

**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF COMPUTER SCIENCES & ENGINEERING**

COMPUTER ENGINEERING DEPARTMENT

**COE-342 – Data and Computer Communication
Programming Assignment #1**

Due Date: Saturday November 9th, 2002

Q.1. Given a square wave signal of period $T = 1$ ms and peak amplitude $A = 1$ V. One period of the signal is shown below:

$$s(t) = \begin{cases} 1 & -T/4 \leq t \leq T/4 \\ 0 & T/4 < t < T/2 \\ 0 & -T/2 < t < -T/4 \end{cases}$$

- (i)** You are required to calculate the Fourier Series of such a signal and determine its frequency components. Limit yourself to the first 6 frequency components.
- (ii)** Using Matlab, plot the signal $s(t)$ and plot its frequency spectrum.
- (iii)** The above signal is sent through different media with different bandwidths. If the media have the following bandwidths, what effects does this have on the signal at the receiver?
 - 1. BW = 20 KHz,
 - 2. BW = 10 KHz.
 - 3. BW = 4.5 KHz.

For each case, reconstruct the signal and plot it using Matlab. What do you conclude?

Solution

i) The Fourier Series expansion for $s(t)$ is given by

$$s(t) = \frac{A_0}{2} + \sum_{n=1,2}^{\infty} A_n \cos(2\pi n f t) + B_n \sin(2\pi n f t)$$

where the coefficients are computed as

$$\begin{aligned} A_0 &= \frac{2}{T} \int_{-T/2}^{T/2} s(t) dt \\ A_n &= \frac{2}{T} \int_{-T/2}^{T/2} s(t) \cos(2\pi n f t) dt \quad n = 1, 2, \dots \\ B_n &= \frac{2}{T} \int_{-T/2}^{T/2} s(t) \sin(2\pi n f t) dt \quad n = 1, 2, \dots \end{aligned}$$

* The coefficient A_0 is equal to:

$$A_0 = \frac{2}{T} \int_{-T/2}^{T/2} s(t) dt = \frac{2A}{T} \times \int_{-T/4}^{T/4} dt = \frac{2A}{T} t \Big|_{t=-T/4}^{t=T/4} = \frac{2A}{T} \frac{T}{2} = A$$

* The coefficients A_n ($n=1,2, 3, \dots$) are computed as

$$\begin{aligned} A_n &= \frac{2}{T} \int_{-T/2}^{T/2} s(t) \cos(2\pi n f t) dt = \frac{2A}{T} \times \int_{-T/4}^{T/4} \cos(2\pi n f t) dt = \frac{2A}{T} \times \frac{1}{2\pi n f} \sin(2\pi n f t) \Big|_{t=-T/4}^{t=T/4} \\ &= \frac{A}{n\pi} \sin(\pi n / 2) = \begin{cases} 0 & n = 2, 4, 6, \dots \\ \frac{2A}{n\pi} (-1)^{(n-1)/2} & n = 1, 3, 5, \dots \end{cases} \end{aligned}$$

* The coefficients B_n ($n=1,2, 3, \dots$) are computed as

$$\begin{aligned} B_n &= \frac{2A}{T} \int_{-T/2}^{T/2} \sin(2\pi n f t) dt = \frac{2A}{T} \times \int_{-T/4}^{T/4} \sin(2\pi n f t) dt = \frac{2A}{T} \times \frac{-1}{2\pi n f} \cos(2\pi n f t) \Big|_{t=-T/4}^{t=T/4} = \\ B_n &= \frac{-A}{\pi n} [\cos(\pi n) - \cos(-\pi n)] = 0 \text{ for all } n \end{aligned}$$

The final Fourier Series expansion for $s(t)$ is given by

$$s(t) = \frac{A}{2} + \frac{2A}{\pi} \sum_{n=1,3,5}^{\infty} (-1)^{(n-1)/2} \frac{\cos(2\pi n f t)}{n}, \text{ or}$$

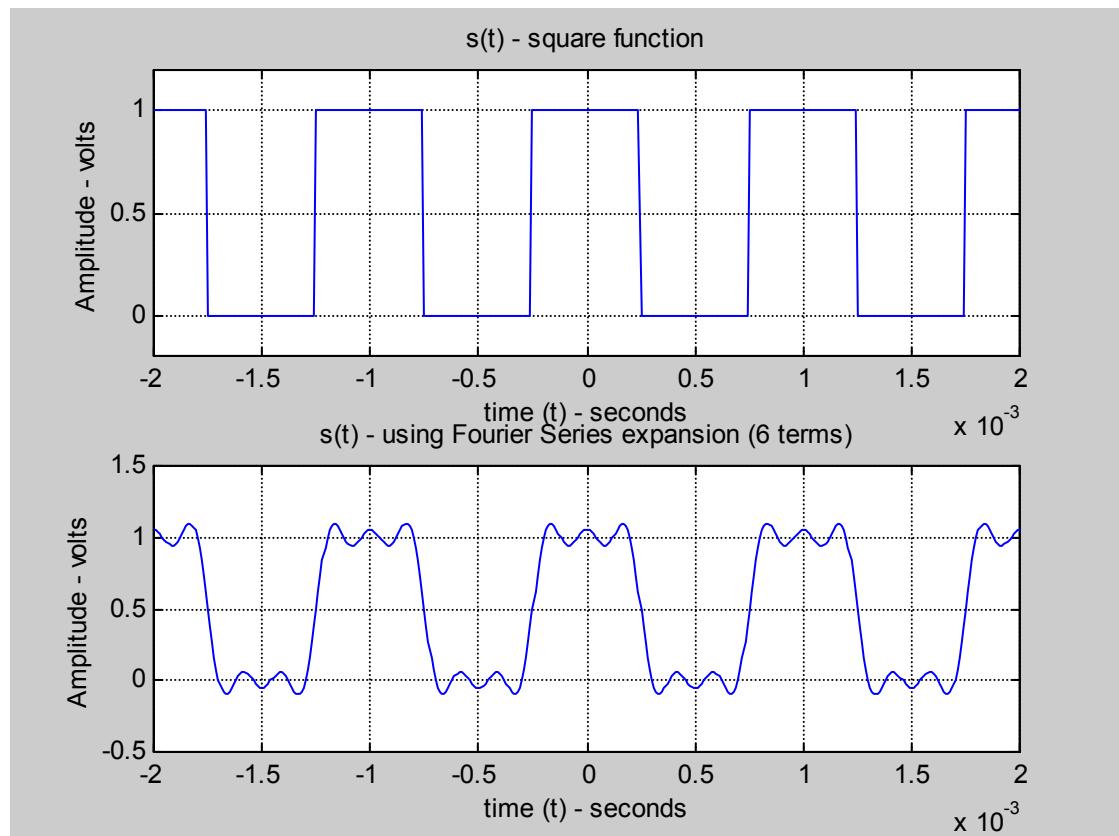
For $A = 1$ volts, $T = 0.001$ second (or $f = 1000$ Hz), $s(t)$ is written as

$$s(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1,3,5}^{\infty} (-1)^{(n-1)/2} \frac{\cos(2000\pi n t)}{n}$$

