King Fahd University of Petroleum & Minerals
Computer Engineering Dept

COE 200 – Fundamentals of Computer Engineering

Term 043

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Karnaugh Map (K-Map)

- A tabular method to simplify function expressions – an alternative to algebraic manipulation
- Produces 2-level (sum of products or product of sums) implementation

1-variable K-map

- Consider the function F(X)

F(X) = 0
(F is NOT dependent on X)

x	0	1
	0	0

F(X) = 1
(F is NOT dependent on X)

x	0	1
	1	1

F(X) = X

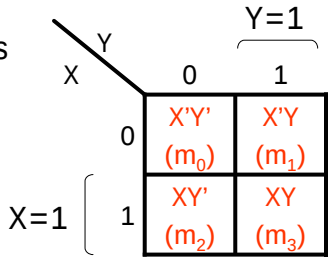
x	0	1
	0	1

F(X) = X'

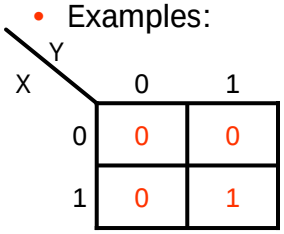
x	0	1
	1	0

2-variable K-map

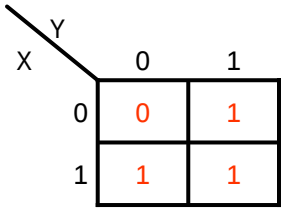
- Consider the function F(X,Y)
- The general 2-variable K-map is as shown
- The map is formed by putting two 1-variable K-maps side by side



- Examples:



F(X,Y) = XY



F(X,Y) = X'Y+XY'+XY
= X +Y

2-variable K-map – cont'd

- Neighbors sharing one literal:
  - X'Y' (or m_0) and X'Y (or m_1) ➔ sharing the literal X'
  - X'Y' (or m_0) and XY' (or m_2) ➔ sharing the literal Y'
  - X'Y (or m_1) and XY (or m_3) ➔ sharing the literal Y
  - XY' (or m_2) and XY (or m_3) ➔ sharing the literal X

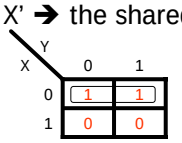
- If for example

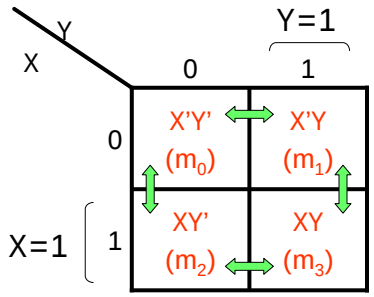
  F(X,Y) = m0 + m1 = X'Y' + X'Y

Then once can simplify F as follows:

F(X,Y) = X'(Y' + Y)

= X' ➔ the shared literal





2-variable K-map – cont'd

- Example2: F(X,Y) = Σm(1,2,3)

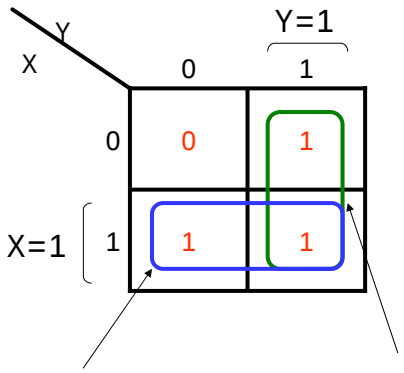
F can be simplified as in

F = X'Y +XY' + XY

= X'Y + XY + XY' + XY

= (X'+X)Y + X(Y'+Y)

= Y + X

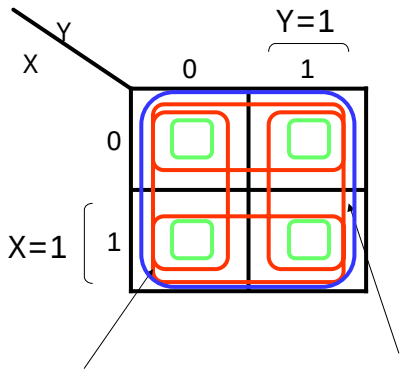


X – the common literal for this neighboring group

Y- the common literal for for this neighboring group

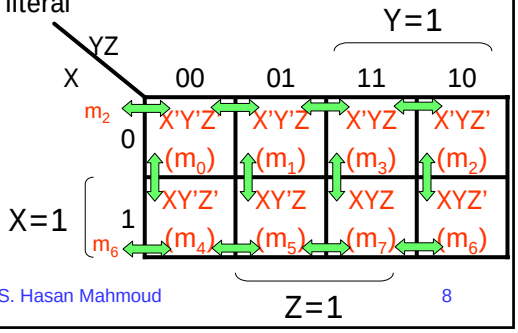
2-variable K-map – All Possible Squares

- 4 Groups each of one minterm
- 4 groups each of two minterms
- 1 group of 4 minterms



