

Numbering Systems

Example: $10 = 1010_2$

Signed-magnitude:

- +10 = 01010_2 (using 5 bits.)
- 10 = 11010_2 (using 5 bits.)
- = 101010_2 (using 6 bits.)
- = $1\ 000\ 1010_2$ (using 8 bits.)

1's complement:

- +10 = 01010_2
- 10 = 10101 (using 5 bits.)

Rule: Given a positive integer N, with an integer part of n digits, and a fraction part of m digits, the (r-1)'s complement of N is given by: $r^n - r^{-m} - N$.

If the number does not contain a fraction part, let m = 0.

In the binary case r = 2

2's complement:

Rule: The r's complement of N is $r^n - N$.

In the binary case r = 2; and 2's complement = 1's complement + 1

Example:

	Binary	Signed Magnitude	1's Complement	2's complement:
Given n = 3 bits.	000	+0	+0	+0
	001	+1	+1	+1
	010	+2	+2	+2
	011	+3	+3	+3
	100	-0	-3	-4
	101	-1	-2	-3
	110	-2	-1	-2
	111	-3	-0	-1

Range:

For an n-bit number:

Representation	Min.	Max.	Note
Unsigned Binary	0	$2^n - 1$	Unique zero representation
Signed Magnitude	$-(2^{n-1} - 1)$	$+(2^n - 1)$	Redundant 0
1's Complement	$-(2^{n-1} - 1)$	$+(2^n - 1)$	Redundant 0
2's complement:	$-(2^{n-1} - 1)$	$+(2^n - 1)$	Unique zero representation

Overflow:

Overflow condition: $OVF = C_{n-1} \text{ XOR } C_n$