

COMPUTER ENGINEERING DEPARTMENT

COE 541 – Local and Metropolitan Area Networks

Assignment 2 – Due Nov 2nd, 2004 in class

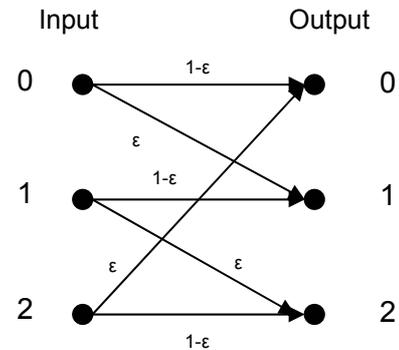
Problem 1:

A ternary communication channel is shown in figure.

Suppose that the input symbols 0, 1, 2 occur with probability 1/2, 1/4, and 1/4, respectively.

- a) Find the probability of the output symbols.
- b) Suppose a 1 is observed as an output. What is the probability that the input was 0, 1, 2? (*Hint: what is required is $P(\text{input} = i/\text{output} = 1)$ for $i=0,1,2$*)

Bonus: 30% - if the channel is modeled the analytical is result is matched to simulation results.



Problem 2:

Let $G_N(z)$ be the probability generating function for the non-negative integer valued random variable N . Prove that the variance of N is given by $Var[N] = G''_N(1) + G'_N(1) - [G'_N(1)]^2$.

Problem 3:

Let N be a binomial random variable with parameters n and p , i.e. the pmf for N is given by

$$p_k = P(N = k) = \binom{n}{k} p^k (1-p)^{n-k} \quad k = 0, 1, \dots, n$$

Find the expression for p_k as n approaches infinity and p approaches zero such that $\alpha = np$ remains fixed.

(Hint: Limit of $(1-x/n)^n = e^{-x}$ as n approaches infinity).

Problem 4:

Suppose that a queueing system is empty at time $t = 0$, and let the arrival times of the first six customers be 1, 3, 4, 7, 8, and let their respective service times be 3.5, 4, 2, 1, 1.5. Tabulate the arrival of i^{th} customer (A_i), service duration of i^{th} customer (τ_i), departure time of i^{th} customer (D_i), waiting time of i^{th} customer (W_i), total delay time of i^{th} customer (T_i) for $i = 1, 2, 3, 4, 5$; Sketch $N(t)$ versus t ; and check Little's formula by computer $\langle N \rangle_t$, $\langle \lambda \rangle_t$, and $\langle T \rangle_t$ for each of the following three service disciplines:

- a) First-come-first-served
- b) Last-come-first served
- c) Shortest-job first

Bonus: 20% for each of the parts a, b, and c (i.e. 60% in total) if the results are produced using code. Note the code has to be correct for any arrival/service sequence and not only for this particular problem.