3-variable K-map

- Consider the function F(X,Y,Z)
- The general 3-variable K-map is as shown
- The map is formed by putting two 2-variable K-maps side by side
- Note:
  - The minterms are ordered such that any two neighboring minterm differ only in one literal
  - The K-Map (the numbering of the minterms) assumes X is the most significant variable and Z is the least significant variabe



3-variable K-map – cont'd

- Example F(X,Y,Z) = $m_5 + m_7$

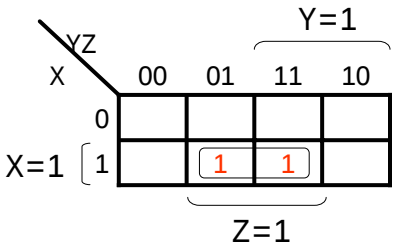
Once can simplify as in

F = $m_5 + m_7$
= XY'Z + XYZ
= XZ(Y'+Y)
= XZ

Or once can use the K-map as shown

The common literals for this group is XZ (they differ in Y)

Therefore: F(X,Y) = XZ



3-variable K-map – cont'd

- Example F(X,Y,Z) = Σm(2,3,4,5)

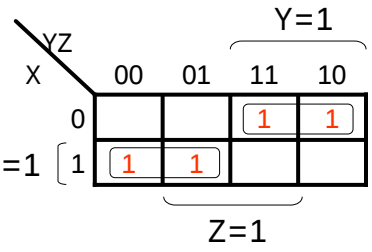
Once can simplify as in

F = $m_2 + m_3 + m_4 + m_5$
= X'YZ' + X'YZ + XY'Z' + XY'Z
= X'Y(Z'+Y) + XY'(Z'+Z)
= X'Y + XY'

Or one can use the K-map as shown

The common literals for the 1st group is X'Y (they differ in Z), while the common literals for the 2nd group is XY' (they differ in Z)

Therefore: F(X,Y) = X'Y + XY'



3-variable K-map – cont'd

- Example F(X,Y,Z) = Σm(0,2,4,5,6)

Once can simplify as in

F = $m_0 + m_2 + m_4 + m_5 + m_6$

= X'Y'Z' +X'YZ' + XY'Z' + XY'Z + XYZ'

=X'Y'Z' +X'YZ' + XY'Z' + XY'Z' + XY'Z + XYZ'

= Y'Z'(X' + X) + YZ'(X'+X) + XY'(Z'+Z')

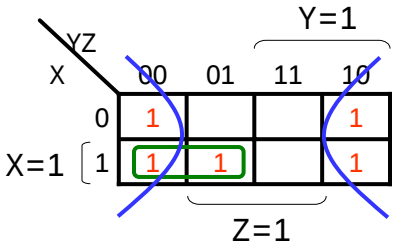
= Z'(Y'+Y) + XY'

= Z' + XY'

Or one can use the K-map as shown

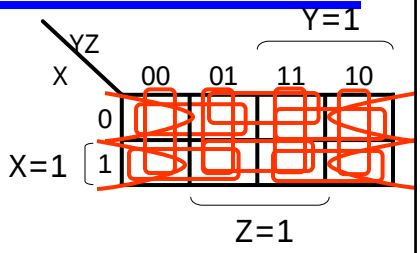
The common literals for the 1st group is XY' (they differ in Z), while the common literal for the 2nd group is Z' (they differ in XY)

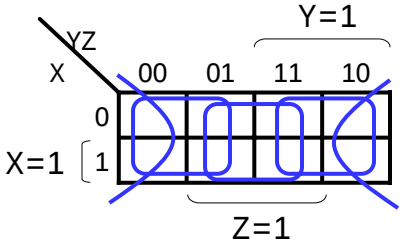
Therefore: F(X,Y) = Z' + XY'



