King Fahd University of Petroleum & Minerals
Computer Engineering Dept

COE 543 – Mobile and Wireless Networks

Term 111

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Material for This Topic

1. Chapter 4: Orthogonal Frequency Division Multiplexing, from "Fundamentals of WiMAX - Understanding Broadband Wireless Networking," by Jeffrey G. Andrews, Arunabha Ghosh, and Rias Muhamed, Printice Hall, 2007.
2. Chapter 6: Orthogonal Frequency Division Multiple Access, from "Fundamentals of WiMAX - Understanding Broadband Wireless Networking," by Jeffrey G. Andrews, Arunabha Ghosh, and Rias Muhamed, Printice Hall, 2007.

The above three references are the minimum required material for this topic. Other related (and used) references are:

1. Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems - Amendment 2: Physical and Medium Access Control Layers for Combined Fixed and Mobile Operation in Licensed Bands, IEEE Std 802.16e™-2005 and IEEE Std 802.16™-2004/Cor1-2005, 28 Feb 2006.
2. Mobile WiMAX – Part I: A Technical Overview and Performance Evaluation – WiMAX forum, August 2006.
3. WiMAX System Evaluation Methodology, WiMAX forum, Sept 2007.
4. Chapter 10: System Level Performance, from "Fundamentals of WiMAX - Understanding Broadband Wireless Networking," by Jeffrey G. Andrews, Arunabha Ghosh, and Rias Muhamed, Printice Hall, 2007.

Chapter 4: Orthogonal Frequency Division Multiplexing -

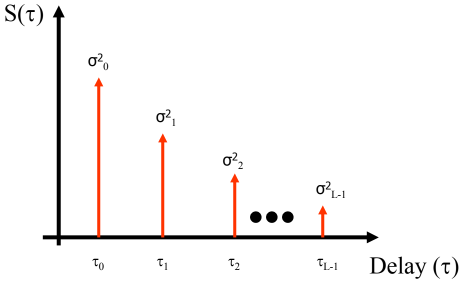
From "Fundamentals of WiMAX - Understanding Broadband Wireless Networking," by Jeffrey G. Andrews, Arunabha Ghosh, and Rias Muhamed, Prentice Hall, 2007.

Orthogonal Frequency Division Multiplexing

- Recently very popular technology choice
    - Can provide very high bit rates over wireless channels
    - Wireless LANs (802.11a/g/n), digital video broadcasting, WiMAX, 3G LTE
- Two main characteristics
    - Efficient
    - Flexible management of intersymbol interference (ISI)

Coherence Bandwidth and Intersymbol Interference

- A mobile/wireless channel is characterized by the mulitpath intensity profile
- The severity of the multipath effect is characterized by the multipath excess delay (T_m) or the RMS delay spread σ_τ



- Multipath excess delay:

$$T_m = \tau_{L-1} - \tau_0$$

- RMS delay:

$$\sigma_\tau = \sqrt{\{E[\tau^2]-(E[\tau])^2\}}$$

$$E[\tau] = \Sigma\,(\sigma_i^2\tau_i) \,/\, \Sigma\,(\sigma_i^2)$$

$$E[\tau^2] = \Sigma\,(\sigma_i^2\tau_i^2) \,/\, \Sigma\,(\sigma_i^2)$$

Coherence Bandwidth and Intersymbol Interference - continued

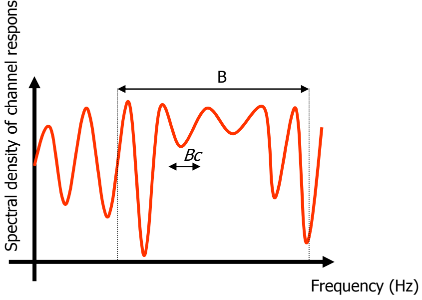
- Channel induced ISI occurs when channel response (characterized by T_m or σ_τ) lasts much longer than the duration of the symbol time, T_s
- In the frequency domain: The coherence bandwidth, Bc is inversely proportional to Tm or σ_τ
  - A good rule of thump is $Bc = 1/(5\sigma_\tau)$
- The coherence bandwidth is the range of frequencies that fade or not fade together (statistically)
- High-speed systems: high bit rate, R ➔ Symbol rate, R_s is high or $T_s = 1/R_s$ is low
  - Bandwidth for such signal, $B \approx R_s = 1/T_s$
- When $Ts << \sigma_\tau$ or equivalently $B >> Bc$ ➔ ISI

Frequency Selective and Frequency Flat Fading

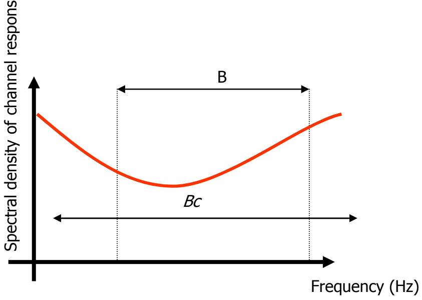
- Frequency Selective and Frequency Flat Fading

Selective fading: Channel induced ISI – leading to irreducible bit error rate (BER)

flat fading: No ISI – but loss of SNR due to Rayleigh fading (Rayleigh limit of BER)



Frequency-selective fading (*Bc* < *B*)



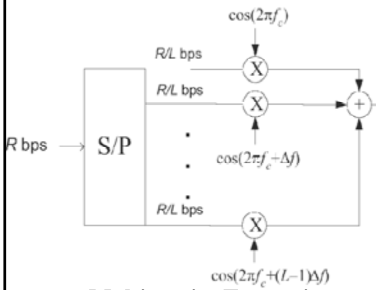
Frequency-flat fading (*Bc* > *B*)

