KFUPM - COMPUTER ENGINEERING DEPARTMENT

COE-241 – Data and Computer Communication
Assignment 2 – Due date: March 9th, 2013

Student Name:
Student Number:

Problem #	Maximum Mark	Mark
1	5	
2	10	
3	20	
4	30 + 10	
5	35 + 10	
Total	100	

Problem 1 (5 points)

2 sin(4πt + π); A = 2, f = 2, φ = π

Problem 2 (10 points)

(1 + 0.1 cos 5t) cos 100t = cos 100t + 0.1 cos 5t cos 100t. From the trigonometric identity cos a cos b = (1/2)(cos(a + b) + cos(a – b)), this equation can be rewritten as the linear combination of three sinusoids: cos 100t + 0.05 cos 105t + 0.05 cos 95t.

Component: cos (100t) ➔ A = 1, f = 100/(2 π) Hz, phase = 0.

Component: 0.05 cos (105t) ➔ A = 0.05, f = 105/(2 π) Hz, phase = 0.

Component: 0.05 cos (95t) ➔ A = 0.05, f = 95/(2 π) Hz, phase = 0.

Problem 3 (20 points):

Consider the periodic signal specified by

$$s(t) = (4\cos(t))^2 \text{ for } t \in \mathbb{R}$$

a) (2 points) Determine the fundamental frequency f_0 for the periodic signal $s(t)$.

b) (3 points) Compute the DC component for $s(t)$.

c) (3 points) Compute total power for $s(t)$.

d) (2 points) Determine the bandwidth for $s(t)$.

e) (10 points) Compute the Fourier Series Expansion (FSE) for $s(t)$.

Solution:

See the distributed example solved for $s(t) = (10\cos(t))^2$ for $t \in \mathbb{R}$. The problem is identical to this one.

Problem 4 (30 points + 10 points bonus):

a) The mathematical expression for p(t) is as follows:

$$p(t) = \begin{cases} A\cos(\pi t/\tau) & |t| \le \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

Note that in general the cosine expression is given by $A\cos(2\pi f t)$ and for this case, f = 1/period = 1/(2τ). To check let t = +/- τ/2 ➔ p(t) = Acos(+/-π/2) = 0. For t = 0, p(0) = A.

b) The F.T is given by

$$P(f) = \int_{-\infty}^{\infty} p(t) e^{-2\pi j f t} dt = \int_{-\tau/2}^{\tau/2} A\cos(\pi t/\tau)\, e^{-2\pi j f t} dt$$

$$= \frac{A e^{-2\pi j f t}}{(\pi/\tau)^2 + (-2\pi j f)^2} \left[(\pi/\tau)\sin(\pi t/\tau) + (-2\pi j f)\cos(\pi t/\tau)\right]\Bigg|_{t=-\tau/2}^{t=\tau/2}$$

Where the following identity has been used:

$$\int \cos ax\, e^{bx}\, dx = \frac{e^{bx}}{a^2 + b^2}(a \sin ax + b \cos ax) + C$$

Therefore, P(f) is given by

$$P(f) = \frac{A}{(\pi/\tau)^2 + (-2\pi jf)^2}\{e^{-\pi jf\tau}[(\pi/\tau)\sin(\pi/2) + (-2\pi jf)\cos(\pi/2)] - e^{\pi jf\tau}[(\pi/\tau)\sin(-\pi/2) + (-2\pi jf)\cos(-\pi/2)]\}$$
$$= \frac{A(\pi/\tau)}{(\pi/\tau)^2 + (-2\pi jf)^2}\{e^{-\pi jf\tau} + e^{\pi jf\tau}\} = \frac{2A\tau}{\pi} \times \frac{\cos(\pi f\tau)}{(1-(2f\tau)^2)}$$

c) P(f) is real-valued – i.e. does not have a complex component. It is defined for all frequency i.e. $f \in (-\infty, +\infty)$.

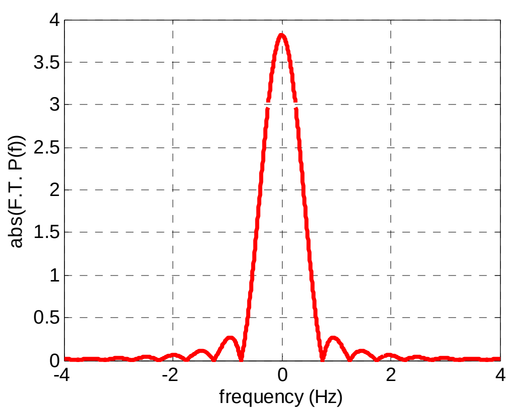
The zeros can be found as $P(f) = 0$ ➔ $\frac{2A\tau}{\pi} \times \frac{\cos(\pi f\tau)}{(1-(2f\tau)^2)} = 0$ ➔ $\cos(\pi f\tau) = 0$ ➔ $\pi f\tau = n\,(\pi/2)$ where n is odd. Therefore, $f = \frac{n}{2\tau}$ for all odd n.

d) The ESDF function is given by

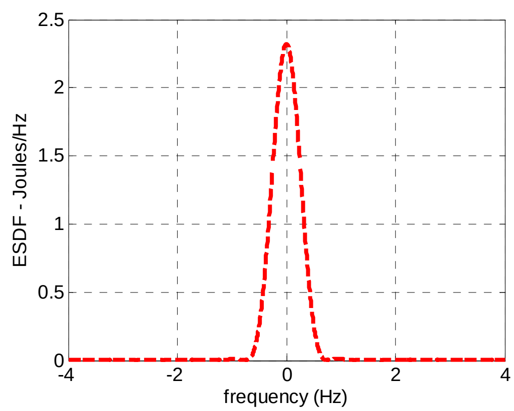
$$ESDF(f) = \frac{1}{2\pi} \times \left\{\frac{2A\tau}{\pi} \times \frac{\cos(\pi f\tau)}{(1-(2f\tau)^2)}\right\} \times \left\{\frac{2A\tau}{\pi} \times \frac{\cos(\pi f\tau)}{(1-(2f\tau)^2)}\right\}^*$$
$$= \frac{2A^2\tau^2}{\pi^3} \times \left\{\frac{\cos(\pi f\tau)}{(1-(2f\tau)^2)}\right\}^2$$

The units of ESDF is Joules per Hz.

e) (bonus part) Plots for $|P(f)|$ and $ESDF(f)$ are as shown.



