

King Fahd University of Petroleum & Minerals Computer Engineering Dept

COE 241 – Data and Computer
Communications

Term 152

Dr. Ashraf S. Hasan Mahmoud

Rm 22-420

Ext. 1724

Email: ashraf@kfupm.edu.sa

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

1

Lecture Contents

1. Fourier Analysis {Please Refer to Fourier Series Expansion Notes}
 - a. Fourier Series Expansion
 - b. Fourier Transform
 - c. Ideal Low/band/high pass filters
2. Analog and Digital Data Transmission
 - a. Analog and Digital Data
 - b. Analog and Digital Signals
 - c. Analog and Digital Transmission
 - d. Asynchronous and Synchronous Transmission
3. Transmission Impairments
 - a. Attenuation and Attenuation Distortion
 - b. Delay Distortion
 - c. Noise
4. Channel Capacity
 - a. Nyquist Formula
 - b. Shannon Capacity Formula
 - c. Eb/No expression

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

2

Fourier Analysis

- **For Material related to**
 - Fourier Series Expansion,
 - Fourier Transform, and
 - Z-Transform

Refer to instructor notes in "Fourier Series Expansion" package

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

3

Analog and Digital Data Transmission

- **The terms:**
 - Analogue ~ continuous
 - Digital ~ discrete
- **They apply to:**
 - A) Info/Data: the information to be delivered
 - B) Signaling: the electrical or electromagnetic wave that propagates carrying the data
 - C) Transmission: the mechanism of delivering the data by processing and propagation of signal

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

4

Examples of Data/Signaling: (1) Voice / AUDIO

- Most familiar type of *analogue* data
- **Audio:** Human Ear (Receiver) bandwidth is ~10 Hz to ~20KHz
 - You can not hear sounds with frequencies much higher than 20KHz or much lower than 10 Hz – Some other animals can do that (bats, whales, etc)
- **Voice:** Human speech (Data) is mostly between 100 Hz and 7K Hz – with most of the energy concentrated in the lower part of this range (300-3400 Hz)

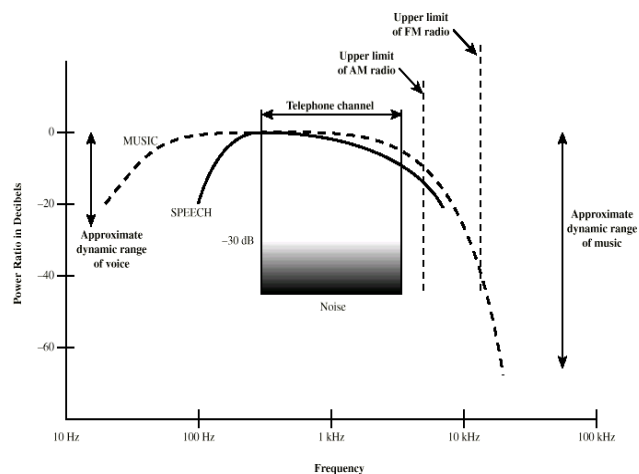
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

5

Examples of Data/Signaling: (1) AUDIO – cont'd

- Typical Speech has a dynamic range of 25 dB – ratio of strongest speech signal to weakest speech signal is 25 dB or $10^{25/10} = 300$ (in linear scale)
- The *Telephone Channel* has a bandwidth of about 3.1KHz (from 300 Hz to 3400 Hz)
- Note that Music has a much wider bandwidth than speech (~10 Hz to ~20 KHz) – Hence a good audio system (CD player, high end speakers, etc) should be able to reproduce these signals
- Music has also a higher dynamic range too – What is the dynamic range of your audio system?



2/17/2016

Dr. Ashraf S. Hasan Mahmoud

6

Examples of Data/Signal VIDEO

Applicable to Cathode Ray Tube (CRT) monitors only; i.e. OLD technology!!
Does not apply to LCD/LED/PLASMA technologies.

- Another common example of analog data
- The original scene (source) is scanned and its image recorded by the camera – RASTER image
- In the TV: a moving electron beam scan the screen producing the picture
 - For black and white: the amount of illumination produced (on a scale from black (lowest) to white (highest) at any point is proportional to the beam intensity
- Hence the original brightness is REPRODUCED on the screen
- Video Image \leftrightarrow Time varying analog signal

2/17/2016

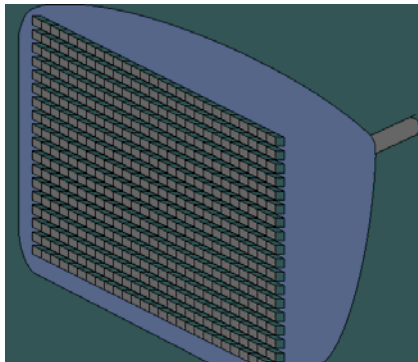
Dr. Ashraf S. Hasan Mahmoud

7

Examples of Data/Signal VIDEO – cont'd

Applicable to Cathode Ray Tube (CRT) monitors only; i.e. OLD technology!!
Does not apply to LCD/LED/PLASMA technologies.

