Adaptive Power Allocation Algorithm to Support Absolute Proportional Rates Constraint for Scalable OFDM Systems

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***Abstract*—It is generally hard to find the optimal solution for sub-channel and power allocation for a multiuser Orthogonal Frequency Division Multiplexing (OFDM) system that maximizes the overall system capacity given the proportional rate constraint. Most existing solutions either utilize a suboptimal sub-channel allocation and attempt to compute the optimal power allocation or assume uniform power distribution among sub-channels and attempt to optimize using the sub-channel allocation. It is observed that derived solutions do not necessarily satisfy the proportional rate constraint, also referred to as the fairness constraint, supplied in the problem formulation. This paper proposes an iterative algorithm that computes the optimal power allocation for a given sub-channel allocation scheme. Unlike previous solutions, the proposed solution does not make any assumptions regarding the channels or regarding the proportionality constants. Furthermore, the proposed solution satisfies the proportional rate constraint in the strictest sense and therefore can provide absolute or hard rate guarantees as opposed to soft ones as the case for previous algorithms. Presented numerical example shows that our algorithm outperforms the original optimal power allocation algorithm and achieves strict satisfaction of the fairness constraint.**

Keywords: capacity; power allocation; dynamic resource allocation; multiuser OFDM.

I. INTRODUCTION

The problem of sub-channel and power allocation for a multiuser Orthogonal Frequency Division Multiplexing (OFDM) system while maximizing the total system throughput and satisfying the typical constraints of total power and fairness can be modeled as a mixed binary integer programming problem. The optimal solution for this problem is generally hard to find. The typical approach is to utilize a sub-optimal sub-channel allocation algorithm and then obtain the optimal power distribution for that specific sub-channel allocation [1]. In the literature, there exist several algorithms that attempt to solve this problem while making some assumptions to reduce the complexity of the problem. Some of these assumptions include shared sub-channel allocations which transforms the problem into a conventional nonlinear optimization problem. Another is the assumption of uniform power allocation cross all the sub-channels which reduces the problem to a sub-channel allocation problem.

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For wireless operators, the ability to specify the amount of allocated capacity for subscribers is an essential function for the network. The allocated capacity for the subscriber is typically proportional to the subscription fee and is an integral part of the overall quality-of-service (QoS) parameters associated with the ongoing data session. To account for this function, a common constraint that is typically included in the problem of sub-channel and power allocation for OFDM systems is the proportional rates constraint, also known as the proportional fairness constraint. With this constraint, the desired capacities resulting after solving the sub-channel and power allocation problem follow some specified ratios. Other definitions for proportional fairness also exist. The studies in [2] and [3] use the term proportional fairness to indicate equal probabilities of accessing the sub-cannels. However, this study is concerned with the former definition involving user capacities rather than the latter.

Examples of studies that tackle the above problem with the consideration of the proportional rates constraint are found in [1], [4]-[9]. In [4], [5], and [6], the authors simplify the problem by assuming that the number of sub-channels allocated to every user follow the proportional ratios. In [4] and [5], this assumption is used to simplify the derived optimal power allocation equations and facilitate the solution. The study in [6], utilizes the proportional number of channels assumption and allocates sub-channels based on the variances of gains for the sub-channels in the allocated sets for the users. For every iteration, the user with the highest variance for the associated set of sub-channels is allocated the sub-channel with the highest gain from the remaining pool of sub-channels. The methods in [4], [5], and [6] may give capacities that are very close to the specified proportional ratios if the gains for the sub-channels as received by different users are similar. However, as the variation in the gains increases between different users, those methods may give a solution that is far from the optimal one.

The above problem has also been tackled by [1], [7], and [8] and the references therein. However, the provided solutions do not necessarily satisfy the proportional rates constraint in general. The algorithm proposed in [1] assumes uniform power distribution, and then sub-channels are allocated iteratively in a round-robin fashion in an attempt to satisfy the proportional rates constraint. A similar approach is followed by the study in [7] where waterfilling is utilize after every sub-channel allocation to redistribute the power on the allocated sub-channels to optimize the capacity solution. The former two methods do not guarantee strict satisfaction of the proportional rates constraint. Furthermore, under some conditions in terms of variation in the sub-channels gains and the specifies ratios in the constraint, the deviation from the specified constraint may be significant. Similarly, the solution provided by [8] relies on finding the optimal power allocation for a given sub-channel distribution. To complete the solution, the proposed method assumes high and comparable sub-channel gains across the system bandwidth. The solutions provided in [5] and [8] are customized recently in [6] to serve real-time and non real-time flows. The original solution of [8] is also enhanced in [3] to provide long-term fairness.