Limiting the expression to first 6 components

$$\hat{s}(t) = \frac{1}{2} + \frac{2}{\pi} \left[\frac{\cos(2000\pi t)}{1} + \frac{-\cos(6000\pi t)}{3} + \frac{\cos(10000\pi t)}{5} + \frac{-\cos(14000\pi t)}{7} + \frac{\cos(18000\pi t)}{9} + \frac{-\cos(22000\pi t)}{11} \right]$$

Graph for $s(t)$ and its Fourier Series Expansion:



Matlab code for part i)

```

clear all

T = 0.001; % period of signal
A = 1; % amplitude of signal

dt = T/100; % step in time domain
t = -2*T:dt:2*T;% define the time axis - up to 4 time periods
%
% define the function s(t) - one period is used
s = A*(1/2 + 1/2*square(2*pi*(t+T/4)./T));

%
% Fourier Series expansion is given by:
%
% infinity
% s(t) = a_0/2 + sum [ a_k * cos(2*pi*k*f*t) + b_k * sin(2*pi*k*f*t) ]
% k=1
%

aa_0          = A;
max_k         = 100;
k             = 1:max_k;
aa_k          = 2*A./(pi*k) .* (-1).^( (k-1)/2 );
aa_k(2:2:max_k) = 0;

bb_k          = zeros(size(k));

f   = 1/T;
ss = A/2 *ones(size(t));
for i=1:6
    ss = ss + aa_k(i)*cos(2*pi*i*f*t) + bb_k(i)*sin(2*pi*f*t);
end

figure(1);
subplot(2,1,1);
plot(t, s);
title('s(t) - square function');
xlabel('time (t) - seconds');
ylabel('Amplitude - volts');
axis([-2*T 2*T -0.2 1.2]);
grid

subplot(2,1,2);
plot(t, ss);
title('s(t) - using Fourier Series expansion (6 terms)');
xlabel('time (t) - seconds');
ylabel('Amplitude - volts');
grid

```

ii) Plotting the signal $s(t)$ and its spectrum

To obtain the spectrum function we will use the FFT routine in Matlab.

```
% ProgAssign01_COE021342_Q1_ii
clear all
%
% N defines the number of samples in the time domain - and the number of
% FFT points
% Since N has to be even, N is made a multiple of 2
M = 10; % The higher the value of M (and consequently N) the better
N = 2^M;% the approximation of the FFT to the Fourier Series expansion

T = 0.001; % period of signal
A = 1; % amplitude of signal

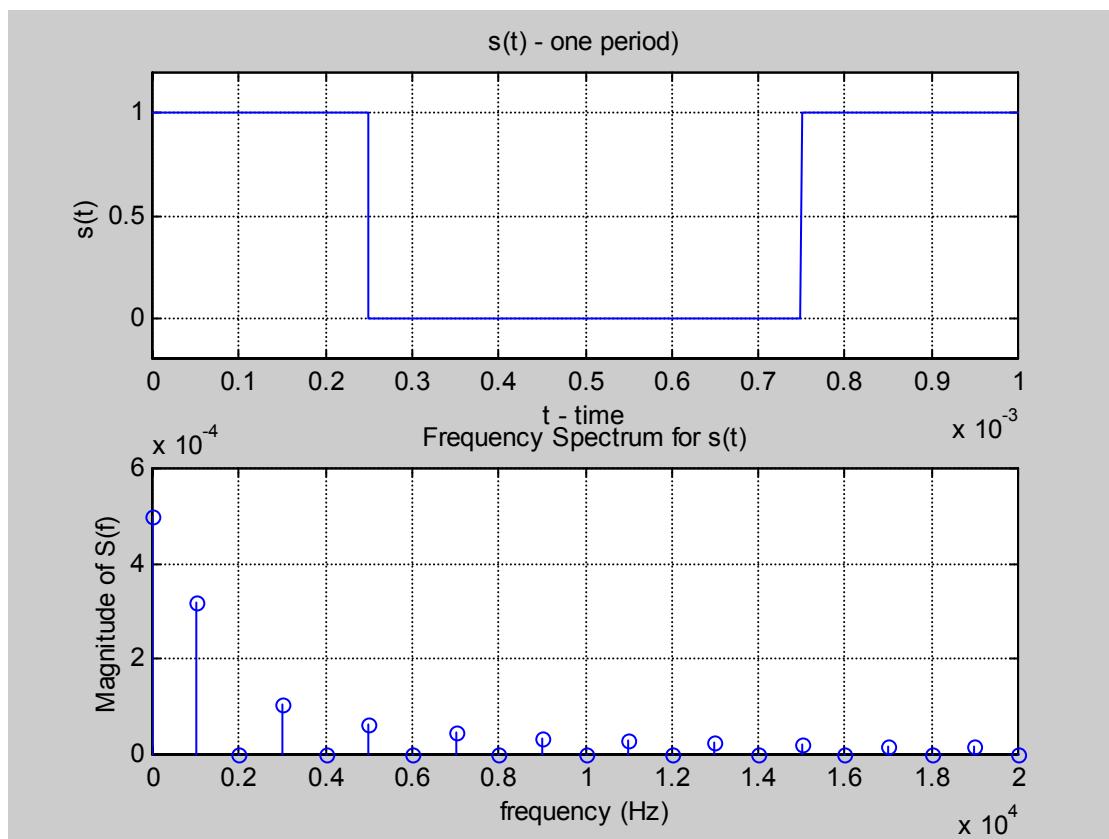
n = 0:1:N-1;
dt = T/N; % step in time domain
t = dt*n; % define the time axis
%
% define the function s(t) - one period is used
s = A*(1/2 + 1/2*square(2*pi*(t+T/4)./T));

%
% fft definition includes averaging over N

S = fft(s)/N;

figure(1);
subplot(2,1,1);
plot(t, s);
title('s(t) - one period');
xlabel('t - time');
ylabel('s(t)');
axis([0 T -0.2 1.2]);
grid
subplot(2,1,2);
f = n./T;
plot(f(1:N/2), abs(T*S(1:N/2)));
title('Frequency Spectrum for s(t)');
xlabel('frequency (Hz)');
ylabel('Magnitude of S(f)');
axis([0 20./T 0 1.2*max(abs(T*S))]);
grid
```