- Scanning Process:
 - Starts at the far left – near top
 - Scans 241.5 lines
 - Ends at middle of screen – lowest part
 - Beam is repositioned at the top again
 - Scanning starts again for the other 241.5 lines (interlaced with the previous lines)



2/17/2016

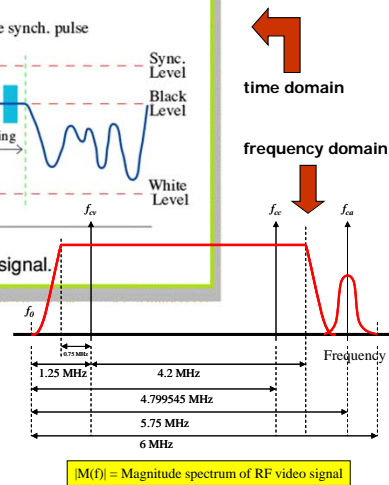
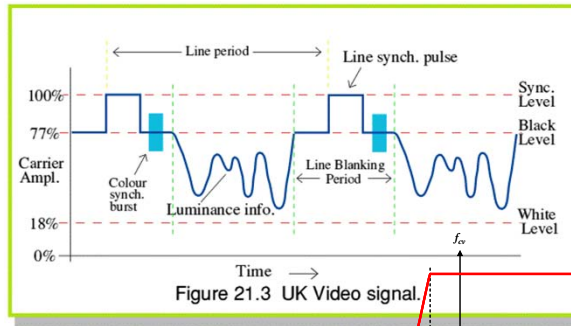
Dr. Ashraf S. Hasan Mahmoud

8

Examples of Data/Signal VIDEO – Example of a Video signal

Applicable to Cathode Ray Tube (CRT) monitors only; i.e. OLD technology!!
Does not apply to LCD/LED/PLASMA technologies.

- For B/W TV you ignore the color synch bursts
- Note
 - the high voltage level corresponds to "BLACK"
 - the low voltage level corresponds to "WHITE"
- What is the line synch pulse?
- What is the purpose of the line blanking period?



2/17/2016

Dr. Ashraf S. Hasan Mahmoud

9

Current Monitor Technologies

- TFT LCD
 - Fluorescent back-lit monitors
 - LED back-lit monitors
- Plasma monitors
 - Higher contrast – Deeper colors – No need to back lighting
- Organic LED (OLED) – organic compound that emits light when current is passed
- Active Matrix OLED (AMOLED)
 - Currently used for mobile devices – no TVs yet!
- These monitors are handled very much similar to ROM cells!

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

10

Examples of Data/Signaling: (5) TEXT

- Digital Data (pre-defined set of symbols)
 - Same as Morse Code
- IRA (or ASCII in the US) define 128 character using 7-bit words
- When transmitted or stored 1Byte or 8-bit words are used
 - A parity bit is added as a simple error detection technique
- The signal representing this data:
 - One voltage level for binary one
 - Another voltage level for binary zero



Figure 3.10 Attenuation of Digital Signals

- What is the bandwidth for such binary signal?

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

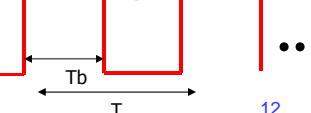
11

Examples of Data/Signaling: (5) TEXT - Bandwidth Estimation for Binary Signal

- Consider a binary signal of rate R_b (i.e. bit duration $T_b = 1/R$)
- Maximum bandwidth is required **when bits alternate** between 0 and 1 → This results in a periodic square waveform whose
 - Period for periodic signal $T = 2T_b$ sec
 - Fundamental frequency $f_0 = 1/T = 1/(2T_b) = R_b/2$ Hz
 - Min frequency may be zero (DC) – depending on the voltage levels and randomization of bits
- Theoretical BW is infinite – Note the sharp corners in the signal!
 - Contains infinite number of multiples of f_0 : $1f_0, 2f_0, 3f_0, \dots \infty$.
 - But most of the energy is located at first few multiples of f_0 (Refer to Fourier Series Expansion Examples)

- Practical bandwidth is
 - $(0, f_0=R_b/2)$ – very conservative, or
 - $(0, 2f_0=R_b)$ – very practical

Binary signal with alternating bits
Rate = R_b bit/sec
Bit duration = $\frac{1}{R_b} = T_b$ sec



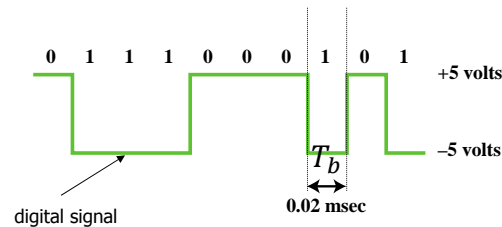
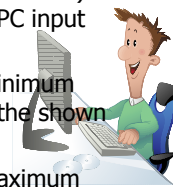
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

12

Examples of Data/Signalling: (6) TEXT

- Figure 3.12 (textbook) – conversion of PC input digital signals.
- What is the minimum frequency for the shown signal?
- What is the maximum frequency for the shown signal?
- What is the PRACTICAL bandwidth for the shown signal?



