KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF COMPUTER SCIENCES & ENGINEERING
COMPUTER ENGINEERING DEPARTMENT

CSE 642 – Computer Systems Performance
December 10th, 2009 – Midterm Exam

Student Name:
Student Number:
Exam Time: 90 mins

- Do not open the exam book until instructed
- The use of programmable calculators and cell phone calculators is not allowed – only basic calculators are permitted
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

Question No.	Max Points	
1	40	
2	40	
3	40	

Total: 120

Q1.) (40 points) For X is an non-negative random variable, the Markov inequality states that the probability of X being greater or equal to some quantity t should be bounded (less or equal) to the expectation of X divided by t, in other words,

$$P(X \geq t) \leq \frac{E[X]}{t} \qquad t \geq 0$$

a) **(15 points)** Use Markov's inequality to prove Chebyshev bound which is given by $P(|X - E[X]| \geq \varepsilon) \leq \frac{Var[X]}{\varepsilon^2}$ for any $\varepsilon \geq 0$.

b) **(25 points)** Let X be uniform random variable in the interval $[-b, b]$.

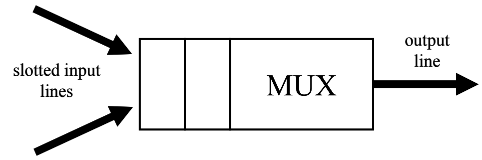
   a. **(5 points)** Specify and plot the cumulative probability distribution function (CDF) for X.

   b. **(5 points)** Compute the mean and variance for X.

   c. **(10 points)** Compute the following quantity $P(|X - E[X]| \geq \varepsilon)$ for some $0 \leq \varepsilon \leq 2b$.

   d. **(5 points)** Compare and *plot* the Chebyshev bound and the exact probability for the event $\{|X - E[X]| \geq \varepsilon\}$ as a function of ε for the random variable X.

Q.2) (40 point) Data in the form of fixed-length packets arrive in slots on the THREE input lines of a multiplexer. A slot contains a packet with probability *p*, independent of the arrivals during other slots or on the other line. The multiplexer transmits one packet per time slot and has the capacity to store THREE packets only. If no room for a packet is found, the packet is dropped.



a) **(10 points)** COMPUTE the probability of *j* (for all possible *j* values) packets arriving on the three input lines during any given time slot. *What* is the distribution of *j*?

b) **(10 points)** DRAW the state transition diagram and DEFINE the transition matrix **P** in terms of *p* – The state is taken to be the number of packets in the multiplexer.

c) **(20 points)** DEFINE, in words, and WRITE an expression for each of the following quantities – define the used terms:

1) Multiplexer average input load in terms of packets per time slot.
2) Multiplexer throughput in terms of packets per time slot.
3) Multiplexer <u>mean number</u> of dropped packets per slot.

Q.3) **(40 points)** Consider a Poisson arrival process that is specified by

$$P_k(t) = P\left(X(t) = k\right) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \qquad k = 0, 1, \ldots$$

a) **(10 points)** Show that the process has independent increments.

b) **(10 points)** Show that the process has stationary increments.

c) **(20 points)** Write the corresponding Kolmogorov forward differential equations *and* show that the given PDF is a solution for these equations.