Solutions for the ISI Problem

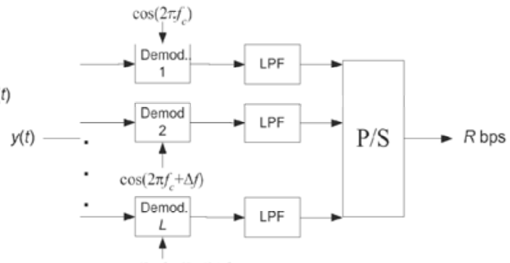
- Equalization
- Decision Feedback Equalizers (DFE)
- Maximum Likelihood Sequence Estimation (MLSE)
- Spread-Spectrum
- **OFDM**
  - Basic idea – split the high speed symbol stream into parallel slower streams – Then the new symbol duration Ts is now shorter Tm or σ_τ ➔ no ISI
  - Multicarrier modulation is one way to achieve this objective
  - OFDM is a special case of multicarrier modulation

Multicarrier Modulation

- Divide the high-rate transmit bit stream into L lower-rate substreams
  - New symbol rate = Ts/L >> σ_τ ➔ No ISI
  - L parallel channels
- Subchannels are typically orthogonal



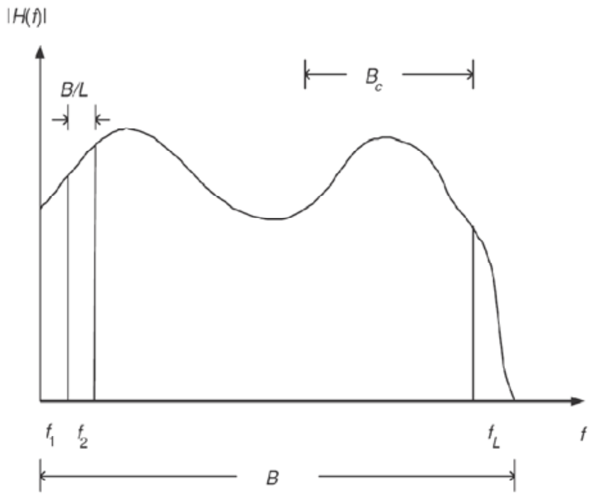
Multicarrier Transmitter



Multicarrier Receiver

Multicarrier Modulation – cont’d

- The transmitted multicarrier signal experiences approximately flat fading on each subchannel since B/L << Bc, even though the overall channel experiences frequency-selective fading (i.e. B > Bc)



Shortcomings of Multicarrier Modulation

- Multicarrier modulation is difficult to implement
- Large bandwidth requirement - needed to preserve orthogonality between sub-channels
- Expensive (and accurate) low pass filters required
- Multiple RF units required

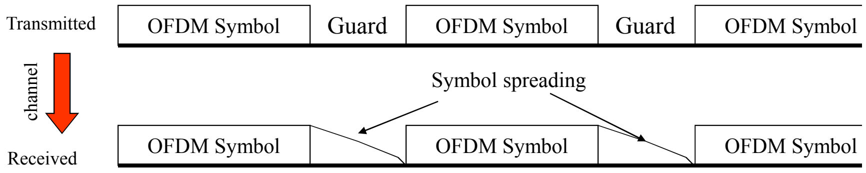
- OFDM alleviates some of these difficulties!

Orthogonal Frequency Division Multiplexing

- No L RF radio in both transmitter and receiver
- OFDM uses efficient computational technique – Discrete Fourier Transform (DFT)
  - Implemented through Fast Fourier Transform (FFT) routines – highly optimized
- The FFT and its inverse (IFFT) create orthogonal sub-carriers using a single radio

Block Transmission with Guard Intervals

- L data symbols grouped into ONE OFDM symbol
- OFDM symbol duration, T = L Ts
- Introduce guard time between OFDM symbols of length Tg
- Since T+Tg > delay spread of channel ➔ subsequent symbols do not interfere with each other
- However, each symbol will interfere with itself (smearing or spreading)
- How to remove this self-symbol interference?
  - Answer – Circular Convolution



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http://cnx.org/content/m10087/latest/
http://cnx.org/content/m12053/latest/

Circular Convolution

- Typical Convolution: (Refer to Linear Time-Invariant Channels)
  - Input signal x(n) of length L, system impulse response h(n)
  - Then output y(n) is given by

$$y(n) = x(n) * h(n) = \sum_{\forall k} h(k)x(n-k)$$

- Circular Convolution:
  - Input signal x(n), system impulse response h(n)
  - Then output y(n) is given by

$$y(n) = x(n) \otimes h(n) = \sum_{k=0}^{L-1} h(k)x(n-k)_L$$

  - $x(n)_L$ is a periodic version of x(n); $x(n)_L$ is defined as x(n mod L)

Circular Convolution Properties

- Has properties similar to those for typical convolution
- Using the DFT operator:

DFT{y(n)} = DFT{x(n)⊗h(n)}
= DFT{x(n)} TIMES DFT{h(n)}

$$\boxed{Y(m) = X(m) \quad H(m)}$$

Key Result

- DFT for x(n) is given by $X(m) \triangleq \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} x(n) e^{-j2\pi nm/L}$
- The IDFT for X(m) is given by $x(n) \triangleq \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} X(m) e^{j2\pi nm/L}$

Recovery of Transmitted Symbol at Receiver

- Using the key result of previous slide

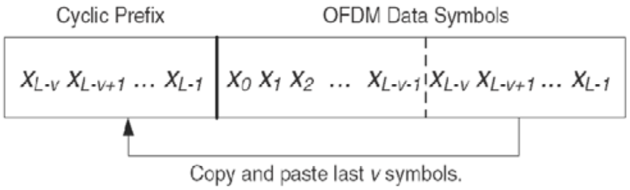
$$\hat{X}(m) = \frac{Y(m)}{H(m)}$$

- Therefore, one need to estimate the channel response H(m) to compute an estimate of the transmitted symbol
  - Effect of noise, interference, etc.
- However, the estimate of the transmitted symbol **is ISI free**
- How to do "produce the effect of" circular convolution?
  - We know channel output is given by the typical convolution summation (integral)
  - We need to make the channel produce the effect of the circular convolution!