Graph for $s(t)$ and its spectrum:



iii) Passing signal through low pass filters

The following matlab code computes the filter output for any cut-off frequency. The variable "Low_Pass_Filter_BW" specifies the cut-off frequency for the filter. For example for part (1) where BW is equal to 20 KHz, the variable "Low_Pass_Filter_BW" should be set to "20000" Hz and so on.

```
% ProgAssign01_coe02342_Q1_1_iii.m
clear all
%
% N defines the number of samples in the time domain - and the number of
% FFT points
% Since N has to be even, N is made a multiple of 2
M = 10; % The higher the value of M (and consequently N) the better
N = 2^M;% the approximation of the FFT to the Fourier Series expansion

n = 0:1:N-1;
T = 0.001; % period of signal
A = 1; % amplitude of signal

dt = T/N; % step in time domain
t = 0:dt:T;% define the time axis - up to 4 time periods
%
% define the function s(t) - one period is used
s = A*(1/2 + 1/2*square(2*pi*(t+T/4)./T));

%
% Fourier Series expansion is given by:
%
%           infinity
%   s(t) = a_0/2 + sum [ a_k * cos(2*pi*k*f*t) + b_k * sin(2*pi*k*f*t) ]
%   k=1
%

aa_0          = A;
max_k         = 100;
k             = 1:max_k;
aa_k          = 2*A./(pi*k) .* (-1).^(k-1)/2;
aa_k(2:2:max_k) = 0;

bb_k          = zeros(size(k));

f_fundamental = 1/T;
Low_Pass_Filter_BW = 20000; % Hz
h = floor(Low_Pass_Filter_BW/ f_fundamental); % decide number of harmonics
ss = A/2 *ones(size(t));
for i=1:h
    ss = ss + aa_k(i)*cos(2*pi*i*f_fundamental*t) + ...
          bb_k(i)*sin(2*pi*f_fundamental*t);
end
```

