KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF COMPUTER SCIENCES & ENGINEERING
COMPUTER ENGINEERING DEPARTMENT
CSE 642 – Computer Systems Performance
Assignment 2 – Due date Dec 7th, 2009

Problem 1 (20 points - Discrete-Time Random Processes):

A discrete-time random process X_n is defined as follows. A fair coin is tossed. If the outcome is heads, $X_n = 1$ for all n; if the outcome is tails, $X_n = -1$ for all n.

a) Sketch some sample paths of the process.

b) Find the pmf for X_n.

c) Find the joint pmf for X_n and X_{n+k}.

d) Find the mean and autocovariance functions of X_n.

Problem 2 (20 points - Continuous-Time Random Processes):

Messages arrive at a computer from two telephone lines according to independent Poisson processes of rates λ_1 and λ_2, respectively.

a) Find the probability that a message arrives first on line 1.

b) Find the pdf for the time until a message arrives on either line.

c) Find the pmf for $N(t)$, the total number of messages that arrive in an interval of length t.

d) Generalize the result of Part c for the “merging” of k independent Poisson processes of rates λ_1, λ_2 λ_3, …, λ_k, respectively: $N(t) = N_1(t) + N_2(t) + \ldots + N_k(t)$.

Problem 3 (20 points - Stationary/Wide-Sense/Ergodic Processes):

Let $X(t) = A\cos(2\pi ft + \Theta)$, where A is a random variable with mean m and variance σ^2. Θ is uniformly distributed in $(0, 2\pi)$. A and Θ are independent random variables.

a) Evaluate $\langle X(t)\rangle_T$, find its limit as $T \rightarrow \infty$, and compare to $m_X(t)$.

b) Evaluate $\langle X(t+\tau)X(t)\rangle_T$, find its limit as $T \rightarrow \infty$, and compare to $R_X(t+\tau,t)$.

Problem 4 (40 points - Markov Chains):

Data in the form of fixed-length packets arrive in slots on the FOUR input lines of a multiplexer. A slot contains a packet with probability p, independent of the arrivals during other slots or on the other line. The multiplexer transmits one packet per time slot and has the capacity to store THREE packets only. If no room for a packet is found, the packet is dropped.

a) COMPUTE the probability of j (for all possible j values) packets arriving on the four input lines during any given time slot.
b) DRAW the state transition diagram and SPECIFY the transition matrix **P** – The state is taken to be the number of packets in the multiplexer.
c) If p is equal to 0.4, what is the probability that the MUX will contain 3 packets after 10 time slot (i.e. at the start of the 11th time slot)? Assume that we start with an empty MUX.
d) Let the load be defined as the mean number of arriving packets per time slot while throughput be defined as the mean number of transmitted packets per time slot. Use Matlab and show the code for:

1) Plot the throughput versus the input load.
2) Evaluate and plot the mean number of dropped packets per time slot.

Problem 5 (40 points - Pure Birth Processes):

Consider the population of bacteria of size $N(t)$ at time t for which $N(0) = 1$, We consider this to be a pure birth process in which any member of the population will split into two new members in the interval $(t,t+\delta)$ with probability $\lambda\delta+O(\delta)$ or will remain unchanged in this interval with probability 1- $\lambda\delta+O(\delta)$ as $\delta \rightarrow$ zero.

a) Let $P_k(t) = \text{P}(N(t) = k)$ and write down the set of differential-difference equations that must be satisfied by these probabilities.

b) From part (a) show that the z-transform $N_N(z,t)$ for $N(t)$ must satisfy $N_N(z,t) = \dfrac{ze^{-\lambda t}}{1-z+ze^{-\lambda t}}$

c) Find $\text{E}[N(t)]$.

d) Solve for $P_k(t)$.

Hint: *if the probability generating function for N(t) is given by* $N_N(z,t) = \sum_{k=0}^{\infty} P_k(t) z^k$ *, then its partial derivative with respect to time is given by* $\dfrac{\partial N_N(z,t)}{\partial t} = \sum_{k=0}^{\infty} \dfrac{dP_k(t)}{dt} z^k$ *. You may need to solve a partial differential equation to arrive at the solution for part b. That can be accomplished using Lagrange's method for solving partial differential equations or maybe through the use symbolic math packages such as Maple, Matlab, etc.*

Problem 6: (40 points - Poisson Random Process)

Consider a Poisson arrival process that is specified by

$$P_k(t) = P\left(X(t) = k\right) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \qquad k = 0,1,...$$

a) Show that the process has independent increments.
b) Show that the process has stationary increments.
c) Write the corresponding Kolmogorov forward differential equations and show that the given PDF is a solution for these equations.
d) What is meant by "Poisson Arrivals See Time Averages" or the PASTA principle? Give a counter example where the PASTA principle does not hold for an arrival process of your choice.