Cyclic Prefix

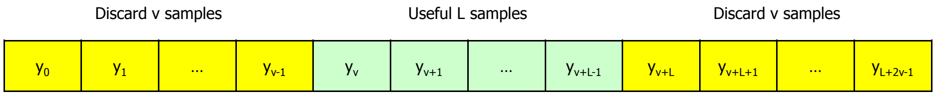
- Circular Convolution is faked by add a specific prefix, referred to Cyclic Prefix (CP) onto the transmitted sequence
- Sequence to transmit x = [x0, x1, …, xL-1]
- Sequence with CP of length v

  xcp = [xL-v, xL-v+1, …, xL-1, x0, x1, …, xL-1]



Cyclic Prefix – cont’d

- Channel output ycp = h * xcp
- h – channel impulse response - length of v+1
- Length of ycp = (L+v)+(v+1)-1 = L+2v
- The first v samples of ycp contain interference from preceding OFDM symbol – discard
- The last v samples of ycp contain interference from self symbol – discard
- Use the remaining L samples of ycp – these are the samples required to recover the L data symbols embedded in x
- The remaining L samples of y are equivalent to x(n)⊗h(n)



Discard v samples				Useful L samples				Discard v samples			
y_0	y_1	…	y_{v-1}	y_v	y_{v+1}	…	y_{v+L-1}	y_{v+L}	y_{v+L+1}	…	y_{L+2v-1}

Refer to proof in chapter 4 of Fundamentals of WiMAX e-book.

Penalties for CP

- CP – v redundant symbols are sent;
- Bandwidth - increase the required bandwidth B → B(L+v/L)
- Power - increase the required transmit power budget by 10log10(L+v/L) dB
- Rate Loss = Power Loss = L / (L + v)

This example is based on Example 4.2 in the reference – but the Presentation below is a little bit different!!

Example: Numbers from WiMAX

- Minimum and maximum data rate loss due to CP for WiMAX
- Delay spread (typical mobile/wireless channel) = 5 μsec
- The system specifies choices for guard band G = {1/4, 1/8, 1/16, 1/32}
- The number of sub-carriers to use L = {128, 256, 512, 1024, 2048}
- For L = 2048 (used for BW = 20 MHz or Ts = 0.05 μsec) – delay spread lasts for about 5/0.05 = 100 symbols
  - Minimum Guard – 1/32 * 2048 = 64 < 100 (some ISI remaining)
  - Maximum Guard – 1/4 * 2048 = 512 > 100 (no ISI)
- For L = 128 (used for BW = 1.25 MHz or Ts = 0.8 μsec) – delay spread lasts for about 5/0.8 = 6.25 symbols
  - Minimum Guard – 1/32 * 128 = 4 < 6.25 (some ISI remaining)
  - Maximum Guard – 1/4 * 128 = 32 > 6.25 (no ISI)

- For mobile WiMAX typical L = 1024 (BW = 10 MHz or Ts = 0.1 μsec) – delay spread last for about 5/0.1 = 50 symbol
- Default G = 1/8
  - Guard – 1/8 *1024 = 128 > 50 → No ISI

G is known as the fractional overhead

Frequency Equalization

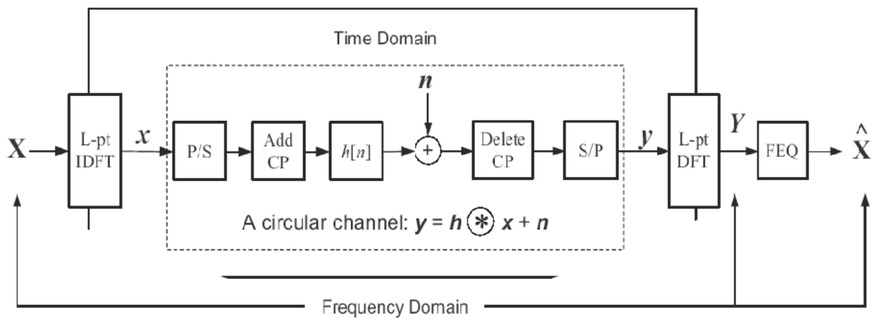
- One-tap frequency equalization (FEQ) is needed to estimate the transmitted symbol on the l^{th} sub-carrier

$$\widehat{X}_l = \frac{Y_l}{H_l}$$

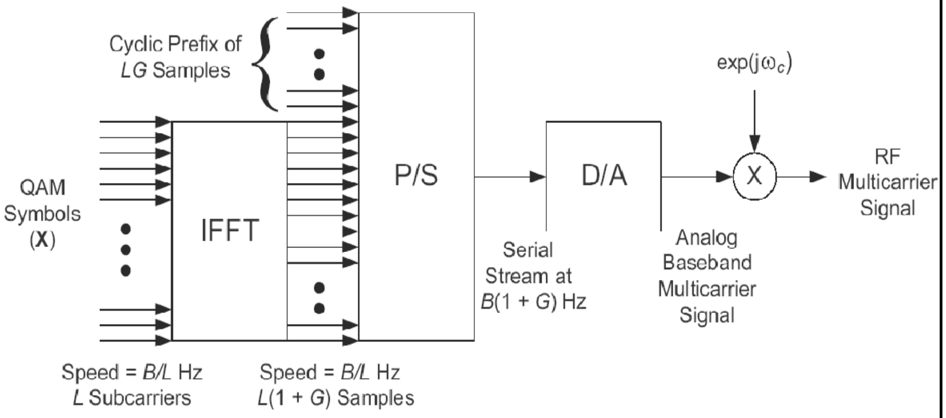
- H_l is the *complex* response of the channel at the frequency $f_c+(l+1)\Delta f$
- Δf is the sub-carrier spacing