COE021-342 - Key solution for Programming Assignment #1

```
% ProgAssign01_coe02342_Q1_1_iii.m - continued

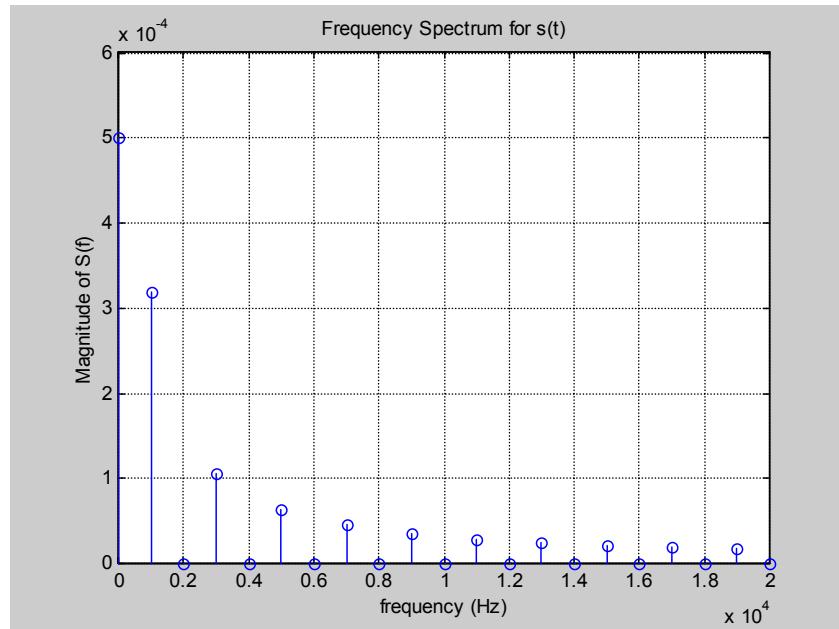
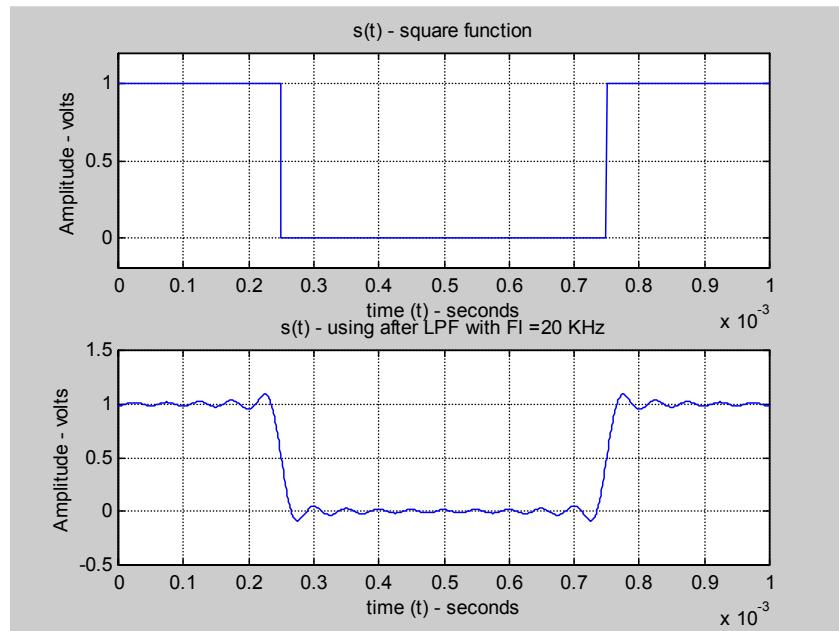
figure(1);
subplot(2,1,1);
plot(t, s);
title('s(t) - square function');
xlabel('time (t) - seconds');
ylabel('Amplitude - volts');
axis([0 T -0.2 1.2]);
grid

subplot(2,1,2);
plot(t, ss);
TitleStr = [ 's(t) - using after LPF with F1 =' num2str(Low_Pass_Filter_BW/1000) ' 
KHz ];
title(TitleStr);
xlabel('time (t) - seconds');
ylabel('Amplitude - volts');
grid

% compute spectrum function
S = fft(ss)/N;

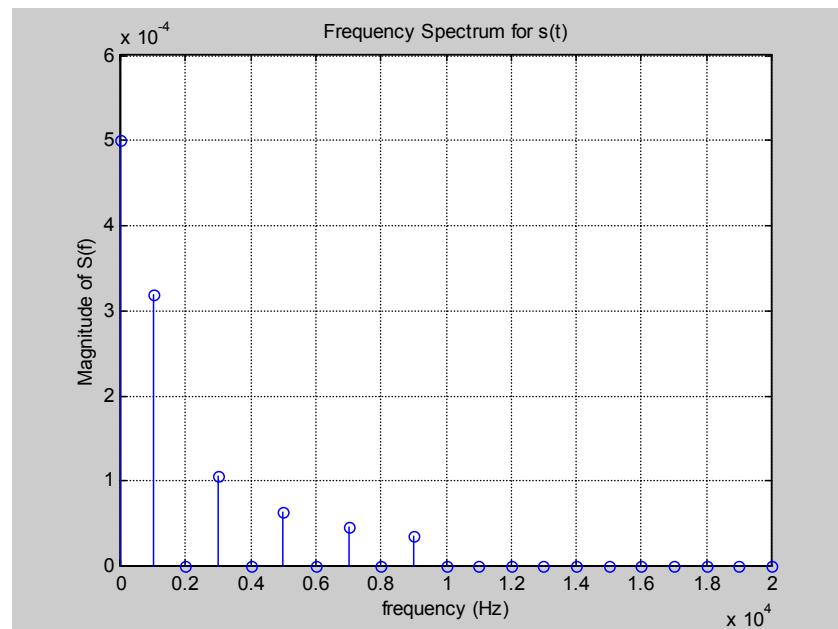
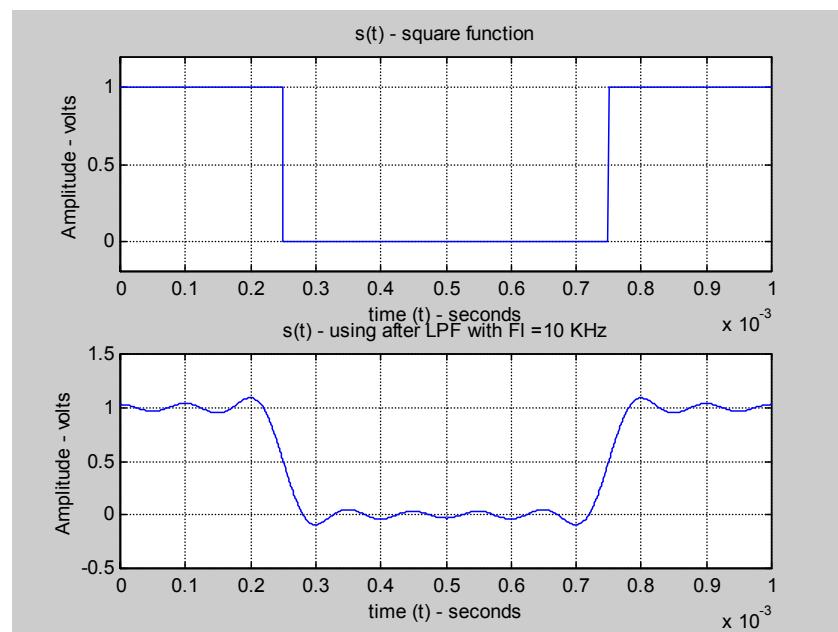
freq = n./T;
figure(2);
stem(freq(1:N/2), abs(T*S(1:N/2)), '');
title('Frequency Spectrum for s(t)');
xlabel('frequency (Hz)');
ylabel('Magnitude of S(f)');
axis([0 20./T 0 1.2*max(abs(T*S))]);
grid
```

1) For BW = 20 KHz:



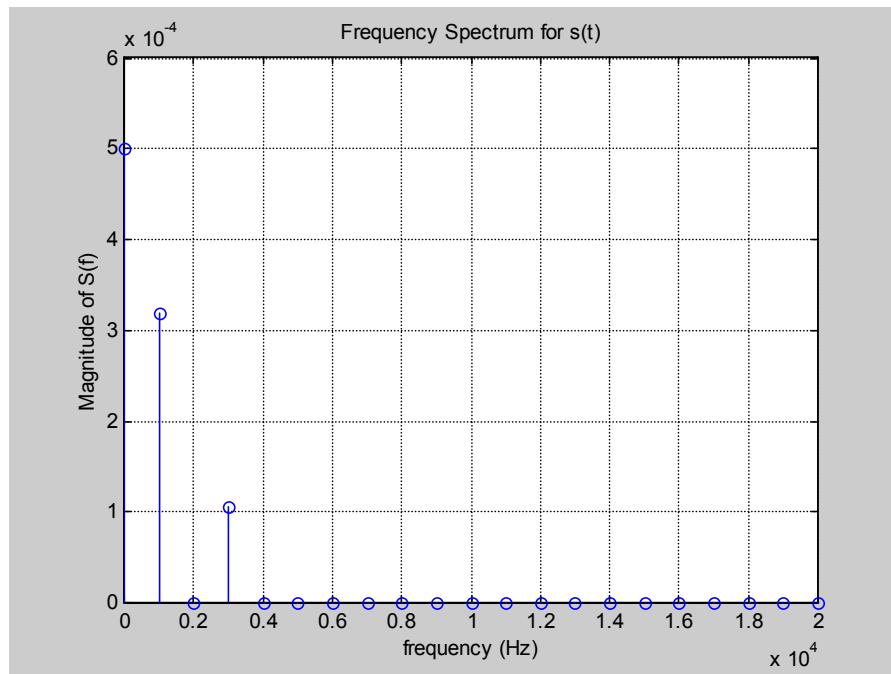
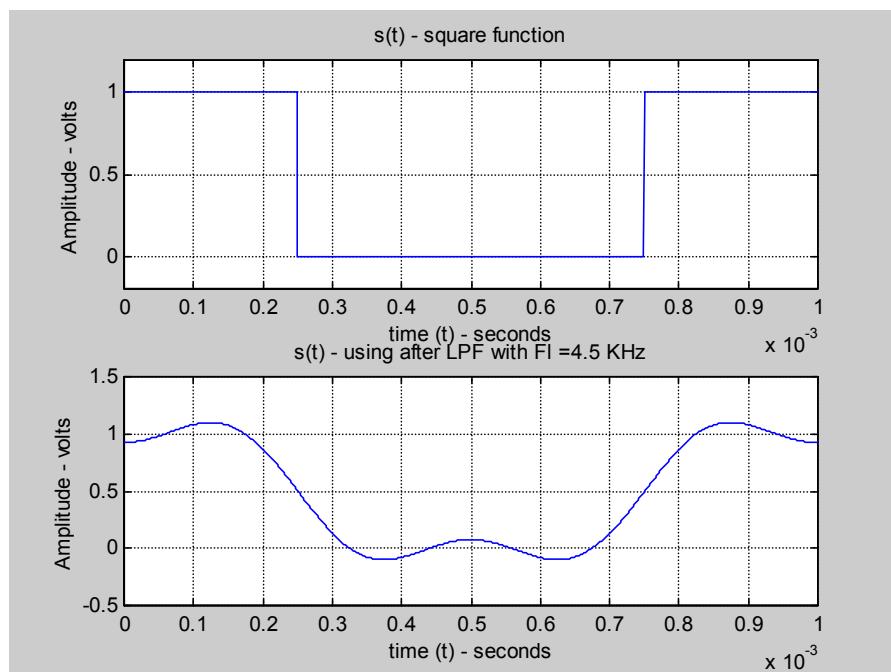
Note the existence of frequency components up to 19 KHz. The filtered signal still very similar to original $s(t)$ since BW = 20 KHz passes most of required components to represent $s(t)$. As BW is decreased, the filter will be rejecting more significant components of $s(t)$ and the filter output will start to differ from the original signal.

