OFDM Basics for Wireless Communications

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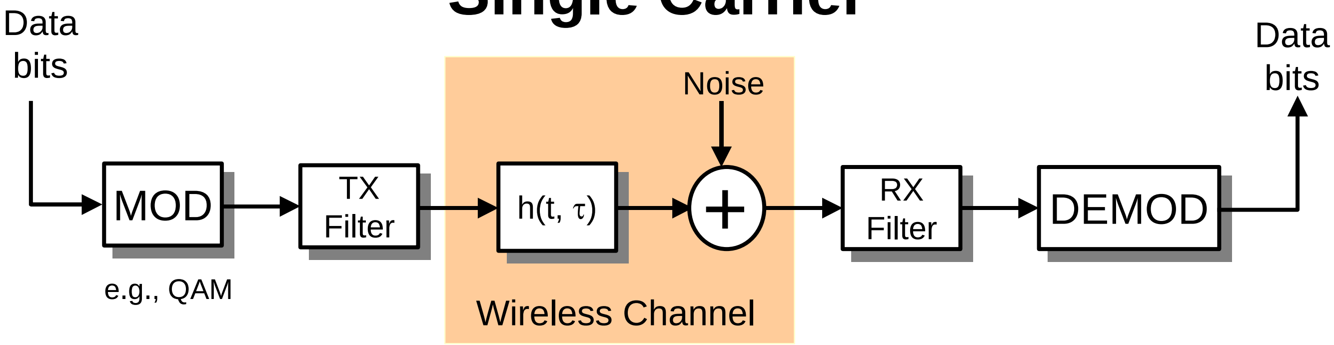
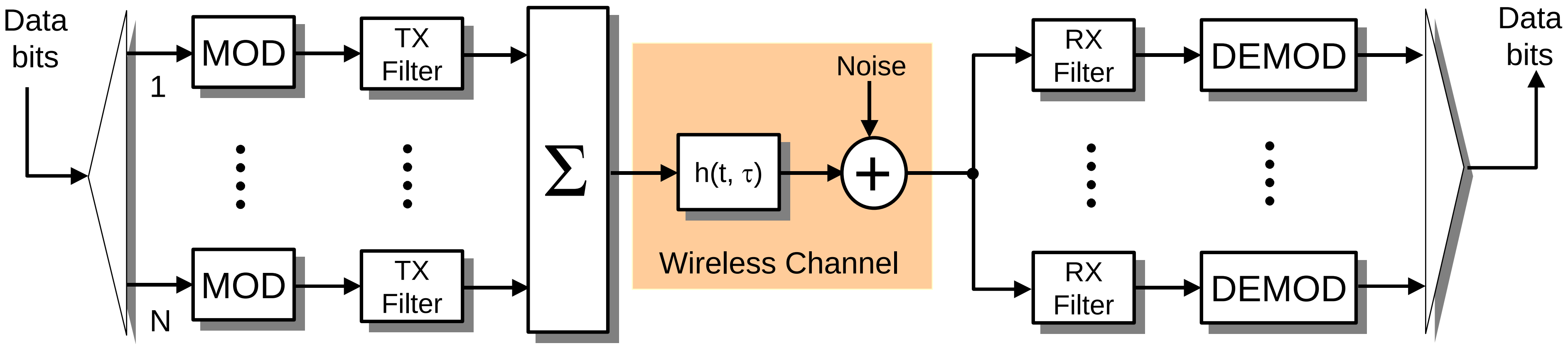
Single Carrier vs. Multicarrier