User input at a PC is converted into a stream of binary digits (1s and 0s). In this graph of a typical digital signal, binary one is represented by -5 volts and binary zero is represented by +5 volts. The signal for each bit has a duration of 0.02 msec, giving a data rate of 50,000 bits per second (50 kbps).

$$\text{Bit rate} \\ R_b = 1/T_b$$

Figure 3.12 Conversion of PC Input to Digital Signal

2/17/2016

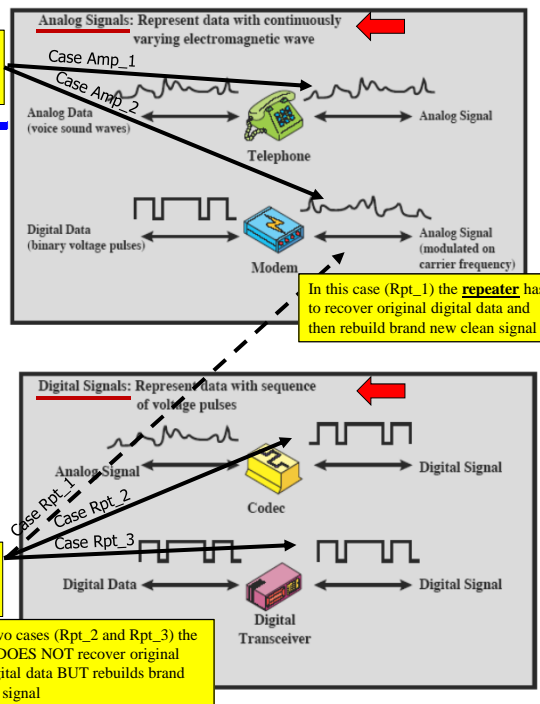
Dr. Ashraf S. Hasan Mahmoud

13

Analog Transmission – Amplifiers are used to transmit analog signals

Analog and Digital Signaling of Analog and Digital Data

Digital Transmission – Repeaters are used to transmit digital signals



2/17/2016

Transmission

- **Analog Transmission:**
 - Analog signal is propagated through amplifiers to compensate for attenuation and to achieve longer distance
 - **Amplifiers:**
 - Boost signal and noise equally
 - May distort original signal
 - Can not be used indefinitely
- **Digital Transmission:**
 - To overcome the higher attenuation, repeaters are used at appropriately spaced points
 - **Repeaters:**
 - Recover original digital data
 - Transmit new signal
 - Can be used indefinitely

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

15

Transmission (2)

- Digital Transmission is the prevailing technology:
 - Digital Technology: Capitalize on advances in digital circuitry
 - Data Integrity: With the use of repeaters, the effects of noise and other signal impairments are not cumulative
 - Capacity Utilization: It is easier to multiplex several digital signals (using TDM) on one high capacity link as opposed to multiplexing analog sources using FDM
 - Security and Privacy: Use of encryption
 - Integration: Provides a uniform vehicle to transport both analog and digital data

very important reasons the popularity of digital transmission

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

16

Asynchronous and Synchronous Transmission - Timing Requirement

- Reception of digital data requires sampling of received signal at receiver → Sampling time should be known
- Clock drift (example):
 - If a receiver clock drifts by 1% every sample time,
 - Then for $T_b = 1\mu\text{sec}$, total drift after 50 bit times = $50 \times 0.01 = 0.5\mu\text{sec}$
 - Hence, instead of sampling at the middle of the bit time, the receiver will sample at the edge of the bit (I.e. receiver is out-of-synch with transmitter clock)
- For correct reception, receiver clock/carrier should be **synchronized** with transmitter

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

17

Asynchronous Transmission

- Exploits: Rx-er can remain for short period in synch with Tx-er
- Used for short stream of bits – data transmitted one character (5 ~ 8 bits) at a time
- Synchronization is needed to be maintained for the length of short transmission
- Character is delimited (start & end) by known signal elements: start bit – stop element
- Rx-er re-synchs with the arrival of new character

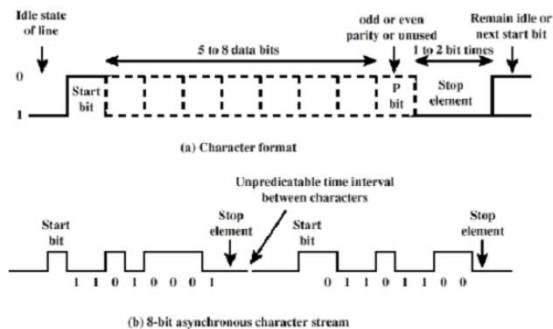
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

18

Asynchronous Transmission (2)

- Simple / Cheap
- Efficiency: transmit 1 start bit + 8 bit of data + 2 stop bits → Efficiency = $8/11 = 72\%$ (or overhead = $3/11 = 28\%$)
- Good for data with large gaps (e.g. keyboard, etc)



2/17/2016

Dr. Ashraf S. Hasan Mahmoud

19

Example:

6-5: An asynchronous transmission scheme uses 8 bits, an even parity, and a stop element of length 2 bits. What percentage of clock inaccuracy can be tolerated at the receiver with respect to the framing error? Assume that the bit samples are taken at the middle of the clock period. Also assume that at the beginning of the start bit the clock and incoming bits are in phase.