OFDM Block Diagram – Operation Steps

1. Break wideband signal into L narrowband sub-carriers (each of B/L)
2. The L sub-carriers for a given OFDM symbol are represented by vector **X**
3. To use a single wideband radio instead of L independent narrowband radios, use IFFT operation
4. For IFFT/FFT to combat ISI – add cyclic prefix (CP) of length v after the IFFT
5. The resulting L+v symbols are sent serially on the wideband channel
6. At the receiver, the CP is removed
7. The L received symbols are demodulated using FFT ➔ L data symbols $Y_l = H_l X_l + N_l$ for sub-carrier l
8. Each sub-carrier is equalized via FEQ to produces an estimate of the transmitted symbol $\widehat{X}_l = Y_l / H_l$



Baseband OFDM Transmitter in WiMAX



The textbook states that Ld is 768; most other references stat that Ld is 720!
There are other discrepancies in other figures such as T and Bsc

Summary of OFDM Parameters

Symbol	Description	Relation	Example WiMAX value
B	System bandwidth	B = 1/Ts	10 MHz
L	Number of sub-carriers	Size of IFFT/FFT	1024
G	Guard fraction	Fraction of L for CP	1/8
Ld	Data sub-carriers	L – (120 pilot + 184 null)	720
Ts	Symbol time	Ts = 1/B	0.1 µsec
Ng	Guard symbols	Ng = GL	128
Tg	Guard time	Tg = TsNg	12.8 µsec
T	OFDM symbol time	T = Ts(L+Ng)	102.9 µsec
Bsc	Sub-carrier spacing	Bsc = B/L	9.76

Example Throughput Calculation

- WiMAX BW = 10 MHz, On downlink there are 720 data carriers
- If we assume 16-QAM modulation (i.e. 4 bits/symbol)
- Downlink throughput $R = \frac{B}{L}\frac{L_d}{1+G}\log_2(M)$

  = 25 Mb/s

- Assuming a coding rate of 1/2 ➔ throughput = 25/2 = 12.5 Mb/s

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Not of interest for the project – for more details Review chapter 4 of the e-book

Issues for OFDM

- Time synchronization
  - Timing offset of the symbol and the optimal timing instants
  - Relatively relaxed requirement since the OFDM symbol structure naturally accommodates reasonable synchronization error
- Frequency synchronization
  - The receiver must align its carrier frequency with that of the transmitter
  - The orthogonality of data symbols is reliant on this perfect alignment
  - Very stringent and significant requirement
- Peak-to-Average Ratio (PAR)
  - In the time-domain the ODFM signal is the sum of a large number of sub-signals ➔ The peak value of the OFDM signal is substantially larger than the average (i.e. high PAR)
  - Presents problems for the power amplifier at the transmitter side
  - One of the very critical problems for OFDM

Chapter 6: Orthogonal Frequency Division Multiple Access

Form “Fundamentals of WiMAX - Understanding Broadband Wireless Networking,” by Jeffrey G. Andrews, Arunabha Ghosh, and Rias Muhamed, Printice Hall, 2007

What is Orthogonal Frequency Division Multiple Access (OFDMA)

- OFDM – a modulation technique that utilizes independent sub-streams of data that can be used by different users
- Single user OFDM systems – all sub-carriers are meant for one user at a time
  - DSL, 802.11a/g and earlier versions of 802.16/WiMAX
- 802.16e-2005 (mobile WiMAX) uses OFDMA – users share sub-carriers and time-slots
- OFDMA allows
  - Increased multiuser diversity,
  - Increased freedom in scheduling (2/3-D scheduling problem)
- OFDMA needs feedback information to determine channel state

Brief Introduction into Multiple Access Strategies – Classic Methods

- Contention (Random) based
- Frequency Division Multiple Access
- Time Division Multiple Access
- Code Division Multiple Access

Contention-Based Random Access

- ALOHA and Slotted ALOHA
  - Decentralized, efficiency 16% and 32%, respectively
- Carrier Sense Multiple Access (CSMA) – a form of random access used for WLANs
  - Distributed Coordination Function (DCF)
  - Theoretical efficiency ~ 60%
  - Practical efficiency < 50% even for single user systems

Frequency Division Multiple Access

- Classically – divide the overall bandwidth into fixed sub bands – each assigned to user
- Can be implemented using OFDM
- Static allocation of mutually exclusive sets of sub-carriers for users: U1→1-16, U2→17-32, U3→33-48, etc.
- Enforced by a multiplexer for various users before the IFFT operation
- High rate users get more sub-carriers?

