

Number Systems Arithmetic

Objectives

- In this lesson, we will study basic arithmetic operations in various number systems with a particular stress on the binary system.

Approach

- Arithmetic in the Binary number system (addition, subtraction and multiplication).
- Arithmetic in other number systems

Binary Addition

$$0 + 0 = 0$$

$$1 + 0 = 1$$

$$0 + 1 = 1$$

$$1 + 1 = 2$$

2 is not an allowed digit in binary

$$1 + 1 = (10)_2$$

$$(3)_{10} + (7)_{10} = (\text{ten})_{10}$$

$$(3)_{10} + (7)_{10} = (10)_{10}$$

Example

Show the result of adding:

$$(27)_{10} + (43)_{10}$$

	<i>Carry</i>	1		
1st Number	2		7	
2nd Number	4		3	+
Result	7		0	

Position	$i+1$	i
weight	$r^{(i+1)}$	$w = r^i$
Digit 1		D₁
Digit 2		D₂ +
Result	D_{Carry}	D_{Sum}

Position	1	$i=0$
weight	$w = 10^1 = 10$	$w = 10^0 = 1$
Digit 1		5
Digit 2		7 +
Result	1	2

1x10

2x1

- Likewise, in case of the binary system, if the weight of the sum bit is 2^i , then the weight of the carry bit is 2^{i+1} .

- Thus, adding $1 + 1$ in the *binary* system results in a Sum bit of 0 and a carry bit of 1.
- The shown table summarizes the *Sum* and *Carry* results for binary addition

Binary Addition Table

	Carry	Sum
Weight	2^1	2^0
$0 + 0$	0	0
$0 + 1$	0	1
$1 + 0$	0	1
$1 + 1$	1	0

$\equiv 1 \times 2^1$

$\equiv 0 \times 2^0$

} $\equiv +2$

Example

5	4	3	2	1	0		
+	1	1	1	1	1		
1	0	1	1	0	1	+	
1	0	0	1	1	1	+	
1	0	1	0	1	0	0	

Carries

Result of Binary Addition (SUM)

Binary Subtraction

$$1 - 0 = 1$$

$$1 - 1 = 0$$

$$0 - 0 = 0$$

$$0 - 1 = ?$$

Position	1	0	
weight	10	1	
1st Number	7	5	
2nd Number		8	-
Result	?	?	

Position	1	0	
weight	10	1	
1st Number	6 7	5	15
2nd Number		8	-
Result	6	7	

$$(5)_{10} - (8)_{10} = (7)_{10} \text{ Borrow 1}$$

➤ For Binary subtraction

$$0 - 1 = 1 \text{ Borrow 1}$$

➤ In general, the result of subtracting two digits each of weight w is two digits. One is the “**Difference**” digit and the other is the “**Borrow**” digit.

- The **difference** digit has the same weight w as the operand digits.
- The **borrow** digit is considered negative and has the weight of the next higher digit (wr).

	Borrow	Difference
Weight	-2^1	$+2^0$
0 - 0	0	0
1 - 1	0	0
1 - 0	0	1
0 - 1	1	1

$\equiv 1x(-2^1)$

$\equiv +1x2^0$

}

$\equiv -1$

Q. What is $1 - 1 - 1 = ?$

A. The answer is **1 borrow 1**.

Explanation: We perform the operation in 2 steps:

- $1 - 1 = 0$
- We then *subtract* **1** from the above result, i.e. $0 - 1$ which is **1 borrow 1**.

Q. What is $0 - 1 - 1 = ?$

A. The answer is **0 borrow 1**.

Explanation: We perform the operation in 2 steps:

- $0 - 1 = 1$ borrow 1
- We then *subtract* 1 from the above result, which yields 0 borrow 1.

Subtraction Example

Col #	5	4	3	2	1	0	
	-	0	1	1	1	1	Borrows
	1	0	1	1	0	0	
	1	0	0	1	1	1	-
	0	0	0	1	0	1	Result of Binary Subtraction (Difference)

Binary Multiplication (example)

Multiplicand	1	0	1	1	
Multiplier		1	0	1	x
		1	0	1	1
	0	0	0	0	+
1	0	1	1		+
1	1	0	1	1	1

Arith. With Bases Other Than 10

Example: Base 5 \rightarrow Digit Set= {0, 1, 2, 3, 4}

$$\begin{aligned}(2)_5 + (3)_5 &= (5)_{10} \\ &= (?)_5 \\ &= (10)_5\end{aligned}$$

Addition Table

+	0	1	2	3	4
0	0				
1	1	2			
2	2	3	4		
3	3	4	10	11	
4	4	10	11	12	13

$$=5 = 0 \times 5^0 + 1 \times 5^1$$

$$=6 = 1 \times 5^0 + 1 \times 5^1$$

$$=8 = 3 \times 5^0 + 1 \times 5^1$$

Multiplication Table

*	0	1	2	3	4
0	0				
1	0	1			
2	0	2	4		
3	0	3	11	14	
4	0	4	13	22	31

$$=6 = 1 \times 5^0 + 1 \times 5^1$$

$$=9 = 4 \times 5^0 + 1 \times 5^1$$

$$=16 = 1 \times 5^0 + 3 \times 5^1$$