Downlink Traffic Power Characterization for Multi-Rate Wireless CDMA Data Networks

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Abstract— **The characterization of downlink traffic power is an important issue for the design of efficient call admission control (CAC) and radio resource management (RRM) procedures. In this paper an analytic model is presented to solve the power-rate assignment problem and to approximate the probability distribution function for the cell site downlink traffic power for a multi-rate CDMA based network. The model accounts for both the radio frequency propagation model and the large signal variations represented by the signal shadowing process. As one application, the model is utilized in calculating the probability of cell site power outage as a function of the system rates and the number of supported bursts. The study compares between model based results and results obtained using conventional Monte-Carlo simulations. Comparisons indicate the model provides an adequate approximation for outage probability figure.**

Keywords- code division multiple access; resource management; scheduling; multi-rate services.

I. INTRODUCTION

CDMA being one of the technologies to be widely deployed as the air interference for next generation networks [1] should support heterogeneous services with variable quality of service (QoS) requirements. Therefore, efficient call admission control (CAC) and resource management (RRM) procedures are critical functions for the optimal performance of these networks. Unlike TDMA/FDMA air interfaces, CDMA does not have a hard capacity limit and therefore not directly amenable to Markovian or queueing analyses. Focusing on multi-rate CDMA services, CAC and RRM studies similar to those in [2][3][4][5] mainly exploited simulation techniques to analyze the performance of the network, whereas those in [6][7] considered pure analytic methods while ignoring radio frequency propagation model (i.e. the power-distance relationship and shadowing) and the soft capacity issue for CDMA.

For single service/rate systems, the study in [8] approximates the interference using a Gaussian distribution and hence an Erlang capacity calculation for the system is possible. The approach is further refined in [9] where the interference is approximated by an equivalent number of voice calls which was found to be closely approximated by a gamma random variable. In these studies, only a single and relatively low service rate is assumed and consequently the number of customers that can be supported is large. This is the reason interference modeling using Gaussian and/or gamma distributions approximations yields acceptable results.

In this study we focus on a multi-rate CMDA network and develop an analytic framework to enable CAC and RRM analyses with the consideration of the radio propagation model and the soft capacity issue for the downlink in the network. The framework is based on approximating the distribution of the cell site downlink traffic power as a function of the supported bursts in terms of their rates and locations of subscribers. As one application of the developed model, we demonstrate the calculation of the power outage probability for the cell site.

The rest of paper is organized as follows. In section II the basic burst power assignment framework is detailed. The proposed approximation and characterization of the cell site downlink traffic power are outlined in section III. In section IV, numerical results obtained through analysis and simulation are discussed and compared. Finally, concluding remarks are made at the end of this paper.

II. BURST POWER ASSIGNMENT FRAMEWORK

Assume a general cellular configuration employing CDMA with arbitrary reuse factor where service areas are numbered from 0 to 18. Following the conventional layout cell 0, the cell of interest, is assumed to be surrounded by two tiers of a total of 18 co-channel interferers. Assume the network supports K arbitrary discrete service rates given by the set $V=\{R_0, R_1, \ldots, R_{K-1}\}$. Using the CDMA access scheme and focusing on downlink quality, power transmitted from the cell of interest as well as power transmitted from the interfering cells affect the quality of ongoing transmission links. Assume the total cell site downlink transmit power is given by P_T Watts and that a fraction $0<\beta<1$ is allocated for overhead channels. That is, the total power for overhead channels, Pov, is equal to βP_T Watts while the maximum traffic power is given by $(1-\beta)P_T$. We further assume that transmissions on the downlink are not perfectly orthogonal with the severity of the intracell interference determined by the orthogonality factor $0<\rho<1$. When the cell site of interest is serving N simultaneous bursts, with the i^{th} burst being at rate $r_i \in V$ for $i = 0, 1, \ldots, N-1$, then the implemented radio resource management procedure must allocate enough average forward link power, P_i, to support an optimal link quality. This is typically accounted for by forcing the corresponding energy-per bit to noise power density, $(E_b/N_0)_i$, to be greater than a given minimum E_b/N_0