At the other end of the spectrum, the authors of [9] focus on satisfying the proportionality constraint with less emphasis on the optimality of the obtained solution. The authors assume that power is allocated to every user is uniformly distributed among the sub-channels, and then an iterative method is employed to determine the power allocation for every user in order to satisfy the proportional ratios constraints. Clearly distributing the power uniformly deteriorates the quality of the obtained solution.

This work proposes an algorithm that is based on the optimal solution proposed in [8] where the proportional rates constraint is satisfied in the strictest sense (i.e. in every time slot) and therefore can provide absolute guarantees for the expected quality of service. In addition, the proposed solution is valid for arbitrary channel conditions and does not make any assumptions in regard to the gains for the sub-channels and the proportionality ratios used in the fairness constraint.

The rest of this paper is organized as follows: Section II defines the problem and the analytical model. The proposed solution is described in section III, while section IV provides and discusses some sample results. Finally, the paper concludes in section V.

II. Problem Definition

Assume the total system bandwidth is divided into N narrowband flat fading sub-channels as in [1], [5]-[8], and let there be K users to serve. Denote the n^{th} sub-channel power gain relative to noise power as received by the k^{th} user by $H_{n,k}$, where $n = 1, 2, ..., N$, and $k = 1, 2, ..., K$. It is desired to distribute the N sub-channels amongst the K users using the sub-channel allocation algorithm derived in [1] or some derivative algorithm. If the sub-channel allocation for the k^{th} user is denoted by Ω_k, then the corresponding power allocations should maximize the overall network throughput as specified by

$$\max_{p_{k,n}} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \frac{1}{N} log_2(1 + p_{k,n} H_{k,n}) \qquad (1)$$

subject to the following three constraints. First, the total power constraint specified by $\sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} \le P_{\text{total}}$ and $p_{k,n} \ge 0$, where P_{total} is the total power budget for the system, and $p_{k,n}$ is the power allocation for the $H_{n,k}$ sub-channel. Second, sub-channel allocations Ω_ks for different users are mutually exclusive, and lastly the proportional rates constraint $R_1/\gamma_1 = R_2/\gamma_2 = ... = R_K/\gamma_K$ is met where R_k is the k^{th} user bit rate, after the allocation process is completed, and is computed by $R_k = \sum_{n\in\Omega_k} 1/N \log_2(1 + p_{k,n} H_{k,n})$ and $\gamma_1, \gamma_2, ..., \gamma_K$ are the proportional rates constraint constants. This paper presents an algorithm for computing the power allocations $p_{k,n}$'s that provides *absolute* or *hard* proportional rate guarantees as opposed to *soft* guarantees while still maximizing the network throughput specified by (1).

The conventional method for solving optimization problem in (1) and the corresponding constraints is to use the Lagrange multipliers as in [5] and [8], and form the cost function that is differentiated and made equal to zero. This yields:

$$\frac{1}{\gamma_1}\frac{N_1}{N}(log_2(1 + H_{1,1}(P_{1,tot} - V_1)/N_1) + log_2 W_1) = \frac{1}{\gamma_k}\frac{N_k}{N}(log_2(1 + H_{k,1}(P_{k,tot} - V_k)/N_k) + log_2 W_k) \qquad (2)$$

for $k = 1, 2, ..., K$. The term $P_{k,\text{total}}$ is the total power allocated to the k^{th} user and should be given by $P_{k,\text{total}} = \sum_{n=1}^{N_k} p_{k,n}$, while the constants V_k and W_k are given by $\sum_{n=2}^{N_k} (H_{k,n} - H_{k,1})/(H_{k,n} H_{k,1})$ and $\prod_{n=2}^{N_k} (H_{k,n}/H_{k,1})^{1/N_k}$, respectively. The quantities V_k and W_k are a manifestation of the sub-channel frequency allocation procedure and depend only on the allocated sub-channel sets Ω_ks only. The above formulation assumes that the channel power gains for the k^{th} user have been ordered such that $H_{k,1} \le H_{k,2} \le ... \le H_{k,N}$ where N_k is the number of sub-channels allocated for the k^{th} user. That is the number of elements in the set Ω_k is equal to N_k. This means the quantity V_k is always positive.

If the relation in (2) is solved for total power allocated for a particular user, $P_{k,\text{total}}$, then the power allocation for the individual sub-channels, $p_{k,n}$, for that particular user can be found using $p_{k,1} = (P_{k,\text{total}} - V_k)/N_k$ and $p_{k,n} = p_{k,1} + (H_{k,n} - H_{k,1})/(H_{k,n} H_{k,1})$ This completely specifies the desired power allocations.