3-variable K-map – All Possible Groups

- 8 groups each of 1 minterms
- 12 groups each of 2 minterms
- 4 groups each of 4 minterms
- 1 group of 8 minterms





Rules for Choosing Groups

- The groups SHOULD cover all minterms
- The groups SHOULD have minimum overlap
- The groups SHOULD be maximized in size (to reduce their number or product terms)

Example

- Consider F(X,Y,Z) = Σm(1,3,4,5,6)

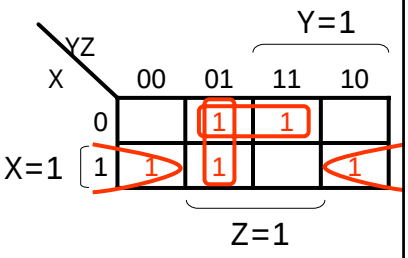
Following the groups selection rules:

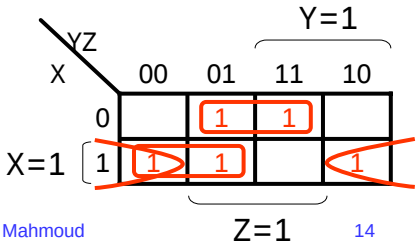
- there is no group of 8 or 4 that can be selected
- there are only groups of 2 that can be selected
- once can select the groups as shown (minimum no of groups)

Therefore F(X,Y,Z) = X'Z+Y'Z+XZ'

OR (See second K-Map)

F(X,Y,Z) = X'Z + XZ' + XY'

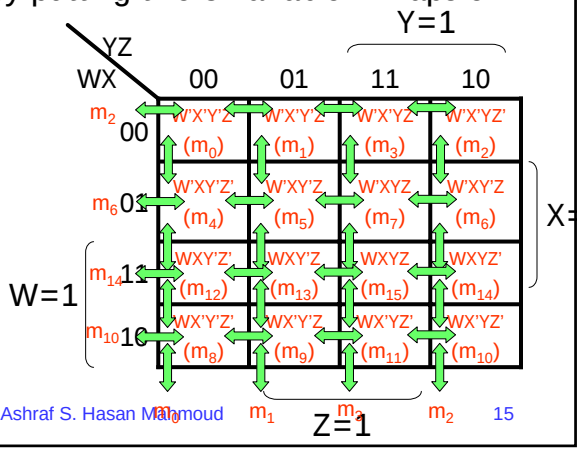




The simplest expression is NOT unique!

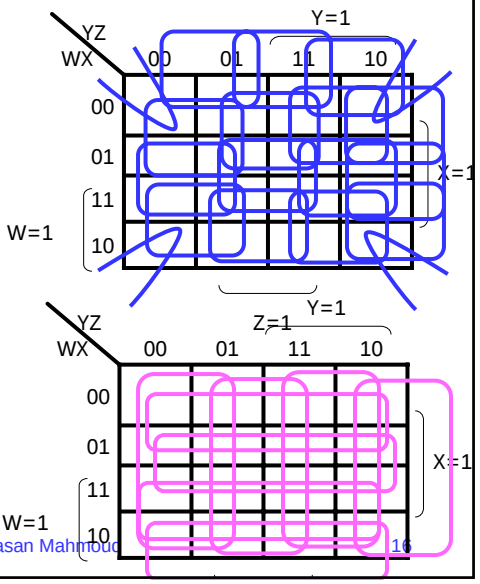
4-variable K-map

- Consider the function F(W,X,Y,Z)
- The general 4-variable K-map is as shown
- The map is formed by putting two 3-variable K-maps on top of each other
- Note:
  - The minterms are ordered such that any two neighboring minterm differ only in one literal
  - The K-Map (the numbering of the minterms) assumes W is the most significant variable and Z is the least significant variable



4-variable K-map – All Possible Groups

- 16 groups of one minterm
- 32 groups of two minterms
- ? groups of four minterms
- 8 groups of 8 minterms
- 1 group of 16 minterms



4-variable K-map – Example

- Consider F(W,X,Y,Z) = Σm(0,1,2,4,5,6,8,9,12,13,14)

F(W,X,Y,Z) = Y' + W'Z' + XZ'