Single Carrier



Multicarrier

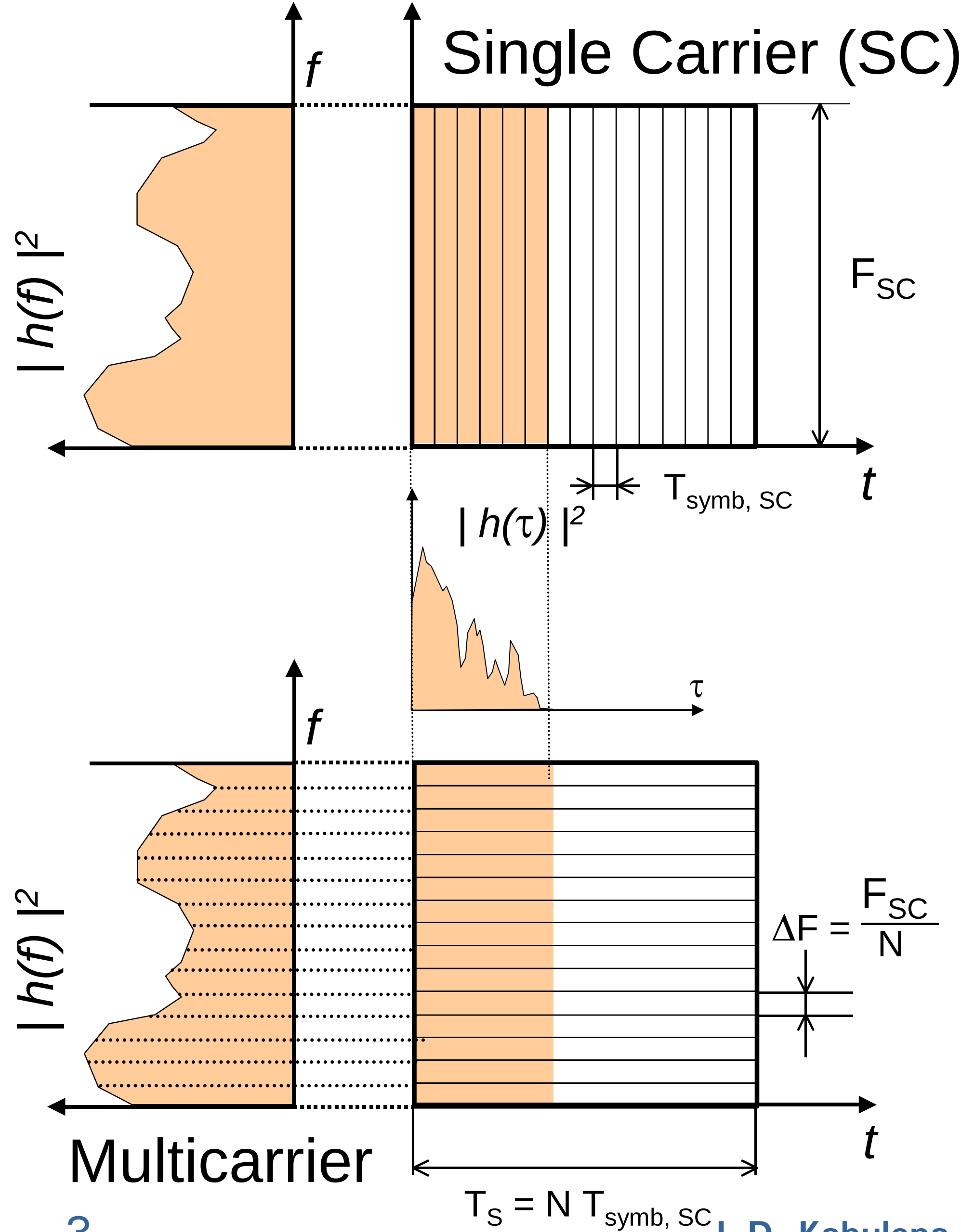


Multicarrier Transmission

Basic principle:

- Split the transmision bandwidth into many narrow subchannels which are transmitted in parallel
- (Ideally) Each subchannel is narrow enough so that it experiences a flat fading although the overall radio propagation environment is frequency-selective.

The time dispersion effects are less significant as the symbol duration increases



Benefit of Multicarrier Transmission

The multicarrier transmission allows to achieve high data rate in frequency-selective radio propagation environment

By assuming the same data rate:

- Single-Carrier

$$\frac{1}{T_{\text{symb,SC}}} > B_C \Rightarrow$$ Distortion, interference (ISI)

⇕

Large amount of signal processing required in the equalizer

- Multicarrier

$$\frac{1}{N\,T_{\text{symb,SC}}} < B_C \Rightarrow$$ No interference

- Data rate can be increased by using a larger number of subcarriers
- Less equalization effort (as ISI is reduced by a factor N)

(B_C = Coherence bandwidth)

Benefit of Multicarrier Transmission: Example

- A data rate of 10 Mbit/s is targeted in a multipath radio environment by using the BPSK modulation. Maximum spread delay = 5 µs