figure. Using, the above definitions, the i^{th} link quality figure is given by

$$\left(\frac{E_b}{N_0}\right)_i = \frac{(W/r_i) P_i L_{i0} 10^{\zeta_{i0}/10}}{(1-\rho) L_{i0} 10^{\zeta_{i0}/10}\left[\sum_{l=0,l\neq i}^{N-1} P_l + Pov\right] + P_T \sum_{k=1}^{18} L_{ik} 10^{\zeta_{ik}/10}} \qquad (1)$$

where W is the system bandwidth, L_{ik} and ζ_{ik} are the path loss coefficient and the shadowing factor, respectively, between the i^{th} user in the cell of interest, and the k^{th} cell site (k=0, 1, 2, …,18). In this study we assume a path loss model where the received power is inversely proportional to distance raised to the path loss exponent, denoted by γ. The standard deviation of the shadowing factor ζ_{ik} is equal to σ_{dB}. It can be seen that the model in (1) accounts for both intracell interference and intercell interference. Furthermore, (1) assumes conservatively that all co-channel cell sites are transmitting at their maximum power level. In (1) only large signal variations are considered where fast fading is assumed to be compensated for by multiple soft handoff legs, if supported, and fast power control. The relation in (1) serves as the core relation to determine whether a particular assignment of service bit rates is feasible or whether an incoming burst request can be supported given the ongoing bursts and their current rates. A feasible solution for (1) is a solution where $P_i \geq 0$ for i=0, 1, …, N-1 and the sum of all bursts power $\sum_{i=0}^{N-1} P_i$ is less or equal to the maximum possible traffic power $(1-\beta)P_T$ for the given bit rate assignments.

The quantity $\sum_{i=0}^{N-1} P_i$ is a function of not only the number of bursts, the corresponding service bit rates, but also the mobile locations and the RF model. In this paper, we use (1) to derive a probabilistic model to characterize the sum of downlink traffic power to enable RRM studies accounting for the RF model and the soft capacity issue for CDMA multi-rate system. We further utilize this model to compute the probability of cell site power outage as a function of the number of supported bursts and the assumed bit rate assignments.

To solve for a feasible solution for a given bit rate assignment, if any, we can utilize (1) to set up a linear system of equations to solve for P_i, i=0, 1, …, N-1. Alternatively, the relation in (1) can be rewritten as

$$\left(\frac{E_b}{N_0}\right)_i = \frac{W}{r_i} \times \frac{P_i}{(1-\rho)\left[\sum_{l=0,l\neq i}^{N-1} P_l + Pov\right] + f_i P_T} \qquad (2)$$

where f_i is given by

$$f_i = \sum_{k=1}^{18}\left(L_{ik} 10^{\zeta_{ik}/10}\right) \Big/ \left(L_{i0} 10^{\zeta_{i0}/10}\right) \qquad (3)$$

for i=0,1, …, N-1. The sum of traffic power is then given by

$$\sum_{i=0}^{N-1} P_i = P_T \frac{\beta \sum_{i=0}^{N-1} G_i + (1-\rho)^{-1} \sum_{i=0}^{N-1} G_i f_i}{1 - \sum_{i=0}^{N-1} G_i} \qquad (4)$$

where $G_i = g_i/(1+g_i)$ and $g_i = (E_b/N_0)_i / (W/r_i)(1-\rho)$. The burst power assignment should be given by

$$P_i = \frac{P_T \times G_i}{(1-\rho)}\left[\frac{\sum_{l=0}^{N-1} G_l f_l + \beta(1-\rho)}{1 - \sum_{l=0}^{N-1} G_l} + f_i\right] \qquad (5)$$

The feasibility of the solution P_i is guaranteed if the following condition holds:

$$\sum_{i=0}^{N-1} G_i \left(1 + \frac{f_i}{1-\rho}\right) \leq (1-\beta) \qquad (6)$$

To characterize the sum of traffic power specified by (4) it is required to characterize first the random variables f_i and in turn the variable $\sum_{i=0}^{N-1} G_i f_i$. It is clear that terms other than $\sum_{i=0}^{N-1} G_i f_i$ are constants and depend only on the network parameters.

