Approximation Techniques For Analytical Characterization Of Downlink Traffic Power For Multi-Service CDMA Networks

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***Abstract*- This paper presents a novel analytical framework for characterizing the downlink power allocations for a multi-rate Code Division Multiple Access (CDMA) network. The characterization takes into account the RF propagation model, the shadowing process and considers both intracell and intercell interference. To arrive at the model, the paper utilizes approximation techniques recently developed in the recent literature to evaluate the sum of lognormal random variables. The overall traffic power is modeled as a random variable that is function of the network parameters and the assigned downlink bit rates. The paper demonstrates the calculation of the forward link power outage probability as one example application for the developed characterization. It also compares between two approximation techniques suggested in this paper: matched first and second order statistics, and the min-max technique. The analytical results are compared to those obtained via Monte-Carlo simulations to assess the accuracy of the adapted approximations. The results indicate the min-max technique produces more accurate results when compared to matched first and second moments approximation scheme.**

I. INTRODUCTION

For wireless communication systems, radio resource management (RRM) is a critical function to the overall performance of the system. This is particularly true for CDMA systems as these systems are limited by interference and require extra care in managing their soft capacity [1]. With the proliferation of wireless internet and the deployment of high-speed 3G system, the optimization of RRM procedures is now even more critical specifically for the downlink [2]. Data burst admission control is at the core of these essential RRM functions. The downlink power allocation, the transmission bit rate assignment, and the coverage are three main aspects that need to be optimized for an efficient data burst control procedure. For example, the studies of [3] and [4] are two samples of numerous recent work in order to analyze and evaluate the performance of CDMA systems in terms of forward link power allocations and coverage. The examples in [5] and [6] and the references therein tackle the transmission bit rate assignment in an attempt to optimize the downlink data throughput.

An important parameter for burst admission control is the total downlink traffic power the system is utilizing in supporting current bursts. An example of this utilization can be found in [7]. This power, in addition of being a function of RRM algorithm, it is also a function of the radio frequency (RF) propagation and the large signal variations represented by the shadowing process. For the same burst rate, mobiles at different locations may require significantly different levels of downlink transmit power from the cell site. Therefore it is important to characterize the total traffic power allocated by the cell site to support data burst and while accounting for the effects of the RF propagation model and the shadowing process.

The purpose of this paper is to develop a novel framework for modeling the cell site power in relation to the burst transmission rate assignment problem for a general multi-rate CDMA system. In this study we propose and assess two approximation methods for modeling the downlink traffic power. The paper calculates the power outage probability as one example application of the developed framework. The developed framework provides a semi-analytic solution taking into account the soft capacity issue, radio propagation model and all the necessary physical parameters such as the shadowing factors and the path loss coefficients.

The rest of this paper is organized as follows. In section II, the overall system configuration is outlined and the primary relations that specify the downlink traffic power are derived. Section III provides the detailed analysis of the sum of traffic power modeling and the required approximations. Numerical illustrative examples of the proposed approach are depicted and discussed in section VI. Finally, the paper provides conclusion and future direction statements.

II. SYSTEM MODEL

Consider a cellular CDMA wireless system with an arbitrary frequency reuse factor. Let the cell of interest, denoted by cell 0, be surrounded by tiers of co-channel cells, numbered 1 to M, where M is the number of co-channel cells. Assume the network supports an arbitrary set of k data rates $V = \{R_0, R_1, ..., R_{k-1}\}$. Here we assume the system executes a burst admission control procedure to decide whether to admit the requested burst or not and what service bit rate is to be used. The design of the burst admission control procedure is not the subject of this paper. While the developed framework can be utilized in designing efficient burst admission control procedures, at this stage we focus on the accuracy of the developed model in terms of the required approximations and calculations.