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

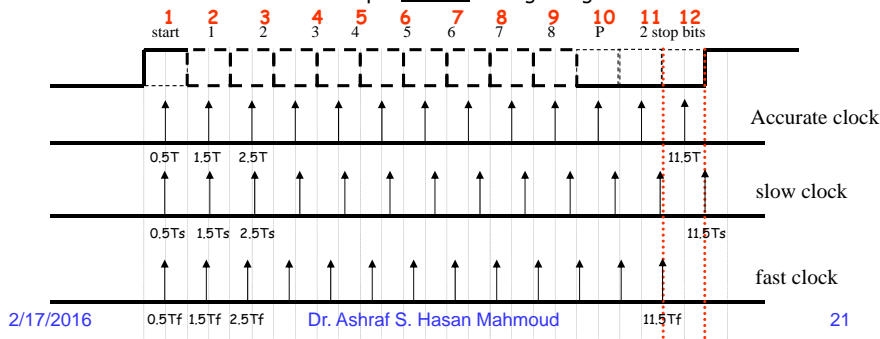
20

Example: solution

An accurate clock will start in phase (middle of first bit) and end in phase (middle of last bit)

However, a slow clock (time between two consecutive samples increases) will start in phase but will sample the last bit away from the middle of the actual bit duration – for this not to make a mistake it should sample at most at end of the last bit duration

For a fast clock (time between two consecutive samples decreases) will start in phase but will sample the last bit before the middle of the actual bit duration – for this clock not to make a mistake it should sample at least at beginning of the last bit duration



Example: solution (2)

Let the bit duration be T . Then a frame is $12T$ long.

Let a clock period be T' . The last bit (bit 12) is sampled at $11.5T'$.

For a fast running clock, the condition to satisfy is

$$11.5T' > 11T \Rightarrow \frac{T}{T'} < \frac{11.5}{11} = 1.045 \Rightarrow f_{clock} < 1.045 f_{bit}$$

For a slow running clock, the condition to satisfy is

$$11.5T' < 12T \Rightarrow \frac{T}{T'} > \frac{11.5}{12} = 0.958 \Rightarrow f_{clock} > 0.958 f_{bit}$$

Therefore, the overall condition: $0.958 f_{bit} < f_{clock} < 1.045 f_{bit}$

Synchronous Transmission

- What if there is a STEADY STREAM of bits between Tx-er and Rx-er
 - Still use the start/stop bits → low efficiency
 - Use synchronous transmission
- Synchronous Techniques:
 - Provide SEPARATE clock signal
 - Expensive and only good for short distances
 - Depend on data encoding to extract clock info
 - E.g. Manchester encoding

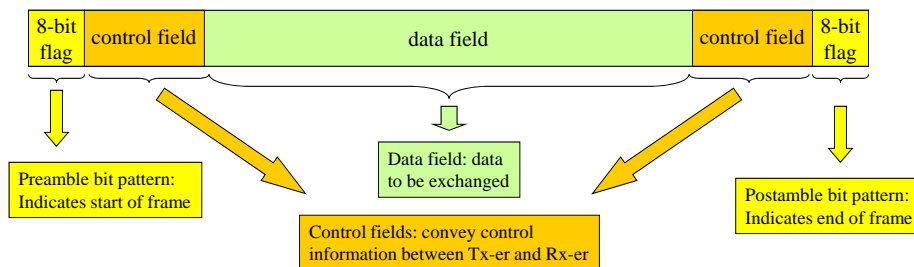
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

23

Synchronous Frame Format

- Typical Frame Structure



- For large data blocks, synchronous transmission is far more efficient than asynchronous:
 - E.g. HDLC frame (to be discussed in Chapter 7): 48 bits are used for control, preamble, and postamble – if 1000 bits are used for data → efficiency = 99.4% (or overhead = 0.6%)

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

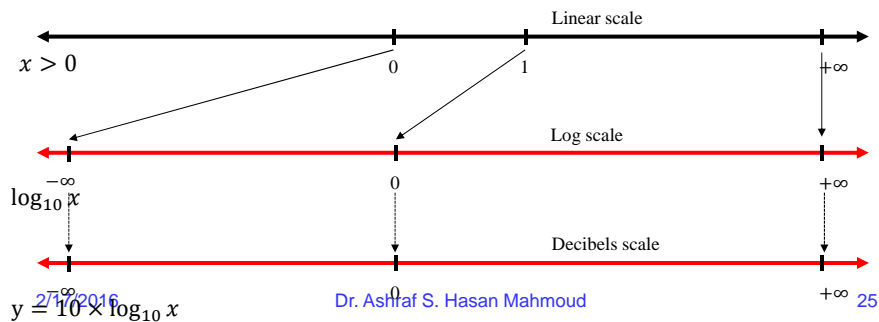
24

Linear, Log10 , and Decibels (dBs)

- Linear $x > 0$ – positive real number \rightarrow Decibels are defined as $y = 10 \times \log_{10}(x)$
- Representation of very small values of x (i.e. $0 < x < 1$) are expanded to $(-\infty, 0)$, and representation of large values of x (i.e. $1 < x < \infty$) are mapped to $(0, \infty)$
- Quantities in engineering are typically reported in Decibels (dBs) – e.g. power
- Conversion:

$$\text{Linear} \rightarrow \text{Decibels: } y = 10 \times \log_{10} x$$

$$\text{Decibels} \rightarrow \text{Linear: } x = 10^{(y/10)}$$



Dr. Ashraf S. Hasan Mahmoud

Power Gain (Loss)