5 Mbit/s with BPSK $\Rightarrow$ Bandwidth = 5 MHz

- Single Carrier Scenario

$T_{symb,SC} = 0.2\ \mu s \Rightarrow \tau_{max} = 25\ T_{symb,SC}$

$\Rightarrow$ Intersymbol-Interference (ISI) is extended over 25 symbols

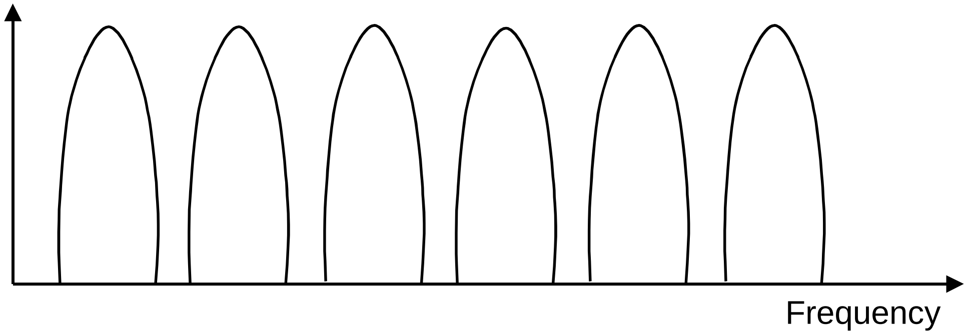
- Multicarrier Scenario

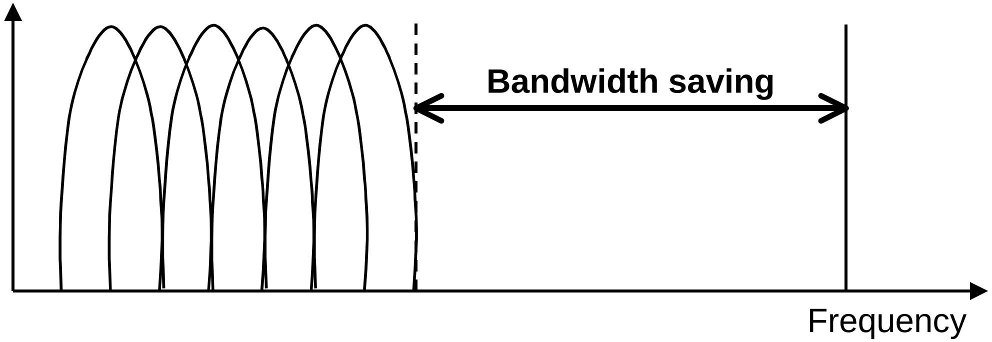
Number of subcarriers: 128

Symbol duration = $N\ T_{symb,SC} \Rightarrow \tau_{max} = 0.039\ NT_{symb,SC}$

$\Rightarrow$ ISI significantly reduced

Orthogonal Multicarrier





Orthogonality between the sub -carriers allows their overlapping while disabling the occurrence of crosstalks.

Thus, a significant power saving can be achieved by using an orthogonal multicarrier technique

Orthogonal Multicarrier (cont'd)

The orthogonality between the subcarriers can be achieved by letting the transmit filters $g_i(t)$ and the receive filters $r_i(t)$ fulfill the following conditions ($i \in \{1, \dots, N\}$)

1. Matched filter condition

$$r_i(t) = K \cdot g_i^*(T_0 - t)$$

2. Convolution condition

$$c_{j,n}(t = 0) = \int_{\tau=-\infty}^{+\infty} g_j(\tau) \cdot h_n(t - \tau)\, d\tau$$

$$= \int_{\tau=-\infty}^{+\infty} g_j(\tau) \cdot g_n^*(t - \tau)\, d\tau = \delta_{j,n} = \begin{cases} 1, & j = n \\ 0, & j \neq n \end{cases}$$

(Assumption: Perfect synchronization, $T_0 = 0$, $K = 1$)

Conventional OFDM

OFDM = Orthogonal Frequency Division Multiplexing

- In a conventional OFDM system, the orthogonality between the subcarriers is achieved by means of the discrete Fourier transform (DFT)

- Baseband OFDM signal

$$s(t) = \sum_{k=0}^{N-1} a_k \;{}^{j2\pi k\,\Delta f\,t} \qquad ,0 \leq t \leq T$$

- Passband OFDM signal

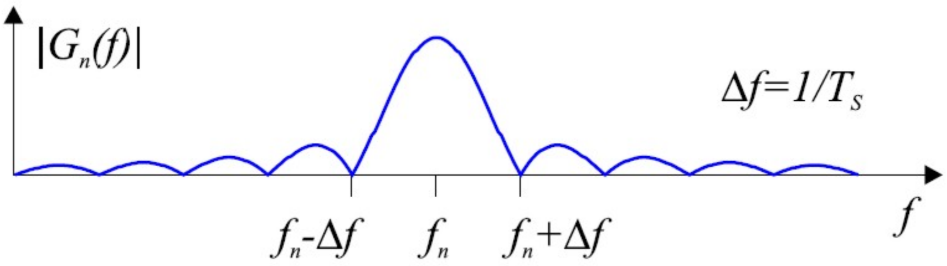
$$s(t) = \mathrm{Re}\left\{ \sum_{k=0}^{N-1} a_k \;{}^{j2\pi\left(f_C + k\,\Delta f\right)t} \right\} \qquad ,0 \leq t \leq T$$

a_k = complex-valued modulated symbols (e.g., QAM)
N = number of subcarriers
f_C = carrier frequency
T_s = sampling period, Δf = subcarrier spacing
The inverse DFT is used at the transmitter side

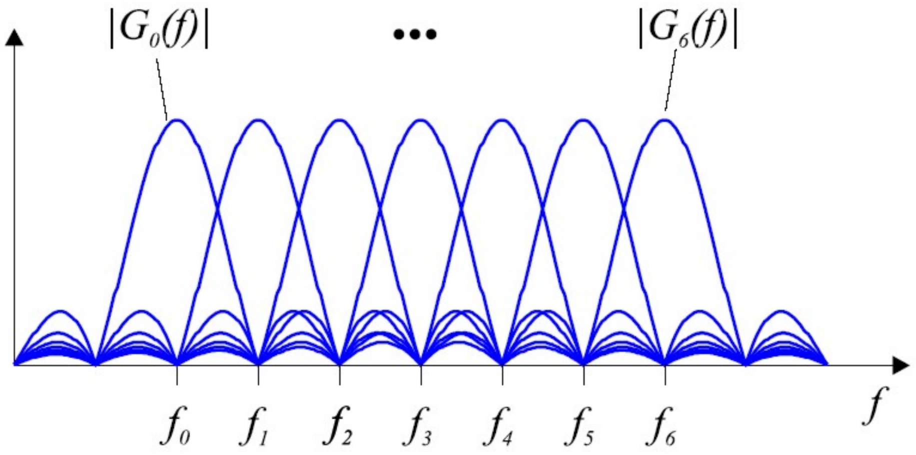
$$\Delta f = \frac{1}{T} = \frac{1}{N\,T_S}$$