Assume the cell site is currently supporting N ongoing downlink simultaneous data bursts where the i^{th} burst is admitted at rate $r_i \in V$ and is allocated a forward link power equal P_i Watts. The link quality for the i^{th} burst (i=0, 1, 2, …, N) is represented by the bit energy to noise power density, $(E_b/N_0)_i$ as

$$\left(\frac{E_b}{N_0}\right)_i = \frac{(W/r_i)P_iL_{i0}10^{\zeta_{i0}/10}}{(1-\rho)L_{i0}10^{\zeta_{i0}/10}\left[\sum_{j\neq i}^{N-1}P_j + P_{ov}\right] + P_T\sum_{\forall k}L_{ik}10^{\zeta_{ik}/10}} \qquad (1)$$

where W is the system bandwidth while L_{ik} and ζ_{ik} are the path loss coefficient and the shadowing factor, respectively, between the i^{th} user in the cell of interest, and the k^{th} co-channel cell site. We assume the total cell site power is equal to P_T Watts with a maximum fraction of β that can be allocated to all traffic transmissions. P_{ov} is the power in overhead transmissions and is given by $(1-\beta)P_T$. The orthogonality factor $0<\rho<1$ determines the severity of the intracell interference, the first terms in the dominator of (1). The above relation assumes conservatively that every co-channel cell site is transmitting at maximum power, P_T.

The path loss model considered in this study assumes the received signal power is inversely proportional to the distance between the subscriber and the base station raised to the path loss exponent, α. We assume the shadowing factor to be a Gaussian random variable with zero mean and standard deviation σ_{dB}, that is $\zeta_{ik} \sim N(0, \sigma_{dB}^2)$.

The burst admission control procedure assigns a power allocation P_i for the i^{th} burst such that the received $\left(E_b/N_0\right)_i$ is greater or equal to a minimum signal quality threshold given by $\left(E_b/N_0\right)_{\min}$. The other power allocation constraint that is relevant to this study is that the sum of all power allocations should be at most $(1-\beta)P_T$. The relation in (1) can be rewritten as

$$\left(\frac{E_b}{N_0}\right)_i = \frac{W}{r_i} \times \frac{P_i}{(1-\rho)[\sum_{j=0,j\neq i}^{N-1}P_j + P_{ov}] + P_T \times f_i} \qquad (2)$$

where the parameter f_i is given by

$$f_i = \sum_{\forall k}(L_{ik}10^{\zeta_{ik}/10}) / \left(L_{i0}10^{\zeta_{i0}/10}\right) \qquad (3)$$

It can be shown through rearrangement of terms in (1) that the sum of power allocated for N bursts in the system is equal to

$$\sum_{i=0}^{N-1}P_i = P_T \times \frac{\beta\sum_{i-0}^{N-1}G_i + \frac{1}{1-\rho}\sum_{i-0}^{N-1}G_if_i}{1-\sum_{i-0}^{N-1}G_i} \qquad (4)$$

where $G_i = g_i/(1+g_i)$ and $g_i = \dfrac{(E_b/N_0)_{\min}}{(W/r_i)(1-\rho)}$.

Furthermore, we can also show that the power allocations are feasible only if the following inequality

$$\sum_{i=0}^{N-1}G_i\left(1+\frac{f_i}{1-\rho}\right) \leq (1-\beta) \qquad (5)$$

is satisfied.

The relation in (4) provides the required cell site transmission power as a function of the network parameters and the RF propagation model. The sum of traffic powers plays a crucial role in the development of call admission control procedures and in the following section we develop a model for the probability distribution function of $\sum_{i=0}^{N-1}P_i$

III. Distribution of the Downlink Traffic Power

The basis for the model derived in this paper is the relation specified by (4). As per that relation, the random variable $Y \triangleq \sum_{i=0}^{N-1}G_if_i$ is a function of the supported rates, the network parameters, and the subscribers' locations and the corresponding shadowing factors. To fully characterize the sum of traffic power, it is required to characterize first Y. Towards this goal we first present approximations for the random variable f_i and then utilize these approximations to compute the cumulative distribution functions (CDFs) for Y and $\sum_{i=0}^{N-1}P_i$.