- *Dynamic sub-carrier allocation* – is an improvement over static allocation
  - Channel state information is used to allocate the sets of sub-carriers

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Time Division Multiple Access

- Time access is divided into slots
- WiMAX uses FDMA and TDMA
  - Users who has data have "slots" in the frame
- Static TDMA – good for circuit-switch applications
- For packet-based applications, a "smart" scheduler is needed - utilize
  - Queue length
  - Channel state
  - Delay constraints

Code Division Multiple Access

- Dominant multiple access technique for present 3G systems
- Data signal is spread over a large frequency range, relative to other systems
- HSDPA (3.5G systems) utilize CDMA and dynamic TDMA to allocate the high bandwidth channel for users based on scheduler objectives
- CDMA and OFDM can be combined
    - Multi-carrier CDMA (MC-CDMA)

Classic FDMA, TDMA, and CDMA

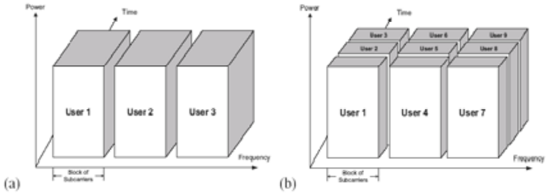


Figure 6.1 (a) FDMA and (b) a combination of FDMA with TDMA

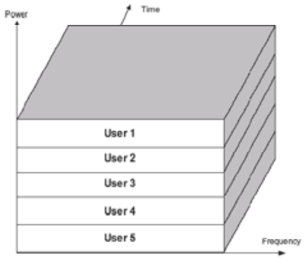
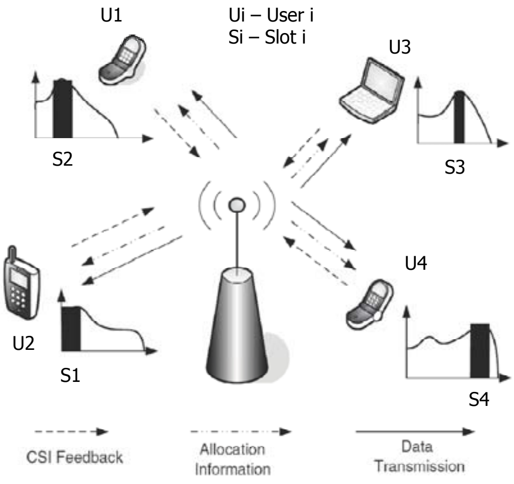


Figure 6.2 CDMA's users share time and frequency slots but use codes that allow the users to be separated by the receiver

For OFDMA – the scheduling problem is a TWO-DIMENSIONAL Problem: Frequency (sub-carriers) and Time (time-slot)
- If you include the MIMO features – it becomes three dimensional!

Scheduler for OFDMA

- OFDMA – combination of FDMA and TDMA
  - Users are DINAMICALLY assigned sub-carriers (FDMA part) in different time slots (TDMA part)
- The scheduler can exploit channel conditions for each use
  - Requires feedback info regarding channel condition
- For a single user
  - Robust multipath suppression
  - Frequency diversity
- The figure assumes the scheduler sends to one user at time – This may not be the case always
  - The scheduler can use differen sub-carriers for different users at the SAME time



Advantages of OFDMA

- OFDMA – flexible, accommodates many users with widely varying applications, data rates, and QoS requirements
- OFDMA can reduce PARP problem – relative to OFDM
- OFDMA provides/facilitates
  - Multiuser diversity
    - The system capacity increases if it has to select a user or a set of users with good channel conditions
  - Adaptive Modulation
    - The modulation and coding rate can adapt to the channel condition

Multiuser Diversity

- The main MOTIVATION for adaptive sub-carrier allocation in OFDMA systems is to exploit multiuser diversity
- Consider a *K*-user system in which the sub-carrier of interest experiences i.i.d. Rayleigh fading.
- Channel gain, h_k, is independent for all sub-carriers
- The pfd for the channel gain is given by

$$p(h_k) = \begin{cases} 2h_k e^{-h_k^2} & \text{if } h_k \ge 0 \\ 0 & \text{if } h_k < 0 \end{cases} \qquad k = 1, 2, \cdots, K$$

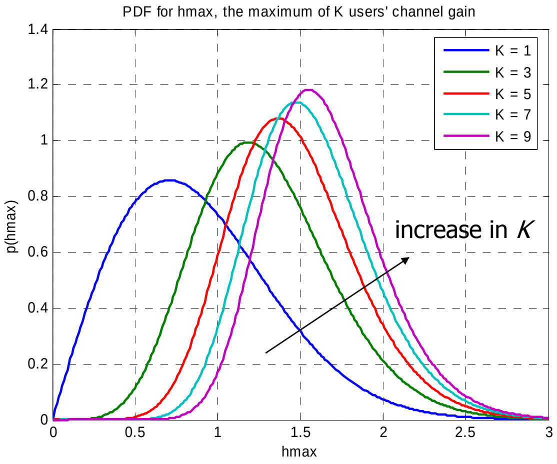
- Define hmax to be max(h1, h2, …, hK), then YOU CAN show that the pdf for hmax is given by

$$p(h_{\max}) = 2Kh_{\max}\left(1 - e^{-h_{\max}^2}\right)^{K-1} e^{-h_{\max}^2}$$

- Observe that for K = 1, the pdf for hmax SHOULD BE the same as the pdf for h1!!

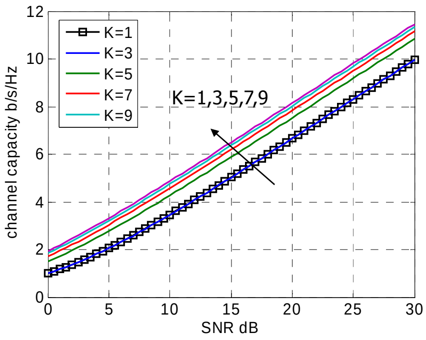
Multiuser Diversity – cont'd

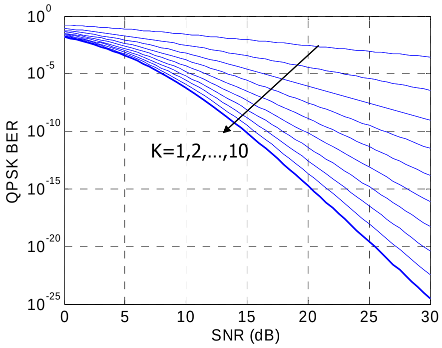
- The PDF for hmax is depicted in figure below
- As the number of population increases, there is a GREATER chance of finding users of GOOD channel conditions → MULTIUSER DIVERSITY



Multiuser Diversity - Capacity/BER Improvement

- Capacity / BER Improvement





The QPSK BER curves may not be as shown in Figure!