Plot for $|P(f)|$.



Plot for $ESDF(f)$.

Problem 5 (35 points + 10 points bonus):

a) Using the slides – the FSE is given by:

$$s(t) = \frac{A}{2} + \frac{-4A}{\pi^2} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2}\cos(2\pi n f_0 t)$$

For this signal $f_0 = \frac{1}{T} = 2$ Hz.

b) The LPF will allow the following components to pass: 0, 1xf0 = 2 Hz, 3xf0 = 6 Hz only. It will reject 5xf0 = 10 Hz and higher frequency components.

Therefore, the output signal is equal to:

$$s_o(t) = \frac{A}{2} + \frac{-4A}{\pi^2}\frac{\cos(2\pi(1)f_0t)}{(1)^2} + \frac{-4A}{\pi^2}\frac{\cos(2\pi(3)f_0t)}{(3)^2}$$

c) Total power for $s_o(t)$ is equal to $\left(\frac{A}{2}\right)^2 + \frac{1}{2}\left(\frac{-4A}{\pi^2(1)^2}\right)^2 + \frac{1}{2}\left(\frac{-4A}{\pi^2(3)^2}\right)^2$ which is equal to 1.3326 Watts

d) The PSD function is given by

$$PSD(f) = \left(\frac{A}{2}\right)^2 \delta(f) + \frac{1}{2}\left(\frac{-4A}{\pi^2(1)^2}\right)^2 \delta(f-2) + \frac{1}{2}\left(\frac{-4A}{\pi^2(3)^2}\right)^2 \delta(f-6)$$

$$= \begin{cases} \left(\frac{A}{2}\right)^2 = 1.0 & f = 0 \text{ Hz} \\ \frac{1}{2}\left(\frac{-4A}{\pi^2(1)^2}\right)^2 = 0.3285 & f = 1f_0 = 1 \text{ Hz} \\ \frac{1}{2}\left(\frac{-4A}{\pi^2(3)^2}\right)^2 = 0.0041 & f = 3f_0 = 6 \text{ Hz} \end{cases}$$

The plot for the PSD function is as shown below.

e) To compute the dynamic range – see the table below

- highest power at f = 0 Hz is 1 Watt

- divide all powers with this 1 Watt → ratios (in dB) are equal to `0.000 -4.834 -23.919`

Then the dynamic range for $s_o(t)$ is 23.9 dB

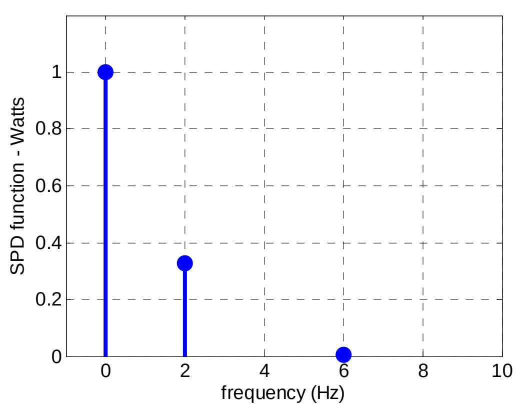


Figure: PSD function for $s_o(t)$.

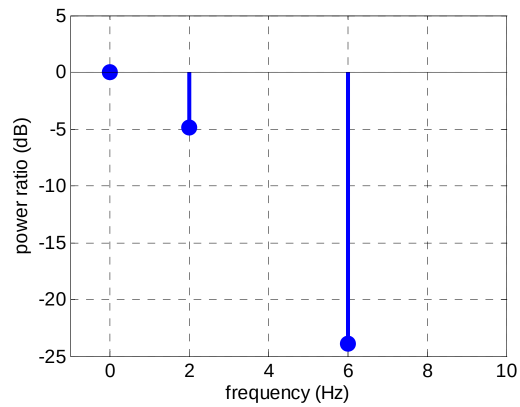


Figure: power ratio in dB for $s_o(t)$.

For parts (d) and (e) – Refer to the calculations shown below:

```
>> Assign2_Problem_5_TraingularFunction
Total power =    1.333 Watts
Max power at f =   0.0 Hz - Power =    1.000 (   0.000 dBW)


Power spectral density function:
Index                :         0          1          3
freq (Hz)            :         0          2          6
PSD (W)              : 1.000000  0.328511  0.004056
PSD-normalized       : 1.000000  0.328511  0.004056
PSD-normalized (dB):     0.000     -4.834    -23.919
```

For parts (f), (g), and (h) we use the following relations:

Noise power = Noise density x BW

SNR = total signal power / noise power

Capacity = BW x log2(1 + SNR)

The minimum number of bits per symbol is found through Nyquist formula: Capacity = 2 BW log2(M) → log2(M) = Capacity / (2 BW)

The corresponding numbers are shown below:

```
Capacity calculations:
Total signal power =    1.333 Watts (  1.247 dBW)
Total noise power  =    0.000 Watts (-60.458 dBW)
           ==> SNR = 1480630.1      ( 61.704 dB)
Shannon capacity   =  184.480 b/s
Min bits/symbol    =   10.249 bits per symbol
```