A. Distribution of the Parameter f_i

The parameter f_i is a positive-valued random variable that depends only on the path loss model and shadowing process. Mathematically, the parameter f_i is a function of two physical parameters: path loss exponent, α, and the standard deviation of the shadowing factor, ζ_{ik}, which is referred to in this paper as σ_{dB}. Close analysis of this variable shows that it is a tedious task to derive the exact distribution of the parameter f_i. However, through Monte-Carlo simulations, it is observed that the distribution for f_i is nearly lognormal for a wide range of applicable values for α and σ_{dB}. Let the mean and standard deviation of the random variable f_i be specified by

$$\mu_{f_i} = E\left[\ln f_i\right] \text{ and } \sigma_{f_i} = \sqrt{Var\left[\ln f_i\right]} \qquad (6)$$

respectively. The $E[x]$ is the expectation of x while $Var[x]$ is the variance of x. It should be noted that the mean and standard deviation for f_i are representative of the environment and not the network physical layer parameters as alluded to earlier. They need to be evaluated only once using Monte-Carlo simulations for the particular environment of interest.

In this paper we approximate the random variable f_i with a lognormal variable. We utilize two techniques for implementation: the matched first and second dB moments, and, the min-max approximation [8]. For a pair of μ_{f_i} and σ_{f_i} given by (6), the lognormal variable distribution with a matched dB mean and dB standard deviation is given

$$\hat{f}_{f_i}(x) = \frac{1}{\sqrt{2\pi}\sigma_{f_i} x} \exp\left[\frac{-\left(\ln x - \mu_{f_i}\right)^2}{2\sigma_{f_i}^2}\right] \tag{7}$$

Fig. 1 shows the CDF for f_i for σ_{dB} equal to 12 dB and 6 dB, and for α equal to 0, 2, 4, and 6. For every combination of σ_{dB} and α, the corresponding f_i random variable is obtained using Monte-Carlo simulations. Employing a conventional cellular layout, a user is located randomly in the cell of interest and the corresponding f_i is computed. Each of the shown curves is the result of 10^6 independent samples of f_i. The straight lines in these curves correspond to the lognormal variable approximation for the variable f_i. The curves are plotted on a lognormal probability paper which is a transformation that makes the CDF of an ideal lognormal random variable plots as a straight line. This serves to emphasize and easily identify deviations from the assumed lognormal distribution. It can be seen the f_i is near lognormal for a good range of α and σ_{dB} especially for large values of σ_{dB}. The approximation is less accurate for small values σ_{dB} and large values of α.

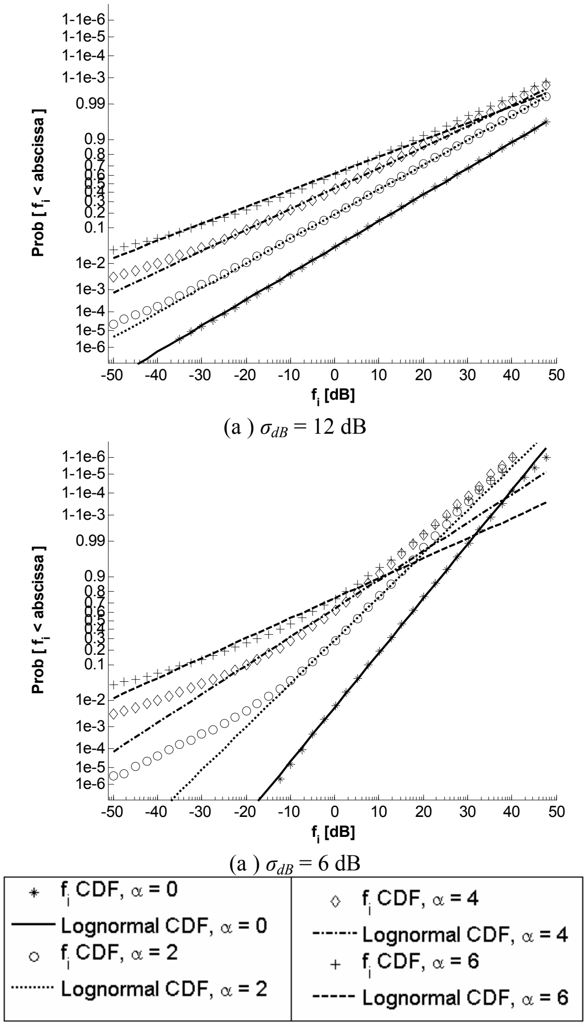
For the second approximation method, we still employ the lognormal random variable, however, we utilize the min-max method [8] to calculate the parameters for the equivalent random variable. In [9] the authors exploit the min-max technique to approximate the sum of lognormal variables while in this study we attempt to utilize the technique to approximate the random variable f_i. Given the empirical distribution of f_i, it is desired to calculate the approximate lognormal CDF specified by as

