  filter is the  $i^{\text{th}}$  signal  $s_i(t)$
- $m_i(t)$  is retrieved by demodulating  $s_i(t)$  using subcarrier  $f_i$

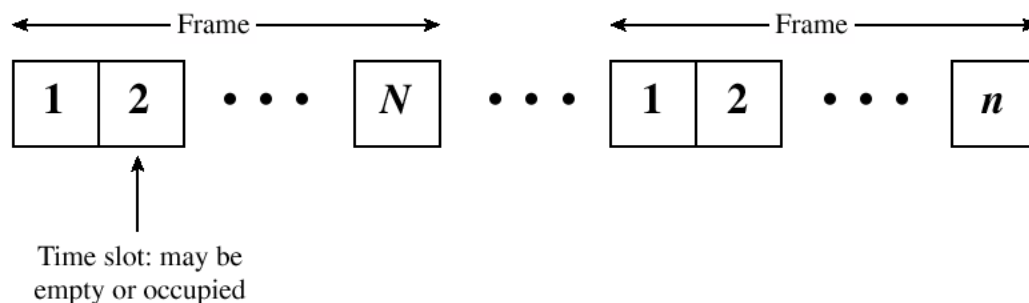


# Synchronous Time-Division Multiplexing - Transmitter

- Digital sources  $m_i(t)$  – usually buffered
- A scanner samples sources in a cyclic manner to form a frame
- $m_c(t)$  is the TDM stream or frame → frame structure is fixed
- Frame  $m_c(t)$  is then transmitted using a modem → resulting analog signal is  $s(t)$



(a) Transmitter

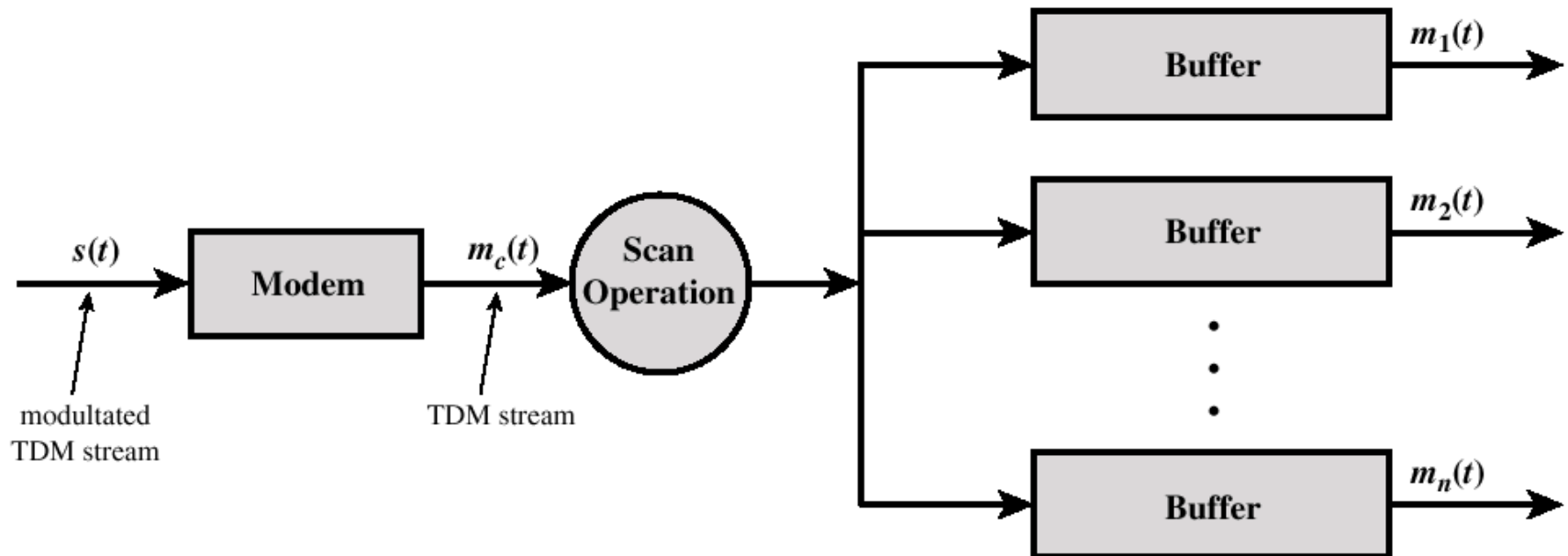


(b) TDM Frames

Fixed assignments – slot left empty if no data  
 No address requirement  
 Bit stuffing

