KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF COMPUTER SCIENCES & ENGINEERING

COMPUTER ENGINEERING DEPARTMENT

COE 540 – Computer Networks
Assignment 2 – Due Date Jan27th, 2009

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Problem 1 (50 points): On the subject of Discrete-Time Markov Chains

Data in the form of fixed-length packets arrive in slots on the THREE input lines of a multiplexer. A slot contains a packet with probability p, independent of the arrivals during other slots or on the other line. The multiplexer transmits one packet per time slot and has the capacity to store THREE packets only. If no room for a packet is found, the packet is dropped.

a) COMPUTE the probability of j (for all possible j values) packets arriving on the THREE input lines during any given time slot.

b) DRAW the state transition diagram and SPECIFY the transition matrix **P** – The state is taken to be the number of packets in the multiplexer.

c) Plot the state probability distribution vector $\underline{P}(n)=\left[p_0(n) \quad p_1(n) \quad p_2(n) \quad p_3(n)\right]$ as a function of the time index n. Plot for $n = 0, 1, \ldots, 10$. Assume the MUX is initially empty.

d) Let the load be defined as the mean number of arriving packets per time slot while throughput be defined as the mean number of transmitted packets per time slot. Use Matlab and show the code for:
   1) Plot the throughput versus the input load.
   2) What is the mean number of dropped packets when the average load is equal to 1 packet per time slot?
   3) What is the mean number of dropped packets when the average load is maximum?

You may use the code routine depicted in Fig Q1.1. The routine takes the one step probability transition matrix and returns the steady state probably distribution.

```
function pis = ComputeSteadyState(P);
L = length(P); % P is of size L by L;
E = zeros(L+1,1);
A(1:L,:) = (P(1:L,1:L) - eye(L))';
A(L+1,:) = ones(1,L); % add sum of pis = 1;
E(L+1)   = 1;
pis      = A\E;
```

Figure P1.1: Routine ComputeSteadyState.m

Hint: Use the code in the slides and modify it to solve this problem.

Problem 2 (40 points): On the subject of Delay and Queueing Models

Assume a small enterprise is installing a PBX telephony system with *c* outgoing phone lines connecting the enterprise with the PSTN. If the population and calling behavior of the enterprise employees are such that calls are generated according to a Poisson arrival process with rate of 5 calls every 4 minutes. The mean call duration is 2 minutes. Assume that calls arriving to the PBX, while the *c* outgoing lines are busy, are buffered and the caller waits till he gets a free line. Let $c = 4$.

(1) Computer the offered load from the enterprise in Erlangs.

(2) What is the probability that a call originating from the enterprise has to wait and what is the average waiting time in seconds.

(3) If it is desired to make the call waiting time not exceed 1 sec, what would be the minimum size (i.e. value of *c*) for the PBX achieving this requirement?

Hint: Use the M/M/c model (Erlang-C Model).

Problem 3 (20 points): On the subject of Probability Theory and Random Variables

Most of the queueing analysis performed in class for single server system assumed exponential service times of mean equal to $E[\tau]$ or $1/\mu$ where μ is the service rate. For a c servers systems, such as those in M/M/c or M/M/c/c systems, we have subsequently assumed that the service rate for the collective c identical but independent servers is $c\mu$ when the servers are busy. Prove mathematically the latter assumption.

Hint: consider the random variable X where X is the time till the first service completion amongst the c busy servers.

Problem 4 (40 points): On the subject of Probability Theory and Random Variables

Consider a binary transmit-receive system where binary bits {0, 1} are transmitted and then detected at the receiver. If the system makes an erroneous decision with probability equal to ε, calculate which input bit is more likely given the receiver output. Assume the 0 bit is transmitted with probability p, while the 1 bit is transmitted with probability 1-p.

Hint: The system behavior is captured by Prob{Out=1/In=0}=Prob{Out=0/In=0} = ε. It is required to compute Prob{In=i/Out=j} for all i and j = 0 and 1 .

Problem 5 (40 points): On the subject of Continuous-Time Markov Chains

Consider a system which alternates between two state 0 and 1. If the system spends an exponentially distributed random time in state 0 with mean $1/\alpha$ while it spends an exponentially distributed random time in state 1 with mean $1/\beta$.

a) Draw the transition rate diagram for the system and specify the rate transition matrix $\underline{\Gamma}$.

b) Write the Chapman-Kolmogorov differential equations for the system in matrix form.

c) Solve the Chapman-Kolmogorov differential equations for the system (show your work) and plot the state probability vector $\underline{P}(t)=\left[p_0(t) \quad p_1(t)\right]^T$ as a function of time for $\underline{P}(0)=\left[p_0(0) \quad p_1(0)\right]^T=\left[2/3 \quad 1/3\right]^T$.

d) Compute the steady state distribution for the system states.