$$\hat{f}_{f_i}(x) = \Phi^{-1}\left(F_{f_i}\left(e^x\right)\right) = \frac{x}{\sigma_{f_i}^*} - \frac{\mu_{f_i}^*}{\sigma_{f_i}^*} \tag{8}$$

where $\Phi^{-1}(x)$ is the inverse standard normal CDF and $F_{f_i}(e^x)$ is the CDF of f_i evaluated at e^x. Here the quantities $\sigma_{f_i}^*$ and $\mu_{f_i}^*$ are calculated as follows [9]:

$$\sigma_{f_i}^* = \frac{\lambda_2 - \lambda_1}{f(\lambda_2) - f(\lambda_2)} \tag{9}$$

$$\mu_{f_i}^* = \frac{\lambda_1 + t_0}{2} - \frac{\sigma_{f_i}^*}{2}[f(\lambda_1) + f(t_0)] \tag{10}$$



(a) σ_{dB} = 12 dB

(a) σ_{dB} = 6 dB

Fig. 1. Sample figures for the CDF of f_i on lognormal probability scale.

t_0 is the unique solution to the equation $f'(x) = 1/\sigma_{f_i}^*$. The study in [9] uses the formulation in (9) and (10) to determine the parameters for the lognormal variable that approximates the sum of lognormal variables. The limits λ_1 and λ_2 are chosen such the CDF provides accurate approximation in the range of 10^{-6} to $1\text{-}10^{-6}$. However, for our purpose of approximating the distribution of f_i, we develop our own limits that serve the objective of evaluating the CDF of the sum of traffic power defined by (4). Towards this goal, examining the constraint given by (5) we observe that the constraint defines an N-dimensional objects whose i^{th} vertex is given when f_j $\forall$ $j \neq i$ are set to zero. This means the largest value of concern for f_i, f_i^H ,is given by

$$f_i^H = \frac{(1-\rho)}{G_{\min}}\left[1 - \beta - G_{\min}\right] \tag{11}$$

where $G_{\min}$ is equal to $\min_{\forall i}\left(G_i\right)$, and ρ, β, and G_i are as defined for (4). For the lower limit of f_i, f_i^L , the minimum theoretical value of f_i is zero and hence one can select a value such that $\hat{f}_{f_i}\left(f_i^L\right) \le \varepsilon$ for some small ε>0. With λ_1 and λ_2 set to f_i^L and f_i^H, respectively, one can use (8) and (9) to determine the parameters of the lognormal variable that approximates f_i for a given α and σ_{dB}.

B. Power Outage Calculation

In the previous subsection, the probability distribution of f_i is approximated by a lognormal probability distribution function (PDF), $\hat{f}_{fi}\left(x\right)$, whose parameters are either given by (6) or by (9) and (10). This makes the random variable $Y=\sum_{i=0}^{N-1} G_i f_i$ a sum of independent lognormal variables. Recently, a variety of relatively accurate methods have been developed to characterize the sum of independent lognormal variables such as those detailed in [9] and [10]. However, these methods remain computationally excessive. In this study we utilize the characteristic function in order to evaluate the CDF of the variable *Y*. To avoid excessive computations we resort to the newly developed technique in [11] to compute the characteristic function of a lognormal variable. The technique has been found to extremely efficient and accurate compared to conventional numerical integration methods and to the modified Clenshaw-Curtis method [12].