Exercise:

- A) Assume a receiver is exploiting multiuser diversity, derive the pdf for hmax – i.e. derive the formula on slide 36.
- B) Use the derived expression in (A) to compute the an expression for the second moment of hmax (i.e. E[hmax^2]) – You may want to consider using Mathcad or Maple.
- C) Use the result of (B) in plotting the capacity (b/s/Hz) curves versus SNR for different K values – i.e. reproduce the left-hand side plot on previous slide (slide 38).
- D) Derive an expression for the pdf of maximum power envelope and use it along with the QPSK BER for AWGN channel formula to plot the QPSK BER curves versus SNR for different K values – i.e. reproduce the right-hand side plot on previous slide (slide 38).

Adaptive Modulation and Coding

- Takes advantage of the fluctuation condition of the channel
- Rule: Transmit as high a data rate as possible when channel is good, and transmit at a lower rate when channel is poor
  - Objective – minimize dropped frames or blocks
- High data rate means:
  - Higher order modulation (i.e. more bits/symbol) and
  - Reduced coding rate (i.e. less coding protection)
  - Example 64-QAM with coding rate 3/4
- Lower bit rate mean:
  - Lower order modulation (i.e. less bits/symbol), and
  - Increased coding rate (i.e. stronger coding protection)
  - Example: QPSK with coding rate ½
- The combination of what modulation/coding to use is called "*burst profile*" in WiMAX

Burst Profiles Supported in WiMAX

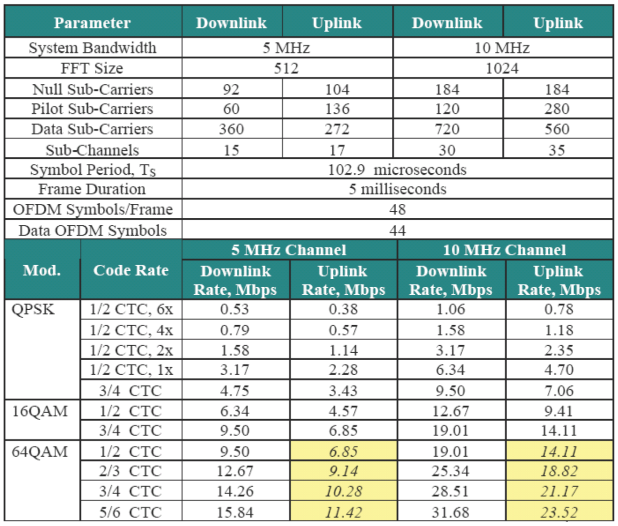
- From the WiMAX overview Part I white paper: "Support for QPSK, 16QAM and 64QAM are mandatory in the DL with Mobile WiMAX. In the UL, 64QAM is optional. Both Convolutional Code (CC) and Convolutional Turbo Code (CTC) with variable code rate and repetition coding are supported. Block Turbo Code and Low Density Parity Check Code (LDPC) are supported as optional features."

	Downlink	**Uplink**
Modulation	BPSK, QPSK, 16 QAM, 64 QAM; BPSK optional for OFDMA-PHY	BPSK, 16 QAM; 64 QAM optional
Coding	Mandatory: convolutional codes at 1/2, 2/3, 3/4, 5/6 Optional: convolutional turbo codes at 1/2, 2/3, 3/4, 5/6; repetition codes at rate 1/2, 1/3, 1/6, LDPC, RS-Codes for OFDM-PHY	Mandatory: convolutional codes at 1/2, 2/3, 3/4, 5/6 Optional: convolutional turbo codes at 1/2, 2/3, 3/4, 5/6; repetition codes at rate 1/2, 1/3, 1/6, LDPC

Burst Profiles and System Throughput

- Refer to slide 25 for how to calculate system throughput taking into account the signal constellation size (M) and the coding rate (r).
- The table shows system throughput increases as the constellation size increases and as r increases

burst profiles

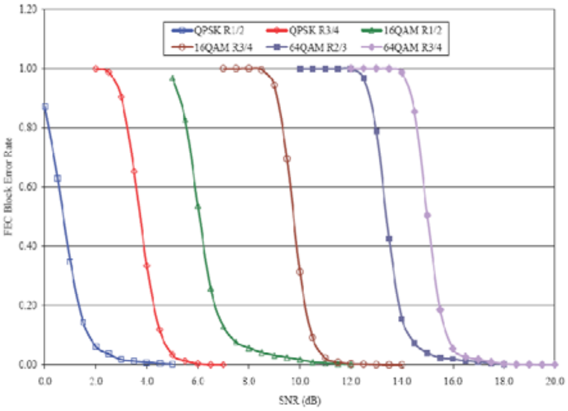


Parameter	Downlink	Uplink	Downlink	Uplink
System Bandwidth	5 MHz		10 MHz	
FFT Size	512		1024	
Null Sub-Carriers	92	104	184	184
Pilot Sub-Carriers	60	136	120	280
Data Sub-Carriers	360	272	720	560
Sub-Channels	15	17	30	35
Symbol Period, T_S	102.9 microseconds			
Frame Duration	5 milliseconds			
OFDM Symbols/Frame	48			
Data OFDM Symbols	44			