Refer to Appendix 3A
Textbook page 131

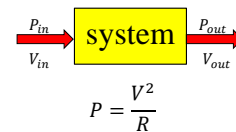
- Consider a system (wire, channel, etc.) where the
 - Power of signal at the input is P_{in}
 - Power of signal at the output is P_{out}
- System power gain in decibels G_{dB} is defined as

$$G_{dB} = 10 \times \log_{10} \left(\frac{P_{out}}{P_{in}} \right)$$

$$= 20 \times \log_{10} \left(\frac{V_{out}}{V_{in}} \right)$$

- Gain of -40 dB is Loss of 40 dB
- Example:** If a signal with power level of 10 mW is inserted into a transmission line and the measured power some distance away is 5 mW – Compute the power loss for such system:
- Solution:**

$$L_{dB} = -10 \times \log_{10} \left(\frac{P_{out}}{P_{in}} \right) = -10 \times \log_{10} \left(\frac{5}{10} \right) = 3 \text{ dB}$$



$$P = \frac{V^2}{R}$$

Amplification - Attenuation (Loss)

Power ratio (P_{out}/P_{in})	Gain in Decibels (G_{dB})	Power ratio (P_{out}/P_{in})	Gain in Decibels (G_{dB})
1	0	0.5	-3.01
2	3.01	10^{-1}	-10
10^1	10	10^{-2}	-20
10^2	20	10^{-3}	-30
10^3	30	10^{-4}	-40
10^4	40	10^{-5}	-50
10^5	50	10^{-6}	-50

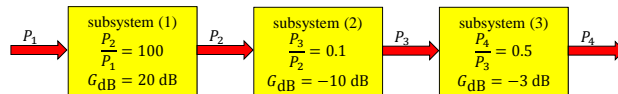
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

26

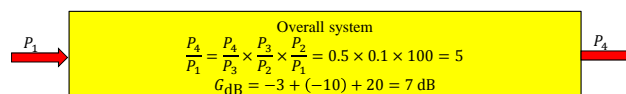
Power Gain (Loss) – cont'd

- **Example:** Consider a system consisting of multiple subsystems as shown below – Compute the overall power gain for the system



- **Solution:**

Recall that:
 $\log(a \times b) = \log(a) + \log(b)$



- Multiplying power ratio is EQUIVALENT to adding decibels (easier)!
- Decibel is a MEASURE OF RELATIVE DIFFERENCE

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

27

Decibel-Watt (dBW) and Decibel-milliwatt (dBm)

- dBW is ratio of power in Watts relative to 1 Watt represented in decibels
- Example:
 - $X = 5 \text{ Watts} \rightarrow X_{\text{dBW}} = 10 \times \log_{10}(5 \text{ Watts}/1 \text{ Watt}) = 10 \times \log_{10}(5) = 7 \text{ dBW}$
 - $X = 20 \text{ mW} \rightarrow X_{\text{dBW}} = 10 \times \log_{10}(0.02 \text{ Watts}/1 \text{ Watt}) = 10 \times \log_{10}(0.02) = -17 \text{ dBW}$
- dBm (or dBmW) is ratio of power in milliwatts relative to 1 milliwatt represented in decibels
- Example:
 - $X = 5 \text{ Watts} \rightarrow X_{\text{dBm}} = 10 \times \log_{10}(5000 \text{ mW}/1 \text{ mW}) = 10 \times \log_{10}(5000) = 37 \text{ dBm}$
 - $X = 20 \text{ mW} \rightarrow X_{\text{dBm}} = 10 \times \log_{10}(20 \text{ mW}/1 \text{ mW}) = 10 \times \log_{10}(20) = 13 \text{ dBm}$
- To convert from
 - dBW to dBm \rightarrow Add 30 (i.e. multiply by 1000)
 - dBm to dBW \rightarrow Subtract 30 (i.e. divide by 1000)

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

28

Transmission Impairments

- Impairments can degrade the quality of an analog signal or cause a bit (symbol) error for a digital signal
- Types of Impairments:
 - Attenuation and Attenuation Distortion
 - Delay Distortion
 - Noise

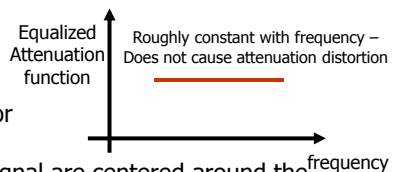
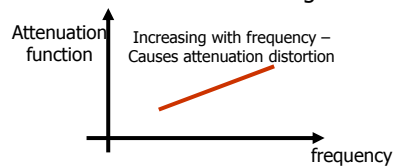
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

29

Attenuation and Attenuation Distortion

- A received signal must have sufficient strength for proper detection at receiver
- For error-free communication signal strength relative to noise must be high
- Attenuation is an increasing function of frequency:
 - Different components of signal are subject to different attenuation → Distortion in time domain
- Solution: Equalize transmission
 - Results in almost equal attenuation (gain) for all frequencies of interest
- Attenuation Distortion is less of a problem for digital signals:
 - Frequencies of interest for a digital signal are centered around the fundamental frequency, f
 - Attenuation function has to be flat around f only



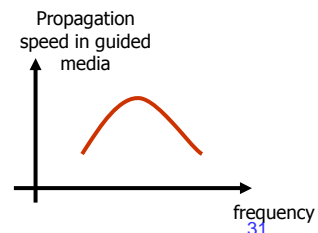
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