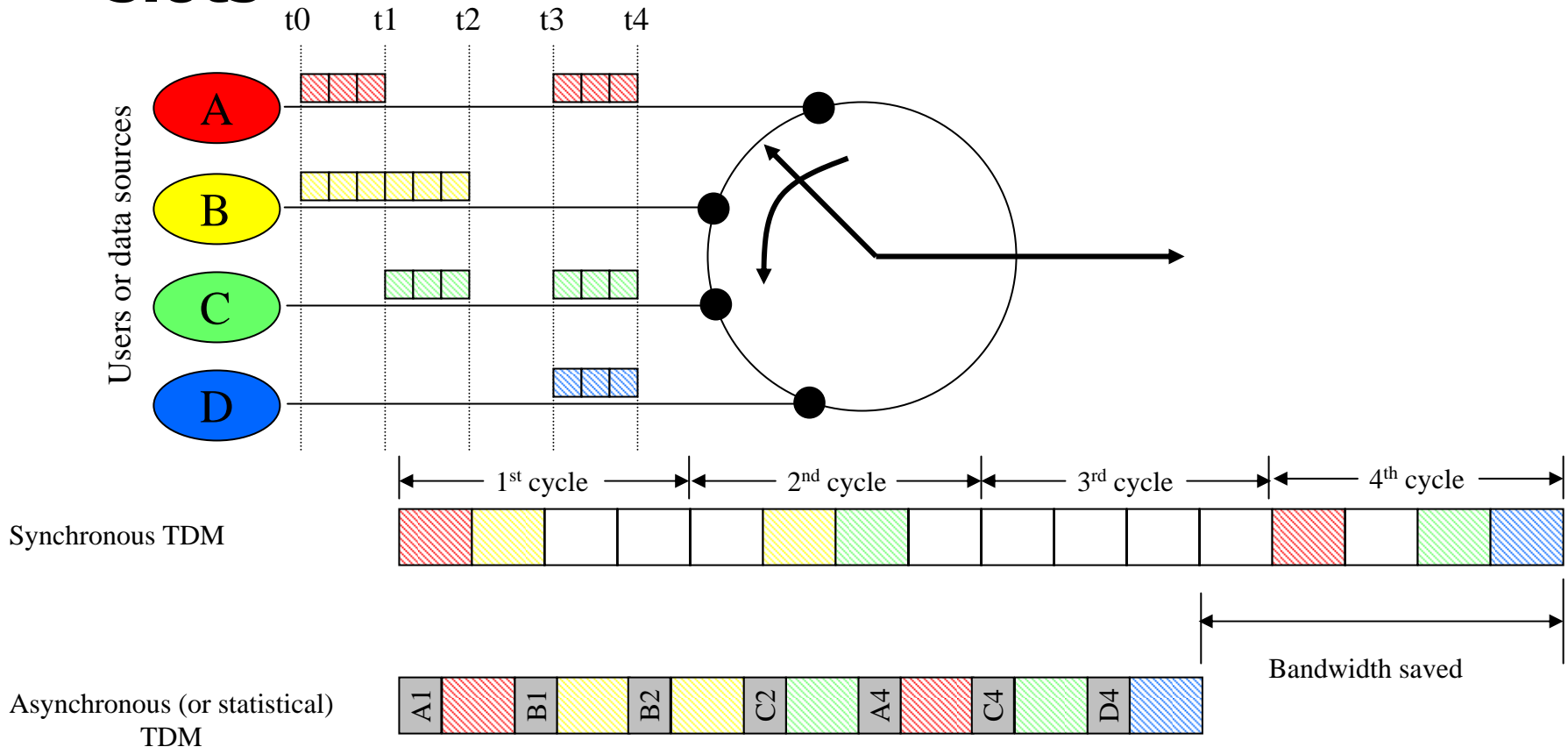
# Synchronous Time-Division Multiplexing - Receiver

- TDM signal  $s(t)$  is demodulated  $\rightarrow$  result is TDM digital frame  $m_c(t)$
- $m_c(t)$  is then scanned into  $n$  parallel buffers;
- The  $i^{\text{th}}$  buffer correspond to the original  $m_i(t)$  digital information



# Statistical Time-Division Multiplexing

- **Dynamic and on-demand allocation of time slots**



# Statistical Time-Division Multiplexing Frame Format

- Clearly, the aim of statistical TDM is increase efficiency by not sending empty slots
- But it requires overhead info to work:
  - Address field
  - Length field



(a) Overall frame



(b) Subframe with one source per frame



(c) Subframe with multiple sources per frame

# Statistical Time-Division Multiplexing – Modeling

- Data items (bits, bytes, etc) are generated at any time – source may be intermittent (bursty) not constant
- $R$  b/s is the peak rate for single source
  - $\alpha R$  b/s is the average rate for single source ( $0 \leq \alpha \leq 1$ )
- The *effective* multiplexing line rate is  $M$  b/s
- Each data item requires  $T_s$  sec to be served or tx-ed
- Data items may accumulate in buffer before server is able to transmit them → Queueing delay