III. Sum Of Traffic Power Characterization

The parameter f_i defined in (3) is a function only of the location of the subscriber, the path loss model, and the shadowing process. Note that f_i does not depend on the network parameters. Let the estimated mean and variance of log(f_i) be equal to m_f and σ_f^2, respectively. The mean and variance of variable f_i are a function of the path loss exponent γ and shadowing process standard deviation σ_{dB} defined in previous section. Performing Monte-Carlo simulations and studying the f_i parameter, it is observed that the random variable f_i is nearly lognormal for a wide and applicable range of γ and σ_{dB}. Therefore, in this paper we approximate the distribution of the parameter f_i by a lognormal distribution function as

$$f_{f_i}(x) = \frac{1}{\sqrt{2\pi}\sigma_f x} \exp\left[\frac{-(\log x - m_f)^2}{2\sigma_f^2}\right] \qquad (7)$$

whose parameters m_f and σ_f^2 are the empirical mean and variance of the variable f_i, respectively. Fig. 1 depicts the cumulative distribution function (CDF) for the random variable f_i and the approximation given by (7) for $\sigma_{dB} = 12$ dB.

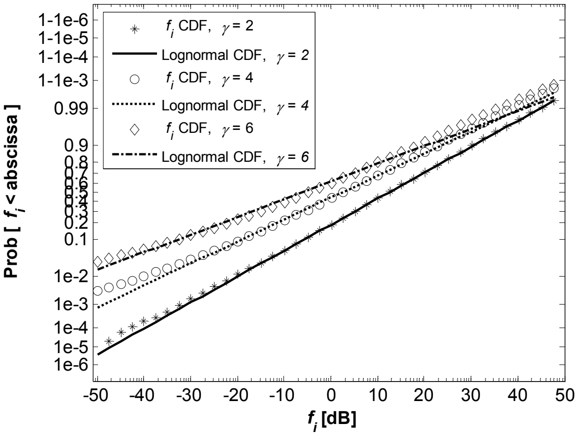

Figure 1. The CDF of the parameter f_i for $\sigma_{\mathrm{dB}} = 12$ dB compared with lognormal CDF plotted on normal probability paper.

As alluded to earlier, the parameters m_f and σ_f^2 are function of the environment and the RF propagation model only. Therefore, they can be made available for a wide range of environments and RF models. For example, Table 1 provides the empirical mean and standard deviations for f_i for a conventional hexagonal cellular service area where the cell of interest is surrounded by 18 co-channel cells. The assumed RF propagation model is the simple path loss exponent model where the signal attenuation is inversely proportional to the distance raised to the exponent γ .

TABLE I. MEAN AND STANDARD DEVIATION FOR THE VARIABLE f_i FOR DIFFERENT RF AND SHADOWING PARAMETERS.

	Mean of log(f_i), m_f			**Standard deviation of log(f_i), σ_f**		
σ_{dB}	$\gamma = 2$	$\gamma = 4$	$\gamma = 6$	$\gamma = 2$	$\gamma = 4$	$\gamma = 6$
6 dB	1.034	-0.988	-2.596	1.832	2.755	3.973
8 dB	1.565	-0.553	-2.267	2.257	3.066	4.173
12 dB	2.857	0.577	-1.386	3.222	3.813	4.712

To compute the probability distribution function (PDF) for the cell site downlink traffic power $\sum_{i=0}^{N-1} P_i$, the relation in (4) is utilized. Let the random variable $\sum_{i=0}^{N-1} P_i$ be denoted by Z while the random variable $\sum_{i=0}^{N-1} G_i f_i$ be denoted by Y. It is clear that Z is a linear transformation of the random variable Y. That is, $Z = aY + b$, where the constants a and b are given by $P_T \Big/ \left[\left(1\text{-}\sum_{i=0}^{N-1} G_i\right)(1\text{-}\rho)\right]$ and $P_T \beta \sum_{i=0}^{N-1} G_i \Big/ (1\text{-}\sum_{i=0}^{N-1} G_i)$, respectively. Therefore, the desired PDF for Z can be written as

$$f_Z(z) = f_Y\left(\frac{z-b}{a}\right) \tag{8}$$

where $f_Y(y)$ is the PDF for the variable Y.