Let the corresponding characteristic function for the approximated random variable f_i be denoted by $\Theta_{\hat{f}}\left(\omega\right)$ defined as $E\left[e^{j\omega f_i}\right]$. Note the subscript *i* is dropped since all f_i's are independent and identically distributed for *i*=0,1, …, *N*-1. It follows that the characteristic function for the random variable $Y=\sum_{i=0}^{N-1} G_i f_i$ is given by

$$\Theta_Y\left(\omega\right)=\prod_{i=0}^{N-1} G_i \prod_{i=0}^{N-1} \Theta_{\hat{f}}\left(G_i\omega\right) \tag{12}$$

The CDF of *Y* can now be computed the inverse transform using conventional techniques. Finally, let the random variable *Z* be defined as the sum of traffic power, i.e. $Z \triangleq \sum_{i-0}^{N-1} P_i$, then using (4), one can note that *Z* is merely a linear transformation of *Y*. That is

$$Z=aY+b \tag{13}$$

Where a and b are constants given by $P_T\Big/\left[\left(1\text{-}\sum_{i=0}^{N-1} G_i\right)(1\text{-}\rho)\right]$ and $P_T\beta\sum_{i=0}^{N-1} G_i\Big/(1\text{-}\sum_{i=0}^{N-1} G_i)$, respectively. Therefore, the desired CDF for Z can be written as

$$F_Z(z)=F_Y\left(\frac{z-b}{a}\right) \tag{14}$$

where $F_Y(y)$ is the CDF for the variable *Y* obtained through (12).

IV. NUMERICAL EXAMPLE AND DISCUSSION

The purpose of this section is to demonstrate the applicability of the developed model and assess the developed approximations relative to results obtained through conventional Monte-Carlo simulations. For the results in this section we assume a cdma2000 1xRTT network [13] where the supported system rates are $V=\{R=2^j R_0 : j=0,1,2,...,k\text{-}1\}$ where the basic rate R_0 is equal to 9.6 kb/s and *k*=5. Furthermore, we assume the cell site maximum transmit power, P_T, is equal to 24 Watts, where a fraction β = 0.2 is allocated for overhead channels. The rest of the physical layer parameters, ρ and E_b/N_0 are assumed to be 0.1 and 12 dB, respectively.

We have shown in Fig. 1 the lognormal approximation for the parameter f_i for a wide range of the path loss exponent α, and the standard deviation of the shadowing process, σ_{dB}. The approximation is relatively accurate for environments with large σ_{dB} while it is less accurate for environments with small σ_{dB} and large α. In wireless communication, indoor propagation suffers from high path loss where α can reach values in the range of 4 to 6 but with the shadowing factor standard deviation σ_{dB} taking values between 8 and 12 dB. On the other hand, for typical outdoor environments α takes values of 2 to 4 while σ_{dB} ranges between 6 and 10 dB [14]. Therefore, the developed approximations are appropriate for both typical indoor and outdoor environments.

To examine the overall accuracy of the developed framework, we utilize the model in computing the forward link outage probability. For a particular users' combination and the corresponding bit rate assignments we compute the probability of downlink traffic power being greater than the maximum possible traffic power. The analytic outage probability, P_{out}, can be obtained using (14) as