Conventional OFDM(cont‘d)

1 subcarrier

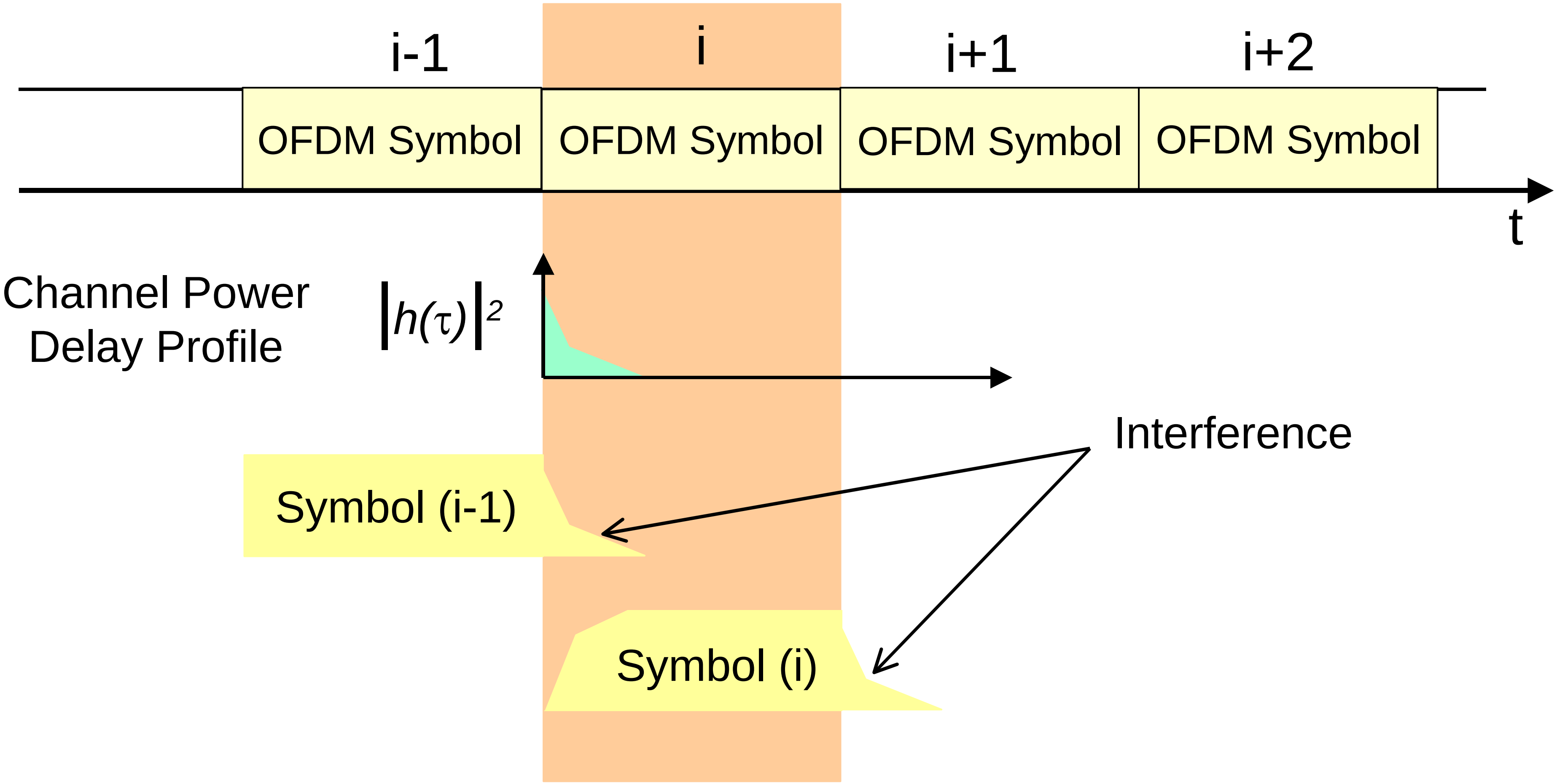


6 subcarriers

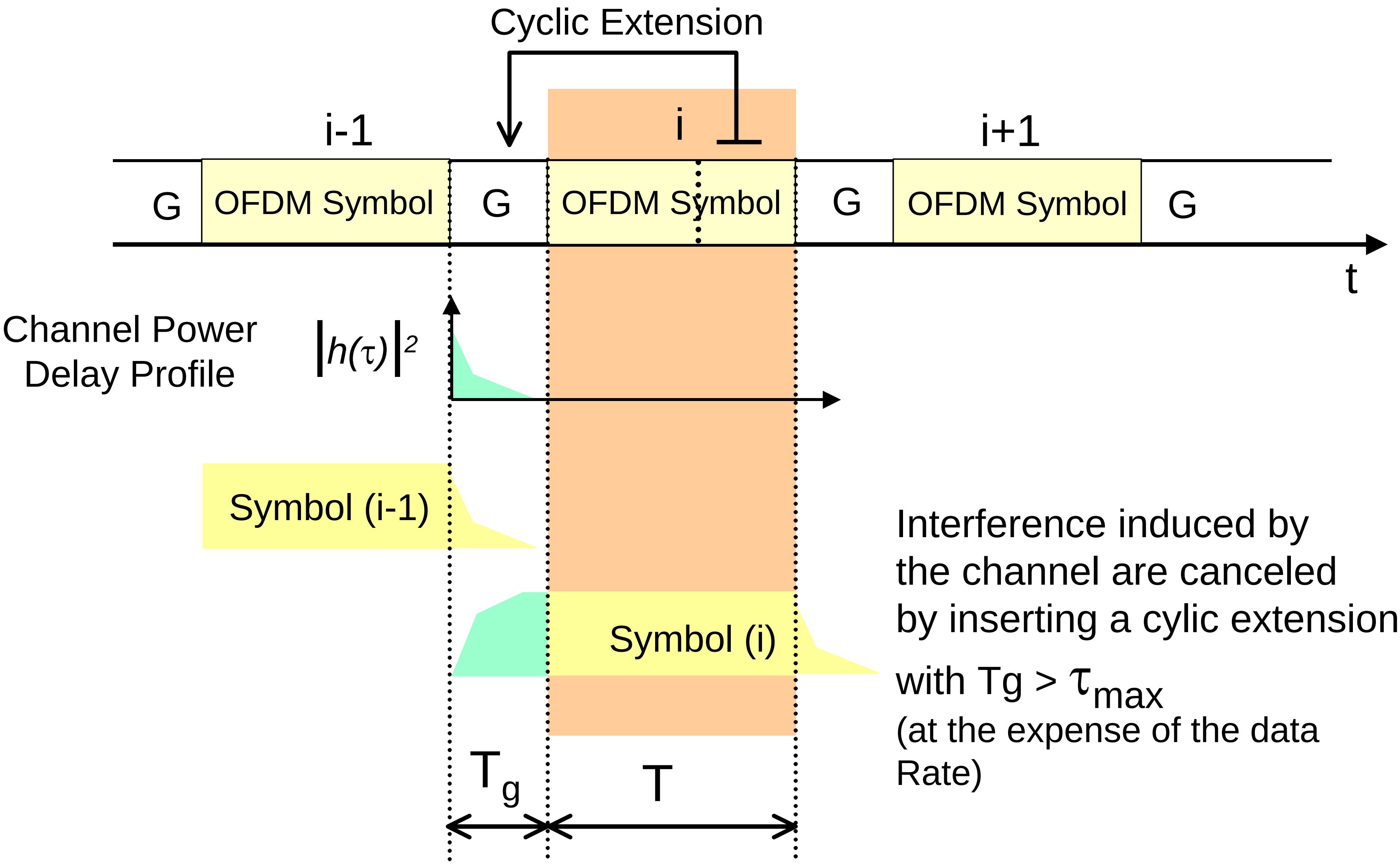


- The receiver is expected to compute the spectra values at those points corresponding to the maxima of individual subcarriers
- As a maximum of a subcarrier corresponds to zeros of other subcarrier, each subcarrier can demolutated independently of the others (by assuming a perfect synchronization)

Impact of a Wireless Channel



Cyclic Extension



Interference induced by the channel are canceled by inserting a cylic extension with Tg > τ_{max}
(at the expense of the data Rate)

Circular Convolution

- In the presence of interference induced by the channel

$$\mathrm{DFT}\{h(k) * s(k)\}_N \neq \mathrm{DFT}\{h(k)\}_N * \mathrm{DFT}\{s(k)\}_N$$

- The cyclic extension (with Tg > τ_{max}) allows to apply the circular convolution

$$\mathrm{DFT}\{h(k) \hat{*} s(k)\}_N = \mathrm{DFT}\{h(k)\}_N \hat{*} \mathrm{DFT}\{s(k)\}_N$$