2) BW = 10 KHz



Note frequency components now exist up to 9 KHz.

3) BW = 4500 Hz



Note in addition to the DC component, only two harmonics exist in the output signal (at $1 \times f$ and $3 \times f$) only. The filter output is shown above is not very close to original signal $s(t)$.

Q.2. Consider the full-wave rectified cosine signal shown in Figure 3.15. Assume that the period of the signal $T = 1\mu s$ and $A = 1V$.

(i) You are required to calculate the Fourier Series of the signal and determine its frequency components. Limit yourself to the first 6 frequency components.

(ii) If you were required to transmit this signal across a transmission media, what bandwidth would you recommend? Draw the received signal using the recommended bandwidth.

Solution

i) For the full-wave rectified cosine signal, the signal $s(t)$ is given by

$$s(t) = A \left| \cos\left(\frac{2\pi}{T}t\right) \right| = \begin{cases} A \cos\left(\frac{2\pi}{T}t\right) & -T/4 < t < T/4 \\ -A \cos\left(\frac{2\pi}{T}t\right) & T/4 < t < 3T/4 \end{cases}$$

The Fourier Series expansion for $s(t)$ is given by

$$s(t) = \frac{A_0}{2} + \sum_{n=1,2}^{\infty} A_n \cos(2\pi n ft) + B_n \sin(2\pi n ft)$$

where the coefficients are computed as

$$\begin{aligned} A_0 &= \frac{2}{T} \int_{-T/4}^{3T/4} s(t) dt \\ A_n &= \frac{2}{T} \int_{-T/4}^{3T/4} s(t) \cos(2\pi n ft) dt \quad n = 1, 2, \dots \\ B_n &= \frac{2}{T} \int_{-T/4}^{3T/4} s(t) \sin(2\pi n ft) dt \quad n = 1, 2, \dots \end{aligned}$$

* The coefficient A_0 is equal to:

$$A_0 = \frac{2}{T} \int_{-T/4}^{3T/4} s(t) dt = \frac{4A}{T} \times \int_{-T/4}^{T/4} \cos\left(\frac{2\pi}{T}t\right) dt = \frac{8A}{T} \times \int_0^{T/4} \cos\left(\frac{2\pi}{T}t\right) dt = \frac{8A}{T} \times \frac{T}{2\pi} \sin\left(\frac{2\pi}{T}t\right) \Big|_{t=0}^{t=T/4}$$

$$A_0 = \frac{4A}{\pi} \times \left[\sin\left(\frac{\pi}{2}\right) - 0 \right] = \frac{4A}{\pi}$$

* The coefficients A_n ($n=1,2, 3, \dots$) are computed as

$$A_n = \frac{2}{T} \int_{-T/4}^{3T/4} s(t) \cos(2\pi n ft) dt = \frac{2A}{T} \times \left[\int_{-T/4}^{T/4} \cos(2\pi ft) \cos(2\pi n ft) dt - \int_{T/4}^{3T/4} \cos(2\pi ft) \cos(2\pi n ft) dt \right]$$

$$= \frac{-A}{\pi(n^2 - 1)} \left[3 \cos\left(\frac{\pi}{2}n\right) + \cos\left(3\frac{\pi}{2}n\right) \right] = \begin{cases} 0 & n = 3, 5, \dots \\ \frac{4A}{\pi(n^2 - 1)} (-1)^{1+n/2} & n = 2, 4, 6, \dots \end{cases}$$

Note that the above expression is not valid for n equal to 1. For n equal to 1, A_1 is equal to

$$A_1 = \frac{2}{T} \int_{-T/4}^{3T/4} s(t) \cos(2\pi \times 1 \times ft) dt = \frac{2A}{T} \times \left[\int_{-T/4}^{T/4} \cos(2\pi ft) \cos(2\pi ft) dt - \int_{T/4}^{3T/4} \cos(2\pi ft) \cos(2\pi ft) dt \right]$$

$$A_1 = 0$$

since $|\cos(2\pi ft)|^2$ has the same area under $(-T/4, T/4)$ and $(T/4, 3T/4)$.