The relation in (2) specifies a set of $K - 1$ simultaneous non-linear equations that must be solved for $P_{k,\text{total}}$'s (or equivalently the power allocations $p_{k,n}$'s) that achieve the maximum throughput and satisfy the constraints. The authors in [5] and [8] do not solve (2) but rather simplified versions of the equation where the channel power gains $H_{k,n}$ are assumed to be large and similar (i.e. V_k's ≈ 0 and or large SNR scenarios). Therefore, the provided solution does not necessarily satisfy the proportional rates constraint for the general case. In this paper we provide a method for solving the original $K - 1$ non-linear equations specified by (2) without making assumptions in regard to the channel power gains. Furthermore, we *augment* the provided solution with an algorithm to ensure that the final output, $p_{k,n}$'s, will also satisfy the proportional rates constraint in the strictest sense.

III. Proposed Method

To solve the $K-1$ non-linear equations specified by (2), let the quantity X_k be define as $X_k = 1 + H_{k,1}(P_{k,\text{total}} - V_k)/N_k$, this means the k^{th} user power allocation, $P_{k,\text{total}}$ can be computed, given X_k, using

$$P_{k,\text{total}} = V_k + \frac{N_k}{H_{k,1}}(X_k - 1) \tag{3}$$

Substituting X_k in (2) and through manipulations, we can write

$$X_k = [(X_i W_i)^{(\gamma_k N_i)/(\gamma_i N_k)}]/W_k \tag{4}$$

for i and $k = 1, 2, ..., K$. To solve for X_i we use (4) and invoke the total power constraint to yield the following

$$\sum_{k=1}^{K} (V_k + \frac{N_k}{H_{k,1}}([(X_i W_i)^{\frac{\gamma_k N_i}{\gamma_i N_k}}]/W_k - 1)) - P_{\text{total}} = 0 \tag{5}$$

The relation (5) specifies one non-linear equation in X_i that can be solved using conventional methods known in the literature or simply by utilizing Matlab's `fsolve` routine [10]. Our algorithm then utilizes (4) and (3) to compute all X_k's and the corresponding total power allocations for the users. Finally the individual power allocations for sub-channels $p_{k,n}$, are calculated as in the original algorithm.

Unfortunately, the solution for (2) and therefore (5), since it is based on (2), can not be guaranteed to exist for any given set of sub-channel frequency allocations. This means the set of sub-channels, Ω_k's, produced by the sub-channel frequency allocation method may not be utilized as is. The conventional method is to drop weak channels from users until a solution can be found as indicated by the flow chart in Fig. 2 of [8]. In this paper we propose a new procedure for modifying the Ω_k's by dropping the weak channels until (2), or equivalently (5), has a valid solution. The devised procedure guarantees that the valid solution found optimizes the system throughput as specified by (1), meets the constraints, and satisfies the proportional rates constraint (4) in the strictest sense.

The key observation related to existence of a solution is the fact that for the given k^{th} user, its sub-channel frequency allocations, Ω_k, should be such that the corresponding V_k is smaller or equal than its final total user power allocation $P_{k,total}$. Therefore to ensure the existence of a valid solution, the following algorithm must first ensure that $\sum_{k=1}^{K} V_k \leq P_{\text{total}}$, and using the iterative procedure we ensure that V_k is smaller or equal than the corresponding total user power allocation $P_{k,total}$. The iterative procedure is as follows:

1) Given the sub-channel frequency allocations sets Ω_k for $k = 1, 2, ..., K$, compute the corresponding V_k and W_k.
2) Check the following inequality: $\sum_{k=1}^{K} V_k \leq P_{\text{total}}$. If the inequality is not satisfied go to step 3 otherwise go to step 4.
3) Select the set Ω_k that correspond to the largest V_k where $k = 1, 2, ..., K$; drop the channel with the smallest power gain $H_{k,n}$. Update set Ω_k, recalculate corresponding V_k and W_k, and go to step 2.
4) Select the user index i such that the corresponding $(W_i)^{N_i/\gamma_i}$ is greater than or equal to $(W_k)^{N_k/\gamma_k}$ for all $k \neq i$ and $k = 1, 2, ..., K$. The theoretical possible range for X_i is all values between 1 and $1 + H_{i,1}(P_{\text{total}} - V_i)/N_i$. Check if (5) has different signs when X_i assumes the two extreme values of its range. If (5) has a sign change, then there is a valid solution X_i between the corresponding extreme values, go to step 5. Otherwise, update the sub-channel frequency allocation sets, go to step 3.
5) A valid solution for (5) is guaranteed. Solve (5) for X_i, and use (4) to compute all X_k for $k \neq i$ and $k = 1, 2, ..., K$.
6) Compute the corresponding total user power allocation $P_{k,total}$ for $k = 1, 2, , K$ using (3) and for the k^{th} user where $k = 1, 2, ..., K$, compute the individual sub-channel power allocations $p_{k,n}$ for $n \in \Omega_k$.