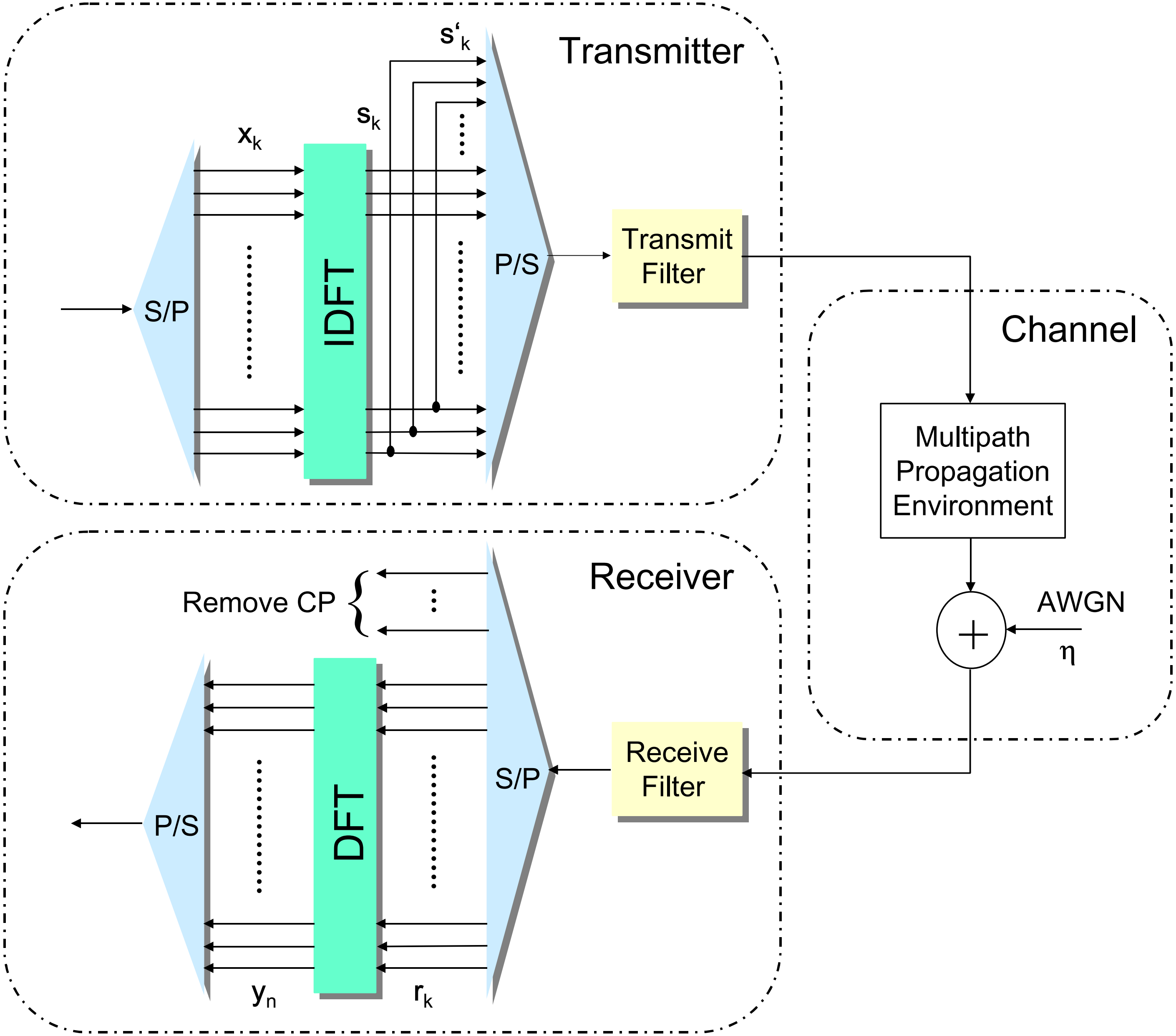
$\hat{*}$ = Circular Convolution

This property allows the use of a simple equalization scheme in the receiver

$$\Rightarrow \qquad \hat{y}(n) = H(n) \cdot y(n)$$

Relationship between transmitted and detected symbol

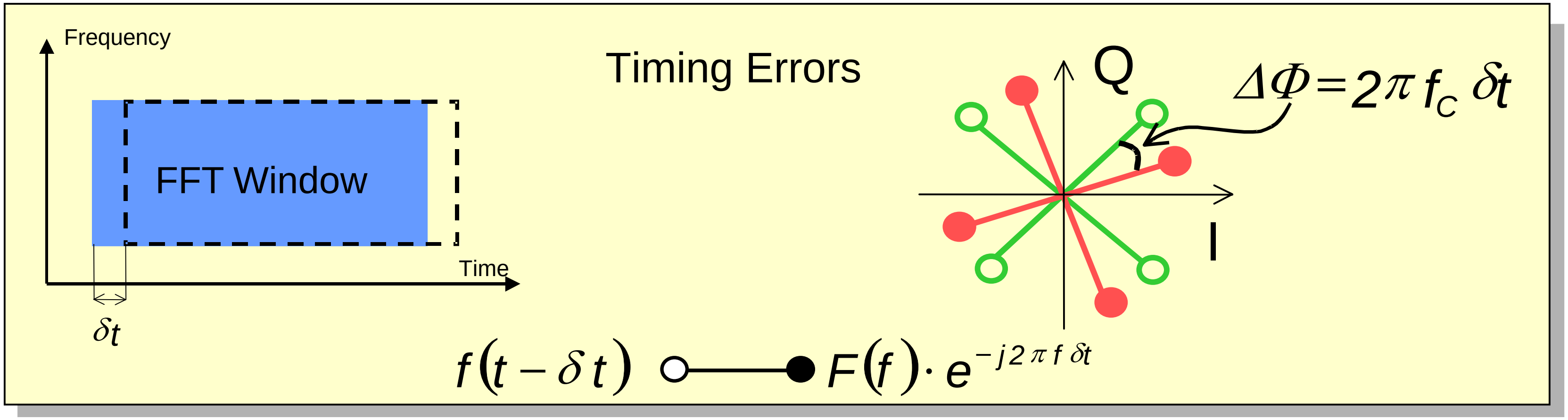
OFDM Transceiver



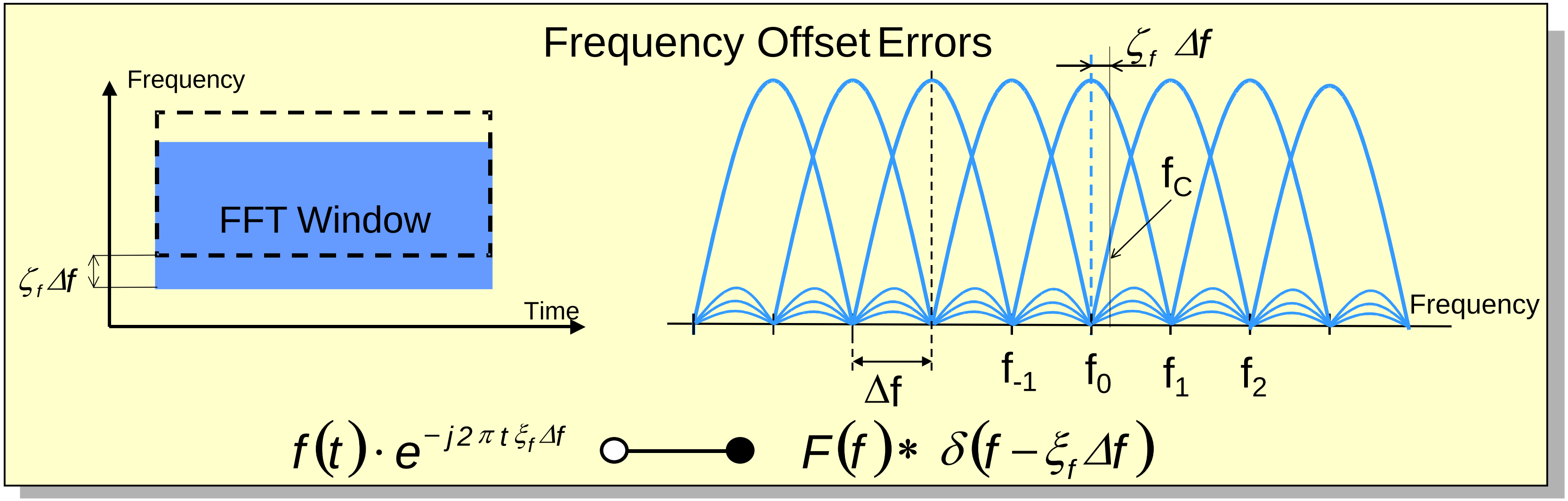