$$P_{out}=1-F_Y(\frac{Z-a}{b}) \tag{15}$$

For a particular assumed combination of users, we vary the number of users assigned the j^{th} system rate R_j, $j=0,1,...,k-1$ from 0 to $n_{max,j}$ where $n_{max,j}$ is obtained using the constraint (4). We then plot the outage probability as a function of the number of users, *X*, assigned to the system rate, R_j. The results are shown in Fig. 2 for four different scenarios: [*X*,0,0,0,0], [3,0,*X*,1,0], [*X*,1,1,0,0] and [*X*,2,0,0,0] where *X* is the range of possible number of users for the corresponding bit rate assignment. The outage probability is evaluated using conventional simulations, the matched dB mean and dB standard deviation approximation, and the min-max approximations. It can be noted that the model closely approximates the statistical outage probability or in other words, the distribution of the downlink traffic power for this multi-rate network is closely approximated by the model developed in this work. Furthermore, the parameters computed utilizing the min-max approximation and the developed limits for the parameter f_i result in relatively more accurate calculations when compared to the pure lognormal approximation.

V. CONCLUSION

In this paper we introduced a novel closed form probabilistic framework that characterizes the sum of downlink traffic power required for data bursts for a multi-rate CDMA network. This framework accounts for the RF propagation model and the shadowing process and accounts for the soft capacity feature of CDMA systems. This paper details the required calculations and approximations to arrive at the required sum of downlink traffic power characterization. The study also shows that with the use of min-max method for approximating a key random variable in the problem formulation, the analytical results match very closely those obtained through conventional Monte-Carlo simulations.

The transmission bit rate assignment problem is a critical optimization problem for data burst scheduling for CDMA networks. The developed model can serve as a first step toward analytical studies for efficient call and burst admission control schemes. In this paper we only gave the calculation of the outage probability as an example application for the developed model. Future studies will attempt to utilize this model for burst scheduling over multi-rate CDMA networks.

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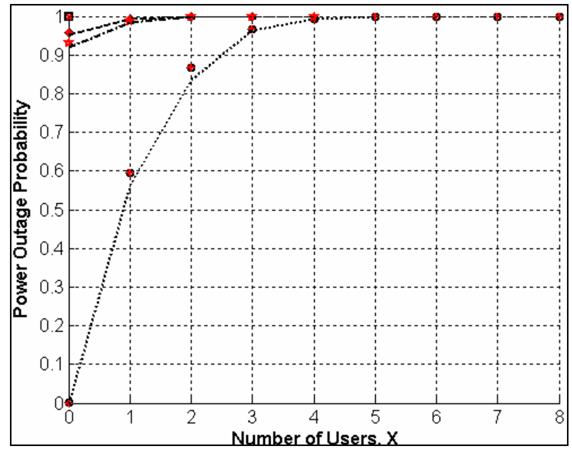
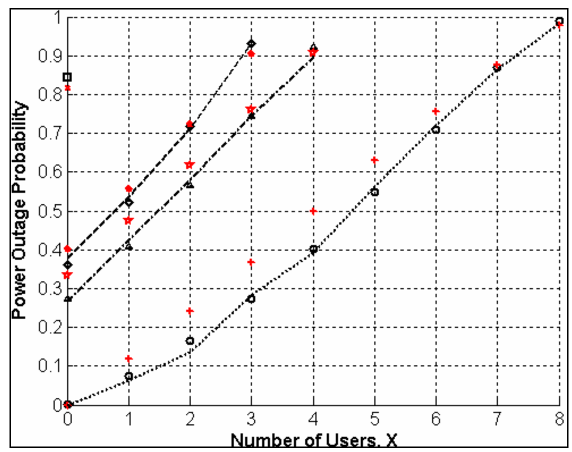


(a) σ_{dB} = 12 dB, α = 2



(b) σ_{dB} = 6 dB, α = 6

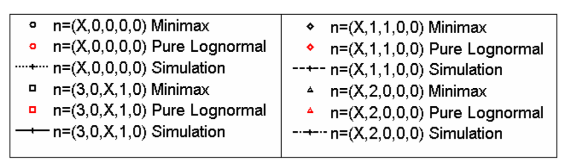


Fig. 2. Outage probability for four different combinations of users.

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