30

Delay Distortion

- For guided media – different frequency components have different propagation speeds
- For unguided media – multipath (signal being received through more than one path) causes delay distortion
- Received signal is distorted due to varying delays experienced at its constituent frequencies
- Critical for digital data: Causes **Intersymbol interference** – a major limitation on maximum bit rate over a transmission channel
- Solution: Equalization



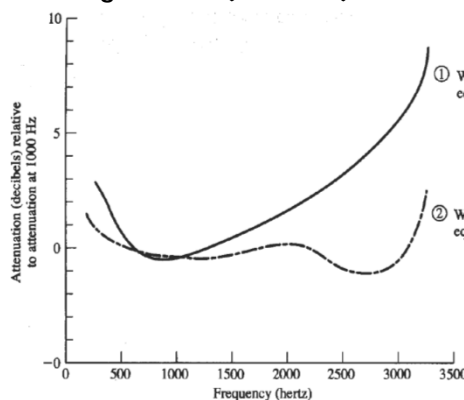
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

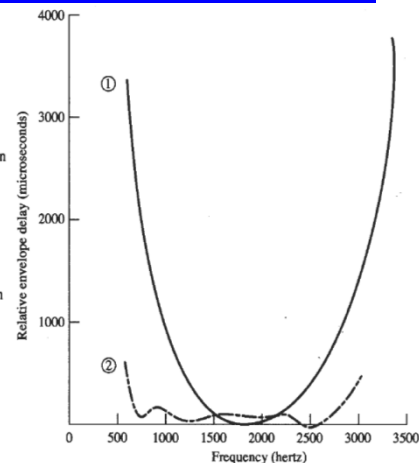
31

Attenuation and Delay Distortion – Effect of Equalization

- **Figure 3.15 (textbook)**



(a) Attenuation



(b) Delay Distortion

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

32

Noise

- Major limiting factor in communication system performance
- Types of Noise:
 - Thermal Noise
 - Intermodulation Noise
 - Crosstalk
 - Impulse Noise

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

33

Thermal Noise

- Due to agitation of electrons
- Function of temperature (energy of electrons)
- Can not be eliminated → limits communication system performance
- Noise power density (noise power found in 1 Hz) , N_0 , is given by

$$N_0 = kT \quad (\text{Watts/Hz})$$

Where k is Boltzman constant = 1.3803×10^{-23} J/degree Kelvin

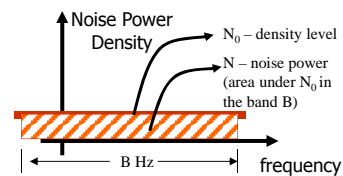
T is the temperature in degrees Kelvin

- Hence, the thermal noise power in a bandwidth B Hz is given by

$$N = N_0 \times B = kT \times B \quad (\text{Watts})$$

- In decibels:

$$\begin{aligned} N_{dB} &= 10\log k + 10\log T + 10\log B \\ &= -228.6 \text{ dBW} + 10\log T + 10\log B \end{aligned}$$



2/17/2016

Dr. Ashraf S. Hasan Mahmoud

34

Examples:

- **Problem:** Given an amplifier with effective noise temperature of 10,000 degrees Kelvin, and a 10-MHz bandwidth, what thermal noise we expect at the output

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

35

Examples:

- **Solution:**

$$N = kT \times B$$

$$k = 1.38 \times 10^{-23} \text{ J/Kelvin}, T = 10,000 \text{ degrees Kelvin}, B = 10 \times 10^6 \text{ Hz},$$

$$N = 1.38 \times 10^{-12} \text{ Watts}$$

$$N_{\text{dBW}} = 10 \log N = -118.6 \text{ dBW}$$

In dBmW, one can write

$$N_{\text{dBmW}} = 10 \log N \times 1000 = -88.6 \text{ dBmW} \text{ or simply, } N_{\text{dBmW}} = N_{\text{dBW}} + 30$$

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

36

Intermodulation Noise

- System:

$$S \rightarrow H(S)$$

- **Linear System:**

$$H_L(S_1 + S_2) = H_L(S_1) + H_L(S_2)$$

- E.g. $H_L(S) = A \times S + B$ – where A and B are constants
- Consider the input $S_1 + S_2$, where $S_1 = \cos(2\pi f_1 t)$ and $S_2 = \cos(2\pi f_2 t)$. The system output is

$$H_L(S_1 + S_2) = A \times \cos(2\pi f_1 t) + A \times \cos(2\pi f_2 t) + B$$
- Note the output signal has frequencies f_1 and f_2 *ONLY*.

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

37

Intermodulation Noise – cont'd

- **NonLinear System (example):** $H_{NL}(S) = A_2 \times S^2 + A_1 \times S + A_0$

The output (for the same input) is

$$H_{NL}(S_1 + S_2) = A \times [\cos(2\pi f_1 t) + \cos(2\pi f_2 t)]^2 + B \times [\cos(2\pi f_1 t) + \cos(2\pi f_2 t)] + B$$

Note that $[\cos(2\pi f t)]^2 = \frac{1}{2} + \frac{1}{2} \cos(2\pi 2f t)$, and

$$\cos(2\pi f_1 t) \cos(2\pi f_2 t) = \frac{1}{2} \cos(2\pi (f_1 + f_2) t) + \frac{1}{2} \cos(2\pi (f_1 - f_2) t)$$