OFDM Drawbacks

1. High sensitivity to synchronization errors

Synchronization errors ⇒ Interference, loss of orthogonality



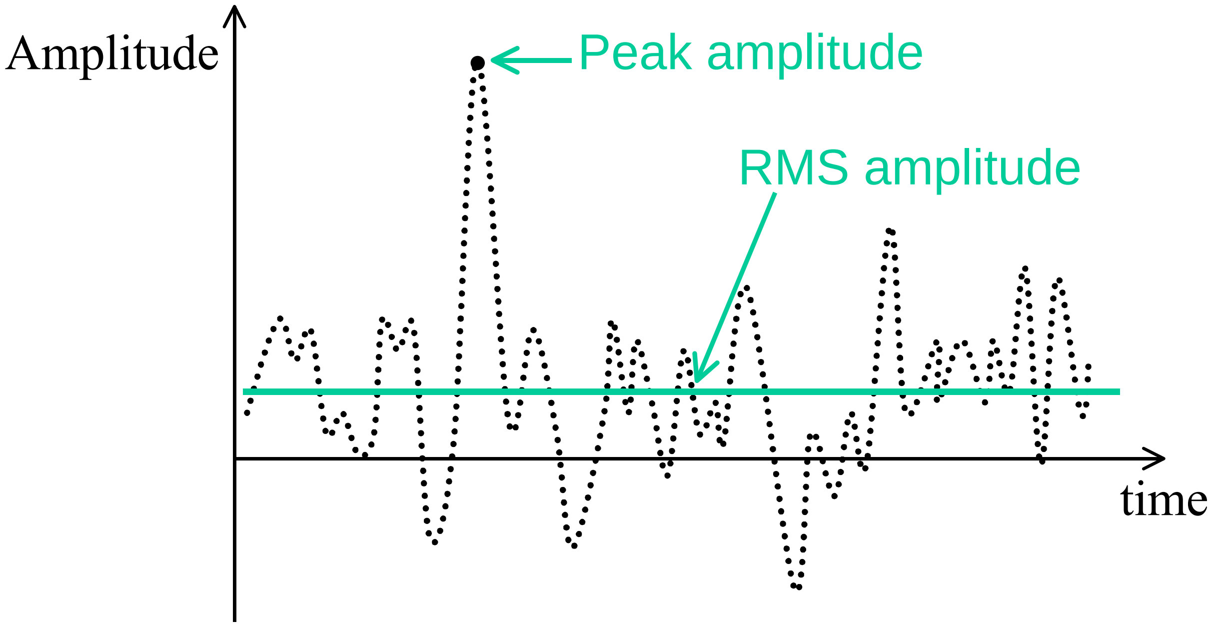
$$f(t-\delta t) \circ\!\!-\!\!-\!\!\bullet\, F(f)\cdot e^{-j2\pi f \delta t}$$



$$f(t)\cdot e^{-j2\pi t \xi_f \Delta f} \circ\!\!-\!\!-\!\!\bullet\, F(f) * \delta(f-\xi_f \Delta f)$$

