KFUPM - COMPUTER ENGINEERING DEPARTMENT

COE-540 – Computer Networks
Quiz 01

Student Name:
Student Number:

Solution:

a) Mathematical expression for s(t) is given by

$$s(t) = \begin{cases} A & 0 \le t < T/2 \\ -A & T/2 \le t < T \end{cases}$$

For this signal T = 1 sec → f0 = 1 Hz.

b) Total power for signal s(t) is given by

$$P_s = \frac{1}{T}\int_0^T |s(t)|^2 dt = \frac{1}{T}\int_0^T A^2 dt = \frac{1}{T} \times A^2 \times T = A^2 \text{ Watts}$$

Therefore, $P_s = A^2 = 4$ Watts.

c) For this signal f_max = infinity ➔ Theoretical bandwidth = infinity

d) The FSE is given by

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(2\pi nt/T) + \sum_{n=1}^{\infty} B_n \sin(2\pi nt/T)$$

The constant A0 is obtained as

$$A_0 = \frac{2}{T}\int_0^T s(t)dt = \frac{2}{T}\left(\int_0^{T/2} Adt - \int_{T/2}^{T} Adt\right) = 0$$

The constants An are obtained as

$$\begin{aligned} A_n &= \frac{2}{T}\int_0^T s(t)\cos\left(\frac{2\pi nt}{T}\right)dt = \frac{2}{T}\left(\int_0^{\frac{T}{2}} A\cos\left(\frac{2\pi nt}{T}\right)dt - \int_{\frac{T}{2}}^{T} A\cos\left(\frac{2\pi nt}{T}\right)dt\right) \\ &= \frac{2}{T}\left(\frac{AT}{2\pi n}\sin\left(\frac{2\pi nt}{T}\right)\Big|_{t=0}^{t=\frac{T}{2}} - \frac{AT}{2\pi n}\sin\left(\frac{2\pi nt}{T}\right)\Big|_{t=\frac{T}{2}}^{t=T}\right) \\ &= \frac{2}{T}\left(\frac{AT}{2\pi n}(\sin(\pi n) - \sin(0)) - \frac{AT}{2\pi n}(\sin(2\pi n) - \sin(\pi n))\right) \\ &= \frac{2}{T}\left(\frac{AT}{2\pi n}(0-0) - \frac{AT}{2\pi n}(0-0)\right) = 0 \end{aligned}$$

The constants Bn are obtained as

$$\begin{aligned} B_n &= \frac{2}{T}\int_0^T s(t)\sin\left(\frac{2\pi nt}{T}\right)dt = \frac{2}{T}\left(\int_0^{\frac{T}{2}} A\sin\left(\frac{2\pi nt}{T}\right)dt - \int_{\frac{T}{2}}^{T} A\sin\left(\frac{2\pi nt}{T}\right)dt\right) \\ &= \frac{2}{T}\left(\frac{-AT}{2\pi n}\cos\left(\frac{2\pi nt}{T}\right)\Big|_{t=0}^{t=\frac{T}{2}} + \frac{AT}{2\pi n}\cos\left(\frac{2\pi nt}{T}\right)\Big|_{t=\frac{T}{2}}^{t=T}\right) \\ &= \frac{2}{T}\left(\frac{-AT}{2\pi n}(\cos(\pi n) - 1) + \frac{AT}{2\pi n}(\cos(2\pi n) - \cos(\pi n))\right) \\ &= \frac{2A}{\pi n}(1 - \cos(\pi n)) = \begin{cases} 0 & n = 2, 4, 6, \ldots \\ \frac{4A}{\pi n} & n = 1, 3, 5, \ldots \end{cases} = \frac{2A}{\pi n}(1 - (-1)^n) \end{aligned}$$

Therefore, the FSE is given by

$$s(t) = \frac{4A}{\pi}\sum_{n=1,3,5,}^{\infty}\frac{\sin\left(\frac{2\pi nt}{T}\right)}{n}$$
$$= \frac{8}{\pi}\sum_{n=1,3,5,}^{\infty}\frac{\sin(2\pi nt)}{n}$$

Note that for this signal there is NO DC component and the An coefficients are zero.

e) To find k* such that s_e(n=k*) has 95% of power, we construct a table similar to that in the slides. Note that the total power is A^2.