* The coefficients B_n ($n=1, 2, 3, \dots$) are computed as

$$B_n = \frac{2}{T} \int_{-T/4}^{3T/4} s(t) \cos(2\pi n ft) dt = \frac{2A}{T} \times \left[\int_{-T/4}^{T/4} \cos(2\pi ft) \sin(2\pi n ft) dt - \int_{T/4}^{3T/4} \cos(2\pi ft) \sin(2\pi n ft) dt \right]$$

$$= \frac{-A}{\pi(n^2 - 1)} \left[\sin\left(3\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2}n\right) \right] = 0 \text{ for all } n$$

The final Fourier Series expansion for $s(t)$ is given by

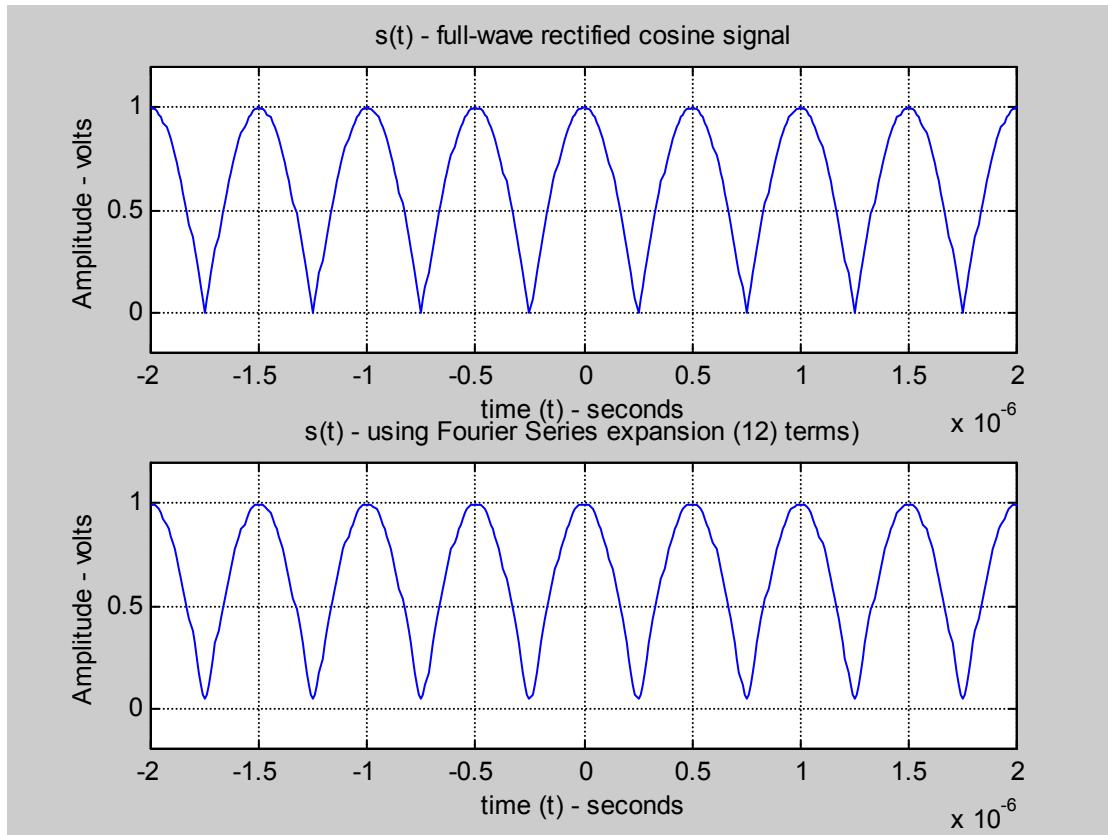
$$s(t) = \frac{4A}{\pi} + \frac{4A}{\pi} \sum_{n=2,4,6}^{\infty} (-1)^{1+n/2} \frac{\cos(2\pi n ft)}{(n^2 - 1)}, \text{ or}$$

For $A = 1$ volts, $T = 1\mu\text{sec}$ (or $f = 10^6$ Hz), $s(t)$ is written as

$$s(t) = \frac{4}{\pi} + \frac{4}{\pi} \sum_{n=2,4,6}^{\infty} (-1)^{1+n/2} \frac{\cos(10^6 \pi n t)}{(n^2 - 1)}$$

Limiting the expression to first 6 components

$$\hat{s}(t) = \frac{4}{\pi} \left[1 + \frac{\cos(2 \times 10^6 \pi t)}{3} + \frac{-\cos(4 \times 10^6 \pi t)}{15} + \frac{\cos(6 \times 10^6 \pi t)}{35} + \frac{-\cos(8 \times 10^6 \pi t)}{63} + \frac{\cos(10 \times 10^6 \pi t)}{99} + \frac{-\cos(12 \times 10^6 \pi t)}{143} \right]$$



