KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF COMPUTER SCIENCES & ENGINEERING
COMPUTER ENGINEERING DEPARTMENT

COE 540 – Computer Networks
Assignment 3 – Due Date May 7th, 2011

Problem 1 (40 points): On the Discrete Time Markov Chains

Data in the form of fixed-length packets arrive in slots on the **two** input lines of a multiplexer. A slot contains a packet with probability p, independent of the arrivals during other slots or on the other line. The multiplexer transmits one packet per time slot and has the capacity to store **two** packets only. If no room for a packet is found, the packet is dropped. **Assume p is equal to 0.4**.

Hint: Define $\underline{p}(n) = [p_0(n) \quad p_1(n) \quad p_2(n)]$ where $p_i(n)$ for $i \in \{0,1,2\}$ is the probability that the mux has i packets at the beginning of slot n. Note that $p_0(n) + p_1(n) + p_2(n)$ is always equal to 1 for any $n = 0, 1, 2, \ldots$. Let the steady-state probability mass function (PMF) be denoted by $\underline{\pi}(n) = [\pi_0 \quad \pi_1 \quad \pi_2]$. Clearly, $\pi_i = \lim_{n\to\infty} p_i(n)$ for $i \in \{0,1,2\}$.

a) **(5 points)** Let N be the number of packets arriving to the multiplexer in a given time slot. Specify the probability distribution for N and its name. Compute the mean for N.
b) **(5 points)** Draw the state transition diagram and specify the probability transition matrix **P** (in terms of p and also after the using the value of p) – The state is taken to be the number of packets in the multiplexer.
c) **(5 points)** Assume the MUX starts with 2 packets in buffer at time slot 0. Plot the components of $\underline{p}(n)$ for the next 10 time slots (i.e. $n = 0, 1, 2, \ldots, 10$).
d) **(5 points)** Compute the steady state pmf for the system.
e) **(5 points)** Compute the mean number of packets in the MUX at any time slot.
f) **(5 points)** Compute the mean MUX throughput in packets per time slot.
g) **(5 points)** Compute the probability of a drop event from the MUX buffer.
h) **(5 points)** Compute the mean number of dropped packets at any time slot.
i) **(20 points – bonus)** Plot the MUX throughput in packets per time slot as a function of load offered to the MUX.

Problem 2 (40 points): Queueing Models

Consider the M/M/1 queue discussed in class.

a. **(5 points)** Draw the state transition diagram depicting states and the corresponding transition rates.
b. **(5 points)** Write and solve the global balance equations to compute the steady state probability of the queueing system having j customers, p_j, for $j = 0, 1, 2, \ldots$.
c. **(10 points)** Compute the mean number of customers in the waiting buffer.
d. (**10 points**) Repeat parts (a) and (b) for M/M/c/c queue.