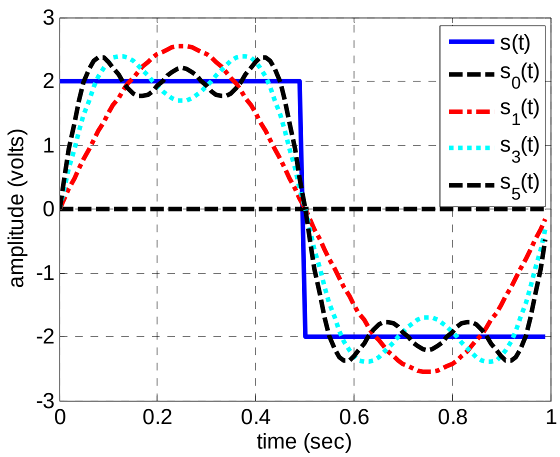
k value	s_e(n=k)	Power (Watts)	Relative to total power
0	0	0	0
1	$\frac{4A}{\pi}\sin(2\pi t)$	$\frac{\left(\frac{4A}{\pi}\right)^2}{2} = 0.8106\,A^2$	$\frac{\left(\frac{8A^2}{\pi^2}\right)}{A^2} = \frac{8}{\pi^2}$ or 81.06%
3	$\frac{4A}{\pi}\sin(2\pi t) + \frac{4A}{3\pi}\sin(2\pi t)$	$\frac{\left(\frac{4A}{\pi}\right)^2}{2} + \frac{\left(\frac{4A}{3\pi}\right)^2}{2} = 0.9006A^2$	$\frac{0.9006A^2}{A^2} = 90.06\%$
5	$\frac{4A}{\pi}\sin(2\pi t) + \frac{4A}{3\pi}\sin(2\pi t) + + \frac{4A}{5\pi}\sin(2\pi t)$	$\frac{\left(\frac{4A}{\pi}\right)^2}{2} + \frac{\left(\frac{4A}{3\pi}\right)^2}{2} + \frac{\left(\frac{4A}{5\pi}\right)^2}{2} = 0.9331A^2$	93.31%
7	$\frac{4A}{\pi}\sin(2\pi t) + \frac{4A}{3\pi}\sin(2\pi t) + \frac{4A}{5\pi}\sin(2\pi t) + \frac{4A}{7\pi}\sin(2\pi t)$	$\frac{\left(\frac{4A}{\pi}\right)^2}{2} + \frac{\left(\frac{4A}{3\pi}\right)^2}{2} + \frac{\left(\frac{4A}{5\pi}\right)^2}{2} + \frac{\left(\frac{4A}{7\pi}\right)^2}{2} = 0.9496A^2$	94.96%
9	$\frac{4A}{\pi}\sin(2\pi t) + \frac{4A}{3\pi}\sin(2\pi t) + \frac{4A}{5\pi}\sin(2\pi t) + \frac{4A}{7\pi}\sin(2\pi t) + \frac{4A}{9\pi}\sin(2\pi t)$	$\frac{\left(\frac{4A}{\pi}\right)^2}{2} + \frac{\left(\frac{4A}{3\pi}\right)^2}{2} + \frac{\left(\frac{4A}{5\pi}\right)^2}{2} + \frac{\left(\frac{4A}{7\pi}\right)^2}{2} + \frac{\left(\frac{4A}{9\pi}\right)^2}{2} = 0.9596A^2$	95.96%

Therefore, k* is equal to 9, and the expansion s_e(n=9) is given by

$$s(t) = \frac{4A}{\pi}\sin(2\pi t) + \frac{4A}{3\pi}\sin(2\pi t) + \frac{4A}{5\pi}\sin(2\pi t) + \frac{4A}{7\pi}\sin(2\pi t) + \frac{4A}{9\pi}\sin(2\pi t)$$

f) The bandwidth for s_e(n=9) is fmax – fmin = 9/T – 1/T = 8/T = 8 Hz.

g) The plot is as shown below:



Plot for part (f)

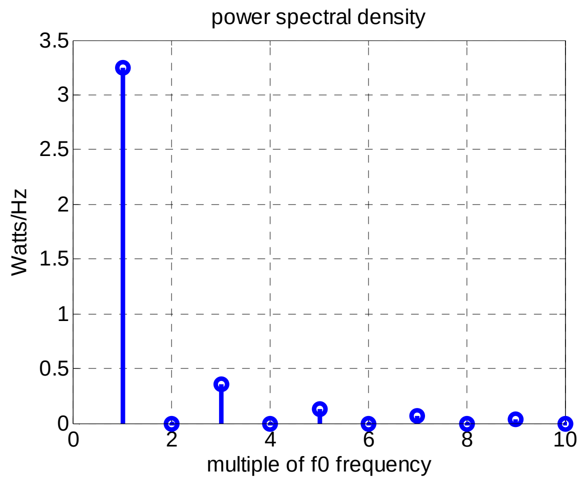
h) The power spectral density function is specified by:

$$PSD(f) = \frac{8A^2}{\pi^2} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} \delta\left(f - \frac{n}{T}\right), or$$

$$= \begin{cases} \frac{8A^2}{\pi^2 n^2} & f = n \times f_0; n = 1,3,5,\ldots \\ 0 & otherwise \end{cases}$$

where $\delta(.)$ is the Dirac delta function.

i) The plot is as shown in Figure.



Plot for part (g)