Output signal contain terms with multiples of $(f_1 + f_2)$ and $(f_1 - f_2)$

- Intermodulation noise: *undesired* signals at the frequency that is multiples of sum or difference of the two original input frequencies
- Caused by **system nonlinearity**

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

38

Crosstalk/Impulse Noise

- Crosstalk:
 - Unwanted coupling between signal paths
 - E.g. electrical coupling between near by twisted pair wires
 - Coax cables are more immune to cross talk compared to twisted pairs
- Impulse Noise:
 - Unlike previous types of noise, this one is:
 - Noncontinuous – irregular pulses or spikes for short duration and high amplitude
 - Causes:
 - Lightening
 - Faults or flaws in communication systems
 - Major concern for digital data

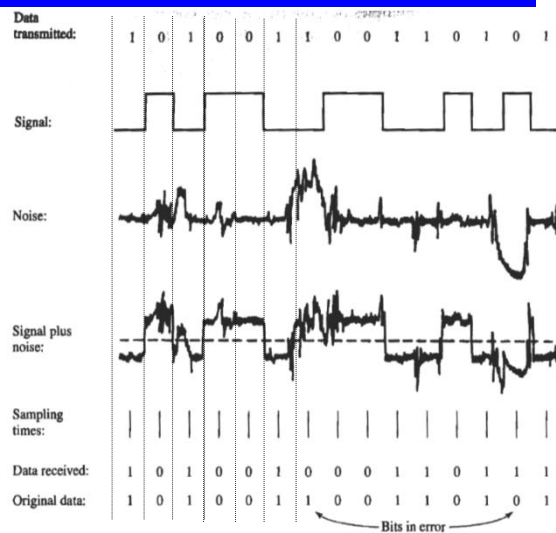
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

39

Effect of Noise on Digital Signal

- Figure 3.15 (textbook)
- Data → Signal: Encoding – to be covered in chapter 5
- Noise – the subject of this section
- Note the sampling times – middle of bit time
- Errors – Error control to be covered in chapter 5



2/17/2016

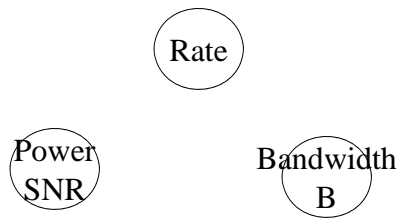
Dr. Ashraf S. Hasan Mahmoud

40

Channel Capacity

- Terminology:

- Data Rate (R): bit rate of channel – bits per second
- Bandwidth (B): bandwidth of transmitted signal – Hz
- Noise power (N): average noise power level for communication channel – Watt/Hz for density or Watt for noise power
- Error rate (Pe): rate at which an erroneous detection is made (detecting 0 for 1 and 1 for 0)



2/17/2016

Dr. Ashraf S. Hasan Mahmoud

41

Nyquist Bandwidth

- For a **noise-free** channel → data rate is limited by B of channel
- A bandwidth of 1Hz is enough to support 2 symbols/second (i.e. bauds)
 - B Hz is enough to support 2B symbols/second or bauds
- Example: B = 3100 Hz (telephone channel) → C = 6200 b/s
- What if we use multilevel signaling :

$$C = 2B \log_2 M$$

where M is the number of discrete levels used

- Example: for M = 8, same telephone channel can support C = 2X3100X $\log_2 8 = 18.6$ kb/s
- M = 2 – receiver recognizes two signal levels 1s and 0s
- M > 2 – receiver recognizes discrete levels other than 1 and 0
- In general, multilevel signaling requires more sophisticated receiver structure and perhaps more power for the same bandwidth

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

42

Shannon Capacity Formula

- Capacity in the **presence of noise**

$$C = B \log_2(1 + SNR)$$

$$SNR_{dB} = 10 \log_{10} \left(\frac{\text{signal power}}{\text{noise power}} \right)$$

where SNR is the ratio of signal power to noise power – a measure of the signal quality

- Example 3.6: $f_{min} = 3 \text{ MHz}$, $f_{max} = 4 \text{ MHz}$, $SNR = 24 \text{ dB}$, $C = ?$

$$B = 4 - 3 = 1 \text{ MHz}$$

$$SNR = 10^{24/10} = 251 \text{ (on the linear scale)}$$

$$C = 1 \times 10^6 \log_2(1 + 251) \sim 8 \times 10^6 \text{ b/s or } 8 \text{ M b/s}$$

One can also calculate the required signaling levels, M , using Nyquist formula: $C = 2B \log_2(M) \rightarrow M = 2^{C/(2B)} = 16$

- Note the C (calculated by Shannon formula) is the theoretical (error-free) limit of the channel for the given B and SNR

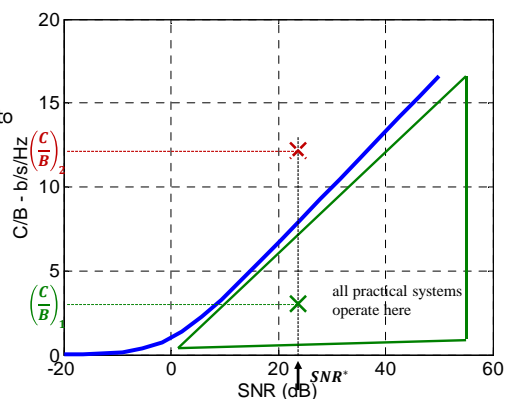
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