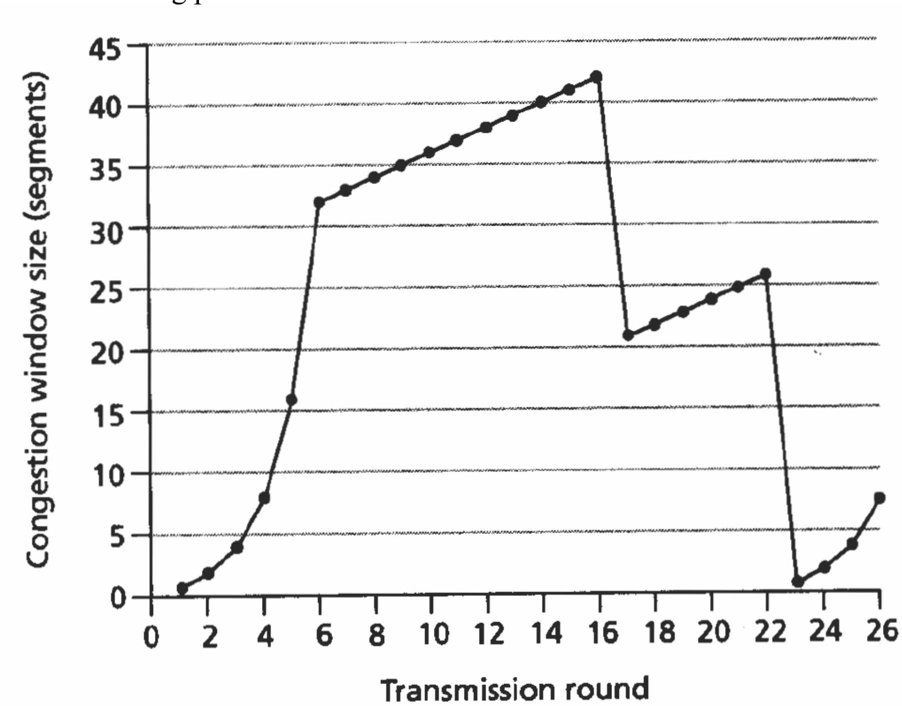
Hint: This question is solved partially in the class notes and was solved on the whiteboard during class time.

Problem 6 (50 points): On the subject of Transport Layer

Kurose's textbook (3^{nd} eddition) Chapter 3 - Problem 27 (page 291).

This is a typical TCP textbook problem. The instruction realizes that probably all students have access to the solution manual or previous offering of this problem and its solution key. However, it is deemed beneficial to include this problem in this homework.

Consider the following plot of TCP window size as a function of time.



Assuming TCP Reno is the protocol experiencing the behavior shown above, answer the following questions. In all cases, you should provide a short discussion justifying your answer.

a. Identify the intervals of time when TCP slow start is operating.

b. Identify the intervals of time when TCP congestion avoidance is operating.

c. After the 16th transmission round, is segment loss detected by a triple duplicate ACK or by a timeout?

d. After the 22nd transmission round, is segment loss detected by a triple duplicate ACK or by a timeout?

e. What is the initial value of `Threshold` at the first transmission round?

f. What is the value of `Threshold` at the 18th transmission round?

g. What is the value of `Threshold` at the 24th transmission round?

h. During what transmission round is the 70th segment sent?

i. Assuming a packet loss is detected after the 26th round by the receipt of a triple duplicate ACK, what will be the values of the congestion-window size and of `Threshold`?

<u>Problem 7 (30 points): On the subject of Internet Applications</u>

Suppose within your Web browser you click on a link to obtain a Web page. Suppose that the IP adderss for the associated URL is not cached in your local host, so that a DNS look-up is necessary to obtain the IP address. Suppose that *n* DNS servers are visited before your local host receives the IP address from DNS; the successive visits incur an RTT of RTT1, …, RTT*n*. Furthermore, suppose that the Web page associated with the link contains exactly one object, a small amount of HTML text. Let RTT0 denote the RTT between the local host and the server containing the object. Assume zero transmission time of the object.

(a) How much time elapses from when the client clicks on the link until the client receives the object.

(b) Suppose the HTML file indexes three very small objects on the same server. Neglecting transmission times, how much time elapses with (1) nonpersistent HTTP with no parallel TCP connections, (2) nonpersistent HTTP with parallel connections, (3) persistant HTTP with piplelining.

Problem 8 (50 points + 20 bonus): On the subject of Network Of Queues – Jackson Theorem

Consider the network of queues depicted in Figure Q6.1. The network consists of two queues: system 1 and system 2. For system 1 there is an external arrival rate of α customers per second. The service rate of system 1 is μ_1. Departures of system 1 are input to system 2 with probability 1-p and are allowed to exit the system completely with probability p. system 2 has a service rate of μ_2. Departures of system 2 are instantaneously input to system 1.

While the total arrival process to system 1 (external plus feedback from system 2) can be shown to be non Poisson, Jackson's theorem states that the number of customers in the queues any time t are independent random variables. In addition, it states that the steady state probabilities of the individual queues are those of an M/M/c_k system, where c_k is the number of servers in the k^{th} system. In the "open network" example, $c_1 = c_2 = 1$ – i.e. both system 1 and system 2 are single server systems.

a) Calculate the net arrival rates λ_1 and λ_2 for system 1 and system 2, respectively.
b) (**20 points – bonus**) Draw the state transition diagram for this system – Take the ordered pair (n_1, n_2) as the state, where n_1 is the number of customers in system 1, and n_2 is the number of customers in system 2.
c) Specify the steady state probability mass function (PMF) for number of customers in system 1, n_1, and system 2, n_2.
d) Write the expression for the joint PMF for n_1 and n_2.
e) Derive an expression for the expected time for a customer from entrance till it exits the system.
Hint: For part (e) use Little's formula and apply it to the entire open network.

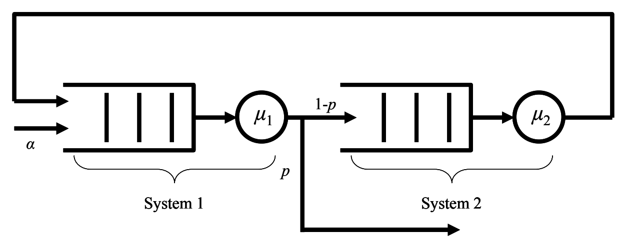


Figure Q8.1: Open queueing network for problem 6.