The matlab code used to plot the function and its Fourier representation is given below:

```
% ProgAssign01_coe021342_Q2_i
clear all

T = 1.0e-6; % period of signal
A = 1; % amplitude of signal

dt = T/100; % step in time domain
t = -2*T:dt:2*T;% define the time axis - up to 4 time periods
%
% define the function s(t) - one period is used
s = A*abs(cos(2*pi*t./T));

%
% Fourier Series expansion is given by:
%
% infinity
% s(t) = a_0/2 + sum [ a_k * cos(2*pi*k*f*t) + b_k * sin(2*pi*k*f*t) ]
% k=1
%

aa_0          = 4*A/pi;
NoOfTerms      = 12;
k              = 1:NoOfTerms;
aa_k          = 4*A./(pi*(k.*k-1)) .* (-1).^(1+k./2);
aa_k(1:2>NoOfTerms) = 0; % odd aa_k coefficients are zeros

bb_k          = zeros(size(k));

f  = 1/T;
ss = aa_0/2 *ones(size(t));

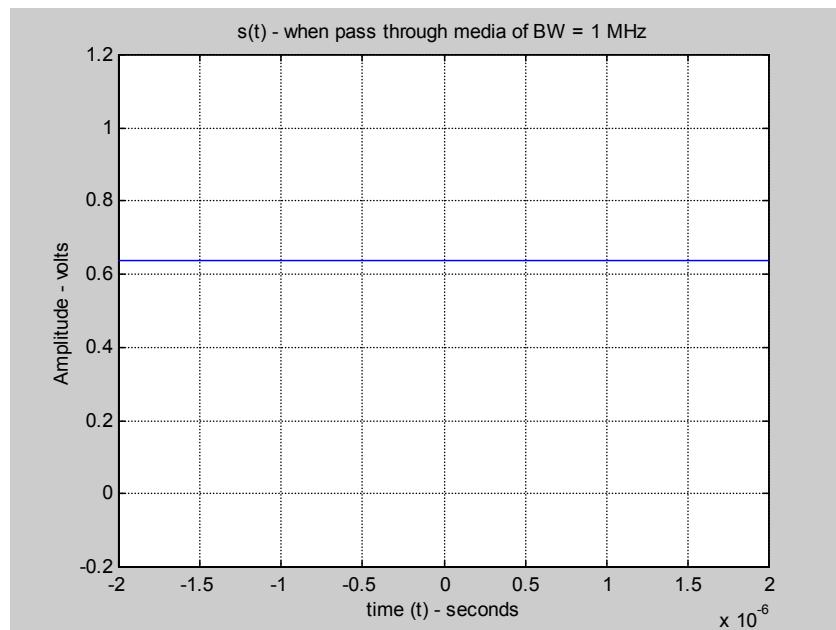
for i=1:NoOfTerms
    ss = ss + aa_k(i)*cos(2*pi*i*f*t) + bb_k(i)*sin(2*pi*f*t);
end

figure(1);
subplot(2,1,1);
plot(t, s);
title('s(t) - full-wave rectified cosine signal');
xlabel('time (t) - seconds');
ylabel('Amplitude - volts');
axis([-2*T 2*T -0.2 1.2]);
grid

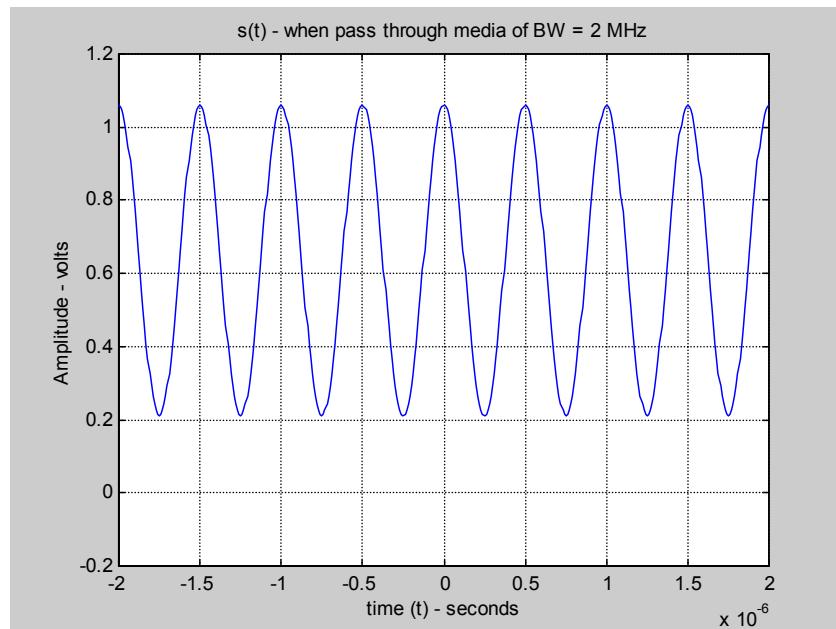
subplot(2,1,2);
plot(t, ss);
TitleStr = ['s(t) - using Fourier Series expansion (' num2str(NoOfTerms) ' terms)'];
title(TitleStr);
xlabel('time (t) - seconds');
ylabel('Amplitude - volts');
axis([-2*T 2*T -0.2 1.2]);
grid
```

ii) Choosing a suitable transmission media

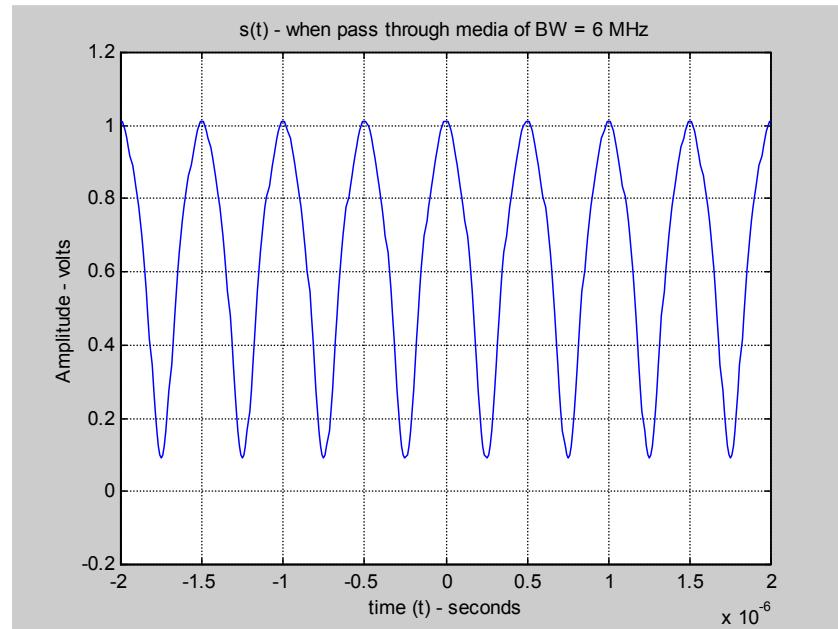
When media bandwidth is small (1 MHz) for example, only the DC component is propagated - higher harmonics are attenuated or rejected. The output signal is as shown below for BW = 1 MHz.



When media bandwidth is increased to 2 MHz; the DC component and the first harmonic at $2Xf$ is propagated - the output signal now looks similar to the input signal $s(t)$ but not yet close (note the lowest amplitude is about 0.2 volts instead of 0)



For a media bandwidth of 6 MHz, the output signal is now more similar to original signal $s(t)$.



One can show that the propagated signal has about 99% of the original power of $s(t)$ when a bandwidth of 4 MHz - i.e. first two harmonics at $2Xf$ and $4Xf$ are passed. This percentage increases to 99.9% when the media allows the first 6 harmonics to pass: $2Xf$, $4Xf$, $6Xf$, $8Xf$, $10Xf$, and $12Xf$ - i.e. when the media bandwidth is equal to 12 MHz.

For part ii) of Q2 a bandwidth higher than 4 MHz seems to be suitable.

Q.3. Given the triangular pulse shown in Figure 3.16, find its Fourier transform and then plot it using Matlab.