OFDM Drawbacks(cont'd)

2. Occurrence of very high peak values



A reduction of the PAPR is highly desirable. The higher the PAPR, the lower The efficiency of circuits such as power amplifiers and analog-to-digital converters

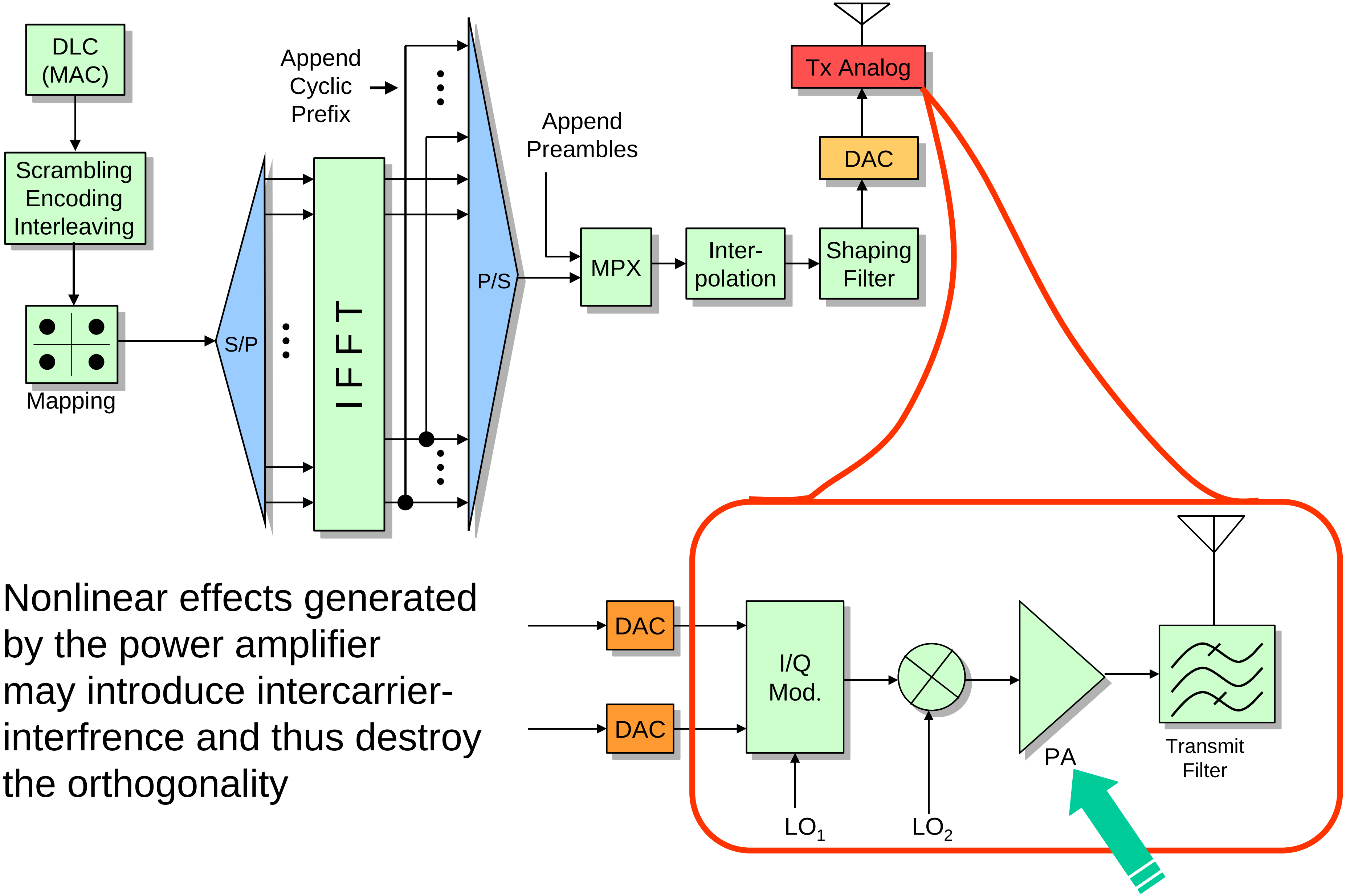
$$CR = \frac{\text{Peak amplitude}}{\text{RMS amplitude}}$$

$$PAPR = CR^2 = \frac{\text{Peak power}}{\text{Average power}}$$

CR: Crest Factor

PAPR: Peak-to-Average Power Ratio

OFDM Drawbacks(cont'd)



Nonlinear effects generated by the power amplifier may introduce intercarrier-interfrence and thus destroy the orthogonality