Examining the random variable Y, it is a sum of N variables each of the form $G_i f_i$. The problem of characterizing the sum of lognormal variables is well studied in the literature. The references [10][11] are examples of two recent approaches for this problem. However, for our purposes, the conventional approach of utilizing the characteristic function of the random variable f_i is employed. To calculate the characteristic function of f_i, we use the efficient method developed in [12]. Finally, the required characteristic function for Y is given by

$$\Theta_Y(\omega) = \prod_{i=0}^{N-1} \Theta_{G_i f_i}(\omega) = \coprod_{i=0}^{N-1} \Theta_f(G_i \omega) \tag{9}$$

where the characteristic function of f_i, $\Theta_f(\omega)$, need to be evaluated only once.

IV. NUMERICAL EXAMPLE

For results in this section we assume a cdma2000 (1xRTT) network [12] where the supported system rates are $V = \left\{ R_j = 2^j R_0, j = 0, 1, \cdots, K-1 \right\}$ where $R_0 = 9.6$ kb/s and $K = 5$. Furthermore, let the cell site maximum transmit power, P_T be equal to 24 Watts, and β is equal to 0.2. The rest of the physical layer parameters ρ and E_b/N_0 are 0.1 and 12 dB, respectively.

We assume a group of users specified by the vector $\overline{n} = (n_0, n_1, \cdots, n_{K-1})$ are positioned randomly in cell of interest for every iteration. n_j is the number of users assigned with the system rate R_j for j=0,1,…, K-1. It should be noted that for a particular $\overline{n}$ to be feasible, the constraint in (6) should be satisfied at least with the f_i parameter set to zero. The power requirement to support $\overline{n}$ is computed using (5) and a power outage event occurs if the computed downlink traffic power is greater than the maximum downlink traffic power $(1-\beta)P_T$ as defined by [6]. For a particular assumed combination of users, we vary the number of users assigned the i^{th} system rate R_i from 0 to $n_{\max,i}$, where $n_{\max,i}$ is specified again by the constraint outlined in (6) when all other burst rates are set to zero, i.e. $R_i \neq 0$ and $R_j = 0$ for j=0,1, …, K-1 and $j \neq i$.

The results obtained through Monte-Carlo simulations are shown in Fig. 2 for four different scenarios: (X,0,0,0,0), (3,0,X,1,0), (X,1,1,0,0), and (X,2,0,0,0) where X is the range of possible number of users for the corresponding bit rate assignment. We also plot the evaluation of the outage probability using the approximate distribution derived in (8). It can be noted that the model closely approximates the statistical outage probability or in other words, the distribution of the downlink traffic power for this multi-rate network is closely approximated by the model developed in this paper.

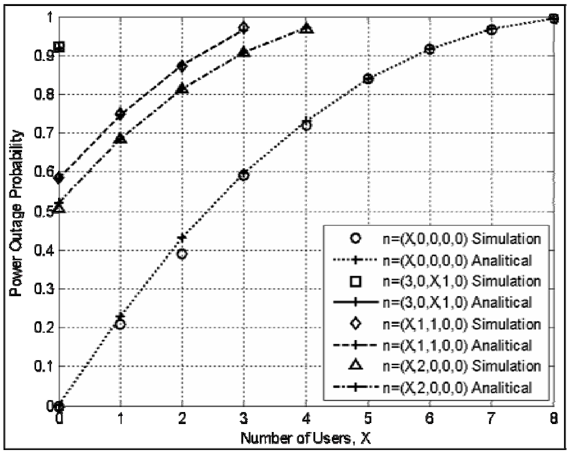

Figure 2. Cell site power outage probability as a function of the number of ongoing bursts (K=5, γ = 4, and σ_{dB}=12 dB)..

V. CONCLUSIONS

This paper presents a method for characterizing the sum of downlink traffic power for supporting data bursts in a multi-rate CDMA system. The method relies on approximating the empirical distribution of the parameter f_i by a lognormal random variable. The developed model accounts for both the path loss exponent and the shadowing process, and it allows the approximation of the probability distribution of the traffic power for a particular bit rate assignment for the ongoing bursts. As an example application of this model, the paper demonstrates the calculation of the cell site power outage. This model should also facilitate the utilization of Markov-based analyses for multi-rate CDMA systems when the downlink traffic power (or number of ongoing bursts) is used a system state variable.

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