Mod.	Code Rate	5 MHz Channel		10 MHz Channel	
		Downlink Rate, Mbps	Uplink Rate, Mbps	Downlink Rate, Mbps	Uplink Rate, Mbps
QPSK	1/2 CTC, 6x	0.53	0.38	1.06	0.78
	1/2 CTC, 4x	0.79	0.57	1.58	1.18
	1/2 CTC, 2x	1.58	1.14	3.17	2.35
	1/2 CTC, 1x	3.17	2.28	6.34	4.70
	3/4 CTC	4.75	3.43	9.50	7.06
16QAM	1/2 CTC	6.34	4.57	12.67	9.41
	3/4 CTC	9.50	6.85	19.01	14.11
64QAM	1/2 CTC	9.50	*6.85*	19.01	*14.11*
	2/3 CTC	12.67	*9.14*	25.34	*18.82*
	3/4 CTC	14.26	*10.28*	28.51	*21.17*
	5/6 CTC	15.84	*11.42*	31.68	*23.52*

Bit Error Rate or Block Error Rate for Different Burst Profiles

- For the same SNR, lower signal constellation size and lower coding rate achieves better BER or BLER



Reference BLER curves in an AWGN channel – from WiMAX System Evaluation Methodology Version 1.7 September 7, 2007Document

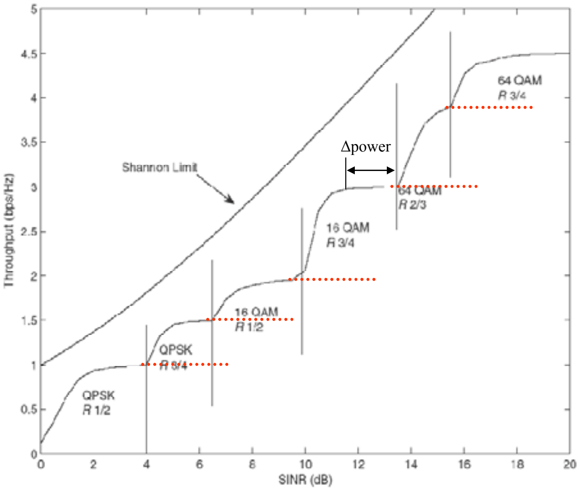
Channel Throughput Using AMC

- Throughput can be computed using:

T = (1-BLER)$r\log_2(M)$ b/s/Hz

- M is the constellation size, and r is the coding rate
- For any given M and r, the maximum possible throughput (when BLER = 0 or at high SINR) is given by $r\log_2(M)$
  - These are the dotted red straight lines super imposed on the plot
- Note that for 16 QAM with r = 3/4 → no capacity is gained when the SINR is increased from 11 dB to 13 dB
  - To the contrary – the increase in transmit power contributes to higher co-channel interference for neighboring cells!

A power not always good!

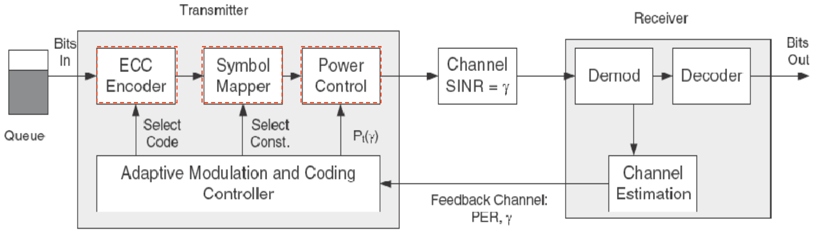


Throughput versus SINR assuming that the best available constellation and coding Configuration are chosen at each SINR.

How To Achieve Adaptability

KEY CHALLANGE

- The transmitter will need to change
  - Coding rate/scheme (ECC Encoder block)
  - Constellation size (Symbol Mapper block)
  - Power assigned to the sub-carrier (Power Control block)
- The change is done based on feedback channel information – reports:
  - BLER (or packet error rate PER) and Channel SINR (γ)



Resource Allocation Techniques for OFDMA

- Objective: Take advantage of multiuser diversity and adaptive modulation and coding in OFDMA systems
- The IEEE802.16e-2005 DOES NOT specify algorithms of how to do that
  - This is left for equipment manufacturers and vendors to differentiate their products
  - The standard specify only the possible configurations and parameters to use
- Resource allocation on downlink (in brief)
  - Users estimate and feedback channel state info (CSI) continuously to basestation (BTS)
  - BTS allocates sub-carriers and power based on CSI using an embedded algorithm
  - BTS informs users of the schedule (map message broadcast to users)
  - Users tune to their sub-carriers and decode required information

OFDMA project

Two Approaches for Resource Allocation

- (1) Minimize total transmit power with a constraint on the user data rate
  - Appropriate for fixed-rate applications such as voice
- (2) Maximize total data rate with a constraint on total transmit power
  - Appropriate for variable-rate applications such as "bursty" data
  - Refer to references in the literature and those sites in chapter 6 of the Fundamentals of WiMAX book.
- The above are formulated a OPTIMIZATION problems with constraints

Focus of project

System Parameters – Focusing on Downlink

Notation	Meaning
K	Number of users
L	Number of sub-carriers
h_{kl}	Envelope of channel gain for user k in sub-carrier l
P_{kl}	Transmit power allocated for user k in sub-carrier l
σ^2	AWGN power spectrum density
Ptot	Total transmit power available at the base station
B	Total transmission bandwidth

Class 1: Maximum Sum Rate (MSR) Algorithm

- The objective is to maximum the sum of rate of all users
- The signal to interference and noise power ratio for user k on the lth sub-carrier is given by

$$SINR_{k,l} = \frac{P_{k,l}h_{k,l}^2}{\sum_{j=1,j\neq k}^{K} P_{j,l}h_{k,l}^2 + \sigma^2 \frac{B}{L}}$$

- Using the Shannon capacity formula as our throughput measure, the MSR should try to maximize the following quantity:

$$\max_{P_{k,l}} \sum_{k=1}^{K}\sum_{l=1}^{L} \frac{B}{L}\log_2\left(1+SINR_{k,l}\right)$$