Solution:

The triangular pulse $s(t)$ can be written as

$$s(t) = \begin{cases} A\left(\frac{t}{\tau} + 1\right) & -\tau < t < 0 \\ A\left(\frac{-t}{\tau} + 1\right) & 0 < t < \tau \end{cases}$$

The Fourier transform for $s(t)$, $S(f)$, is given by

$$S(f) = \int_{-\infty}^{+\infty} s(t) \exp(-j2\pi f t) dt$$

Therefore $S(f)$ is computed as

$$S(f) = \int_{-\infty}^{+\infty} s(t) \exp(-j2\pi f t) dt = \int_{-\tau}^0 A\left(\frac{t}{\tau} + 1\right) \exp(-j2\pi f t) dt + \int_0^\tau A\left(\frac{-t}{\tau} + 1\right) \exp(-j2\pi f t) dt$$

$$\text{Let } I_1 = \int_{-\tau}^0 A\left(\frac{t}{\tau} + 1\right) \exp(-j2\pi f t) dt \text{ while } I_2 = \int_0^\tau A\left(\frac{-t}{\tau} + 1\right) \exp(-j2\pi f t) dt$$

$$I_1 = \int_{-\tau}^0 A\left(\frac{t}{\tau} + 1\right) \exp(-j2\pi f t) dt = \int_{-\tau}^0 \frac{A}{\tau} t \exp(-j2\pi f t) dt + \int_{-\tau}^0 A \exp(-j2\pi f t) dt$$

$$\begin{aligned} &= \frac{A}{\tau} \left[\frac{t}{(-j2\pi f)} \exp(-j2\pi f t) - \frac{1}{(-j2\pi f)^2} \exp(-j2\pi f t) \right] \Big|_{t=-\tau}^{t=0} + \left[\frac{A}{(-j2\pi f)} \exp(-j2\pi f t) \right] \Big|_{t=-\tau}^{t=0} \\ &= \frac{A}{\tau} \left[\left(0 - \frac{1}{(-j2\pi f)^2} \right) - \left(\frac{-\tau}{(-j2\pi f)} \exp(j2\pi f \tau) - \frac{1}{(-j2\pi f)^2} \exp(j2\pi f \tau) \right) \right] + \left[\frac{A}{(-j2\pi f)} (1 - \exp(j2\pi f \tau)) \right] \\ &= \frac{jA}{(2\pi f)} + \frac{A/\tau}{(2\pi f)^2} [1 - \exp(j2\pi f \tau)] \end{aligned}$$

Similarly, the quantity I_2 is equal to

$$I_2 = \int_0^\tau A\left(\frac{-t}{\tau} + 1\right) \exp(-j2\pi f t) dt = \int_0^\tau \frac{-A}{\tau} t \exp(-j2\pi f t) dt + \int_0^\tau A \exp(-j2\pi f t) dt$$

$$\begin{aligned}
 &= \frac{-A}{\tau} \left[\frac{t}{(-j2\pi f)} \exp(-j2\pi f t) - \frac{1}{(-j2\pi f)^2} \exp(-j2\pi f t) \right]_{t=0}^{t=\tau} + \left[\frac{A}{(-j2\pi f)} \exp(-j2\pi f t) \right]_{t=0}^{t=\tau} \\
 &= \frac{-A}{\tau} \left[\left(\frac{\tau}{(-j2\pi f)} \exp(-j2\pi f \tau) - \frac{1}{(-j2\pi f)^2} \exp(-j2\pi f \tau) \right) - \left(0 - \frac{1}{(-j2\pi f)^2} \right) \right] \\
 &\quad + \left[\frac{A}{(-j2\pi f)} (\exp(-j2\pi f \tau) - 1) \right] \\
 &= \frac{-jA}{(2\pi f)} + \frac{A/\tau}{(2\pi f)^2} [1 - \exp(-j2\pi f \tau)]
 \end{aligned}$$

Adding the two integrals

$$\begin{aligned}
 S(f) = I_1 + I_2 &= \frac{A/\tau}{(2\pi f)^2} [1 - \exp(j2\pi f \tau)] + \frac{A/\tau}{(2\pi f)^2} [1 - \exp(-j2\pi f \tau)] \\
 &= \frac{2A/\tau}{(2\pi f)^2} \left[1 - \frac{\exp(j2\pi f \tau) + \exp(-j2\pi f \tau)}{2} \right] = \frac{A}{(2\pi^2 f^2 \tau)} [1 - \cos(2\pi f \tau)] \\
 &= \frac{A}{(\pi^2 f^2 \tau)} \sin^2(\pi f \tau) = A \tau \left[\frac{\sin(\pi f \tau)}{\pi f \tau} \right]^2
 \end{aligned}$$

The following code plots the pulse $s(t)$ and its frequency spectrum function $S(f)$. The frequency spectrum function $S(f)$ is graphed using the analytic equation obtained above and also using the approximation obtained through the use of the FFT routine. The graphs are generated for $\tau = 1$ second, and $A = 1$ volt.

COE021-342 - Key solution for Programming Assignment #1

```
% ProgAssig01_coe021coe342_Q3_i
clear all
%
% N defines the number of samples in the time domain - and the number of
% FFT points
% Since N has to be even, N is made a multiple of 2
M = 10; % The higher the value of M (and consequently N) the better
N = 2^M;% the approximation of the FFT to the Fourier Series expansion

%
% Specification for the triangular pulse
Taw = 1; % Signal is defined on (-Taw, Taw)
A = 1; % amplitude of signal
n = 0:1:N-1;
dt = Taw/N; % step in time domain

tneg = -Taw: dt: 0;
tpos = dt:dt:Taw;
t = [tneg tpos]; % define the time axis
%
% define the function s(t) - one period is used
s = A*(-abs(t)/Taw + 1);

%
% fft definition includes averaging over N

freq = 0:0.01:20;
S1 = A*Taw*(sin(pi*freq*Taw)./(pi*freq*Taw)).^2;

S2 = fft(s)/N;

figure(1);
plot(t, s);
title('s(t) - pulse');
xlabel('t - time');
ylabel('s(t)');
axis([-Taw Taw -0.2 1.2]);
grid

figure(2);
subplot(2,1,1);
f = n.^(2*Taw); % 2Taw is the width of the pulse in time-domain
plot(f(1:N/2), abs(Taw*S2(1:N/2)));
title('Frequency Spectrum for s(t) - using FFT');
xlabel('frequency (Hz)');
ylabel('Magnitude of S(f)');
axis([0 10./Taw 0 1.2*max(abs(Taw*S2))]);
grid

subplot(2,1,2);
plot(freq, S1);
title('Frequency Spectrum for s(t) - using Fourier Transform');
xlabel('frequency (Hz)');
ylabel('Magnitude of S(f)');
axis([0 10./Taw 0 1.2*max(abs(S1))]);
grid
```