43

Shannon Capacity Formula – cont'd

- Plot of $\frac{C}{B} = \log_2(1 + SNR)$
- C/B – spectral efficiency – bits/sec/Hz
- Practical systems
 - For $SNR = SNR^*$ - it possible to build a system that will produce $(C/B)_1$ bits/sec/Hz
 - For $SNR = SNR^*$ - it is impossible to build a system that will produce $(C/B)_2$ bits/sec/Hz
- Different systems use different signals and bandwidth to achieve communication – How to compare cost and performance?
 - Unified Performance C/B
 - Unified Cost – E_b/N_0



2/17/2016

Dr. Ashraf S. Hasan Mahmoud

44

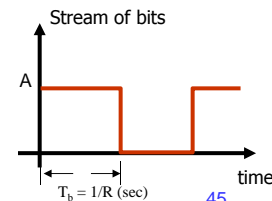
E_b/N_0 Expression

- An alternative representation of SNR
- Consider the bit stream shown in figure – for bit of rate R , then each bit duration is equal to $T_b = 1/R$ seconds
- Energy of signal for the bit duration is equal to $A^2 \times T_b$, where its power is equal to bit energy / T_b or A^2 .
- Noise power is equal to $N_0 \times B$ (refer to thermal noise section)
- Hence, SNR is given by signal power / noise power or

$$SNR = \frac{\text{signal power}}{N_0 B} = \frac{E_b}{N_0} \times \frac{R}{B}$$

- One can also write

$$\left(\frac{E_b}{N_0} \right)_{dB} = \text{SignalPower}(dBW) - 10 \log R - 10 \log k - 10 \log T$$



2/17/2016

Dr. Ashraf S. Hasan Mahmoud

45

Examples:

- Example 3.7: For a binary phase-shift keying (defined in Chapter5) E_b/N_0 is 8.4 dB is required for a bit error rate of 10^{-4} (1 bit error out of 10,000). If the effective noise temperature is 290K (room temperature) and the data rate is 2400 bps, what received signal level is required?

- Solution: We have

$$\left(\frac{E_b}{N_0} \right)_{dB} = \text{SignalPower}(dBW) - 10 \log R - 10 \log k - 10 \log T$$

$$8.4 = \text{SignalPower}_{dB} - 10 \log 10(2400) - 228.6 \text{ dBW} - 10 \log 10(290)$$

$$\rightarrow \text{SignalPower}_{dB} = -161.8 \text{ dBW}$$

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

46

Examples:

- **Problem:** If the received signal level for a particular digital system is -151 dBW and the receiver system effective noise temperature is 1500 degrees Kelvin. What is the E_b/N_0 for a link transmitting 2400 b/s

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

47

Examples:

- **Solution:**

$$E_b/N_0 = (\text{signal power} / \text{noise power}) * (B/R)$$

$$\text{Noise power} = kT \times B$$

$$\text{Hence } E_b/N_0 = (\text{signal power}) / (kT \times R)$$

$$= 10^{-151/10} / (1.38 \times 10^{-23} \times 1500 \times 2400)$$

$$= 15.99$$

$$= 12 \text{ dB}$$

$$\text{Or } (E_b/N_0)_{\text{dB}} = \text{Signalpower_dBW} - 10\log k - 10\log T - 10\log R$$

$$= -151 - 10\log(1.38 \times 10^{-23}) - 10\log 1500 - 10\log 2400$$

$$= 12 \text{ dB}$$

2/17/2016

Dr. Ashraf S. Hasan Mahmoud

48

Spectral Efficiency

- Spectral Efficiency = ratio of useful bits/sec (capacity, C) to channel bandwidth, B in Hz.
- Therefore, Spectral Efficiency = C/B (b/s/Hz)
- Remember that
$$E_b/N_0 = S/(N_0 \times R) = S/N \times B/C$$
- But using Shannon $\rightarrow S/N = 2^{C/B} - 1$, or
$$E_b/N_0 = B/C (2^{C/B} - 1)$$
- Very useful formula relating the achievable spectral efficiency for a given E_b/N_0 .

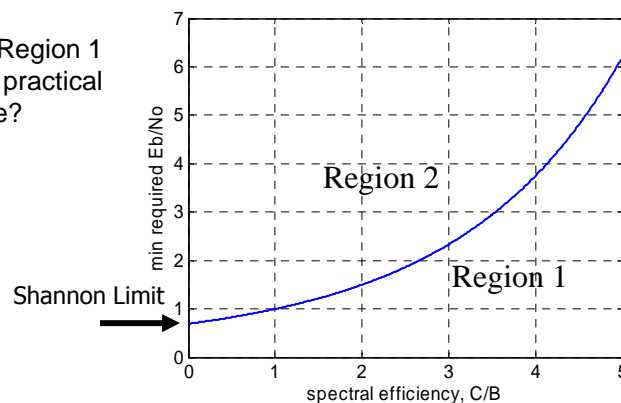
2/17/2016

Dr. Ashraf S. Hasan Mahmoud

49

Spectral Efficiency – cont'd

- Note that the minimum required E_b/N_0 for ANY communication system to work error-free is 0.6956 or -1.6 dB \rightarrow This is known as the SHANNON LIMIT.
- In what region (Region 1 or Region 2) do practical systems operate?



2/17/2016

Dr. Ashraf S. Hasan Mahmoud

50