The convergence of the above iterative procedure specified by steps 2, 3, and 4 is ensured through updating the sub-channel frequency allocation sets until a valid solution for (5) is found. The obtained solution maximizes the system throughput and also guarantees that the provided users' rates R_k's satisfy the proportional rates constraint such that $R_1 : R_2 : ... : R_K = \gamma_1 : \gamma_2 : ... : \gamma_K$.

IV. Results

To test the proposed algorithm we utilize the same 6-tap channel model employed in [5] and [8], and assume a total of 1 Watt for P_{total}. The power spectral density for noise is taken to -65 dBW per Hz. The minimum user capacity in b/s/Hz results are shown in Fig. 1 for 64 sub-channels in a 1 MHz bandwidth configuration. The results show the proposed algorithm provides significantly higher capacity than the original algorithm in [8]. The figure also includes that capacity results as computed by the classical time-division multiple access (TDMA) scheme. The capacity increase is about 26% for $K = 2$ and as high as 50% for $K = 14$. In Fig. 2 we evaluate fairness as defined by Jains fairness index stated in relation (3) of [8]. The index is a measure of adherence to the proportional rates constraint. It is clear that the proposed algorithm satisfies the proportional rates constraint in the strictest sense, while the original algorithm in [8] does not. The same behavior in terms of higher capacity and fairness is observed for other values of number of sub-channels and other channel model parameters.

V. Conclusion

This paper has presented an algorithm for solving the sub-channel and power allocation for a multiuser OFDM system. The solution utilizes a sub-channel allocation and computes the optimal power allocation for the given sub-channel distribution without making any assumptions regarding the

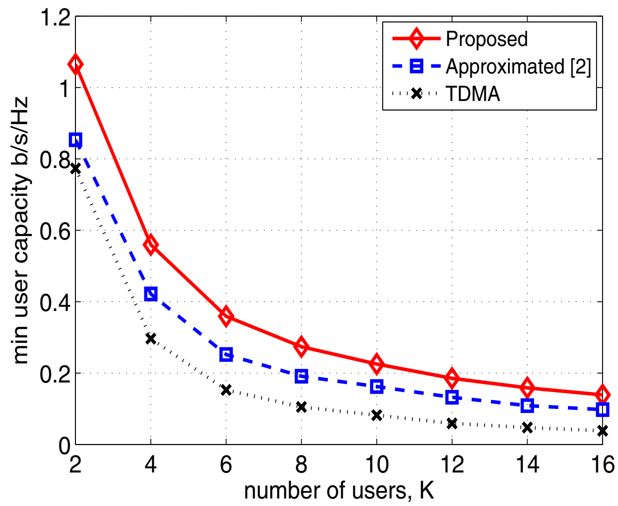

Fig. 1. minimum user capacity for multiuser OFDM versus number of users for proposed algorithm and other methods

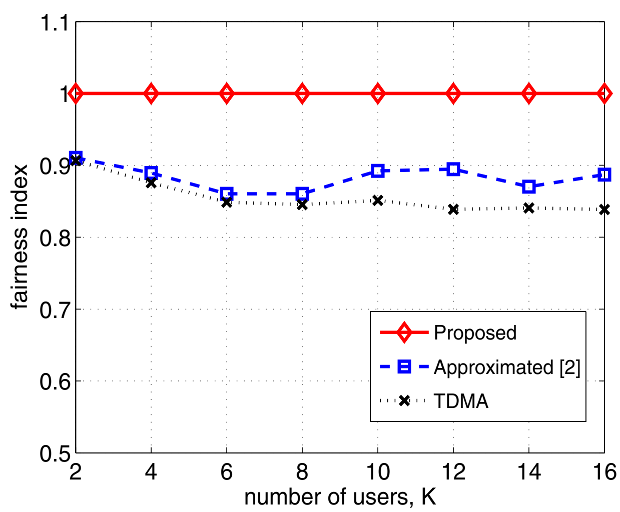

Fig. 2. Fairness index and adherence to the proportional rates constraint for proposed algorithm and original algorithm

channel characteristics and for arbitrary proportional rate constraint. The proposed algorithm guarantees the solution to satisfy the proportional rate constraint in the strictest sense and therefore can provide absolute or hard rate guarantees as opposed to soft ones as it is generally the case with previous solutions.

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