- The total power constraint is give by $\sum_{k=1}^{K}\sum_{l=1}^{L} P_{k,l} \leq Ptot$

Class 1: Maximum Sum Rate (MSR) Algorithm – cont'd

- The sum of sub-carriers capacity is maximized if the capacity per sub-carrier is maximized
- Denote the capacity per sub-carrier by C_l, then

$$C_l = \sum_{k=1}^{K} \log_2 \left(1 + \frac{P_{k,l}}{\sum_{j=1, j \neq k}^{K} P_{j,l} + \sigma^2 \frac{B}{h_{k,l}^2 L}} \right) = \sum_{k=1}^{K} \log_2 \left(1 + \frac{P_{k,l}}{P_{tot,l} - P_{k,l} + \sigma^2 \frac{B}{h_{k,l}^2 L}} \right)$$

- $P_{tot,l} - P_{k,l}$ denotes the other users' interference to user k in sub-carrier l.
- It is easy to show that C_l is maximized if all power $P_{tot,l}$ is allocated to the single user with the largest channel gain in sub-carrier l.
  - "Greedy" optimization

Class 2: Maximum Fairness (MF) Algorithm

- Since channel gain varies by orders of magnitude, there will be users that are starved with MSR
- Maximum fairness (MF) algorithm aims to allocate sub-carriers and power such that *minimum* user's data is maximized.
- Equivalent to equalizing the data rates of all users → maximum fairness
- Problem referred to as max-min problem (maximize the minimum data rate).
- MF is considerable more difficult to solve the MSR
  - Objective function is not concaved
  - Usually sup-optimal solutions are sought – the solution for sub-carrier and power allocation are done SEPARATELY.
- Common solution steps:
  - Assign all sub-carriers have equal power
  - iteratively each available sub-carrier to a low-rate user with the best channel on it
  - Perform optimum water-filling (power allocation) for remaining power budget

Class 2: Maximum Fairness (MF) Algorithm – cont'd

- Common solution steps:
  - Assign all sub-carriers have equal power
  - iteratively each available sub-carrier to a low-rate user with the best channel on it
  - Perform optimum water-filling (power allocation) for remaining power budget
- Points of weakness:
  - Rate distribution amongst users is not flexible
  - The total BTS throughput is limited by the worst SINR

Class 3: Proportional Rate Constraints (PRC) Algorithm

PRC is a generalization of MF

- Typically, different users have different rate constraints
- Objective: maximize the sum of throughput, with the additional constraint that each user's data rate is proportional to a set of predetermined systems parameters $\{\beta_k\}$ for k=1,2, …, K.
- The proportional data rates constraint can be expresses as

$$\frac{R_1}{\beta_1} = \frac{R_2}{\beta_2} = \cdots = \frac{R_K}{\beta_K}$$

where each user's achieved data rate, R_k is

$$R_k = \sum_{l=1}^{L} \frac{\rho_{k,l} B}{L} \log_2\left(1 + \frac{P_{k,l} h_{k,l}^2}{\sigma^2 \frac{B}{L}}\right)$$

$\rho_{k,l}$ can be either 1 or 0, indicating whether sub-carrier l is used by user k or not, respectively.

Class 3: Proportional Rate Constraints (PRC) Algorithm – cont’d

- For MF – set all β_k’s to 1.
- The PRC optimization problem is generally not easy to solve
  - Mix of variables: continuous ($P_{k,l}$) and binary ($\rho_{k,l}$)
  - Feasible set is not convex
- Suboptimal solutions are possible:
  - Do sub-carrier allocation first
  - Next step to assign power

Class 4: Proportional Fairness (PF) Scheduling

- All previous algorithms attempt to achieve objective instantaneously
  - i.e. The BTS will solve the assignment problem every slot
  - Every slot, the problem is resolved
- An alternative approach is to attempt to achieve the objective OVER TIME
  - Therefore, the objective is not met PER slot or frame
  - But the average performance of user over the entire session duration satisfies the desired objective
- This formulation adds a third parameter:
  - Throughput,
  - Fairness, and
  - Latency ← The new added parameter

Class 4: Proportional Fairness (PF) Scheduling – cont'd

- Let $R_k(t)$ be instantaneous (requested) data rate for user k at time slot t
- Let $T(t)$ be the average throughput for user k up to time slot t.
- At every slot, the scheduler selects the user with the highest $R_k(t)/T_k(t)$ for transmission – denote this user by k^*
- The average throughput for all users is then updated as

$$T_k(t+1) = \begin{cases} (1-1/t_c)T_k(t) + \frac{1}{t_c}R_k(t) & k = k^* \\ (1-1/t_c)T_k(t) & k \neq k^* \end{cases}$$

- t_c is the latency time-scale – controls the latency in the system

Class 4: Proportional Fairness (PF) Scheduling – cont'd

- Note that only the average throughput of the user k^* is increased (since it is selected) – the average throughput for all other users decreases

- Adaptation for OFDMA
  - Let Rk(t,n) be the data rate for user k in sub-carrier n at time slot t.
  - Let Ωk(t) be the set of sub-carriers in which user k is scheduled for transmit at time slot t
  - The average throughput is updated as follows:

Class 4: Proportional Fairness (PF) Scheduling – Adaptation for OFDMA

- Let $R_k(t,n)$ be the data rate for user k in sub-carrier n at time slot t.
- Let $\Omega_k(t)$ be the set of sub-carriers in which user k is scheduled for transmit at time slot t
- The average throughput is updated as follows:

$$T_k(t+1) = (1 - 1/t_c)T_k(t) + \frac{1}{t_c}\sum_{n\in\Omega_k(t)} R_k(t) \qquad k = 1, 2, \cdots, K.$$