

# Number Systems Arithmetic

## Objectives

- In this lesson, we will study basic arithmetic operations in various number systems with a particular stress on the binary system.

## Approach

- Arithmetic in the Binary number system (addition, subtraction and multiplication).
- Arithmetic in other number systems

## Binary Addition

$$0 + 0 = 0$$

$$1 + 0 = 1$$

$$0 + 1 = 1$$

$$1 + 1 = 2$$

2 is not an allowed digit in binary

$$1 + 1 = (10)_2$$

$$(3)_{10} + (7)_{10} = (\text{ten})_{10}$$

$$(3)_{10} + (7)_{10} = (10)_{10}$$

**Example**

Show the result of adding:

$$(27)_{10} + (43)_{10}$$

|                              |              |          |   |   |
|------------------------------|--------------|----------|---|---|
|                              | <i>Carry</i> | <b>1</b> |   |   |
| <b>1<sup>st</sup> Number</b> | 2            |          | 7 |   |
| <b>2<sup>nd</sup> Number</b> | 4            |          | 3 | + |
| <b>Result</b>                | 7            |          | 0 |   |

|                 |                          |                        |
|-----------------|--------------------------|------------------------|
| <b>Position</b> | $i+1$                    | $i$                    |
| <b>weight</b>   | $r^{(i+1)}$              | $w = r^i$              |
| <b>Digit 1</b>  |                          | <b>D<sub>1</sub></b>   |
| <b>Digit 2</b>  |                          | <b>D<sub>2</sub></b> + |
| <b>Result</b>   | <b>D<sub>Carry</sub></b> | <b>D<sub>Sum</sub></b> |

|                 |                 |                |
|-----------------|-----------------|----------------|
| <b>Position</b> | $1$             | $i=0$          |
| <b>weight</b>   | $w = 10^1 = 10$ | $w = 10^0 = 1$ |
| <b>Digit 1</b>  |                 | <b>5</b>       |
| <b>Digit 2</b>  |                 | <b>7</b> +     |
| <b>Result</b>   | <b>1</b>        | <b>2</b>       |

1x10

2x1

- Likewise, in case of the binary system, if the weight of the sum bit is  $2^i$ , then the weight of the carry bit is  $2^{i+1}$ .

- Thus, adding  $1 + 1$  in the *binary* system results in a Sum bit of 0 and a carry bit of 1.
- The shown table summarizes the *Sum* and *Carry* results for binary addition

**Binary Addition Table**

|         | Carry | Sum   |
|---------|-------|-------|
| Weight  | $2^1$ | $2^0$ |
| $0 + 0$ | 0     | 0     |
| $0 + 1$ | 0     | 1     |
| $1 + 0$ | 0     | 1     |
| $1 + 1$ | 1     | 0     |

  

|   |                       |
|---|-----------------------|
| $\equiv 1 \times 2^1$   | $\equiv 0 \times 2^0$ |
| }<br><div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"><math>\equiv +2</math></div> |                       |

**Example**

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
|   | 5 | 4 | 3 | 2 | 1 | 0 |   |
| + |   | 1 | 1 | 1 | 1 |   |   |
|   | 1 | 0 | 1 | 1 | 0 | 1 |   |
|   | 1 | 0 | 0 | 1 | 1 | 1 | + |
|   | 1 | 0 | 1 | 0 | 1 | 0 |   |

  

Carries

Result of Binary Addition (SUM)

## Binary Subtraction

$$1 - 0 = 1$$

$$1 - 1 = 0$$

$$0 - 0 = 0$$

$$0 - 1 = ?$$

|                              |    |   |   |
|------------------------------|----|---|---|
| <b>Position</b>              | 1  | 0 |   |
| <b>weight</b>                | 10 | 1 |   |
| <b>1<sup>st</sup> Number</b> | 7  | 5 |   |
| <b>2<sup>nd</sup> Number</b> |    | 8 | - |
| <b>Result</b>                | ?  | ? |   |

|                              |                           |              |    |
|------------------------------|---------------------------|--------------|----|
| <b>Position</b>              | 1                         | 0            |    |
| <b>weight</b>                | 10                        | 1            |    |
| <b>1<sup>st</sup> Number</b> | <del>6</del> <del>7</del> | <del>5</del> | 15 |
| <b>2<sup>nd</sup> Number</b> |                           | 8            | -  |
| <b>Result</b>                | 6                         | 7            |    |

$$(5)_{10} - (8)_{10} = (7)_{10} \text{ Borrow 1}$$

➤ For Binary subtraction

$$0 - 1 = 1 \text{ Borrow 1}$$

➤ In general, the result of subtracting two digits each of weight  $w$  is two digits. One is the “**Difference**” digit and the other is the “**Borrow**” digit.

- The **difference** digit has the same weight  $w$  as the operand digits.
- The **borrow** digit is considered negative and has the weight of the next higher digit ( $wr$ ).

|               | <b>Borrow</b> | <b>Difference</b> |
|---------------|---------------|-------------------|
| <b>Weight</b> | $-2^1$        | $+2^0$            |
| <b>0 - 0</b>  | <b>0</b>      | <b>0</b>          |
| <b>1 - 1</b>  | <b>0</b>      | <b>0</b>          |
| <b>1 - 0</b>  | <b>0</b>      | <b>1</b>          |
| <b>0 - 1</b>  | <b>1</b>      | <b>1</b>          |

$\equiv 1x(-2^1)$

$\equiv +1x2^0$

}

$\equiv -1$

**Q.** What is  $1 - 1 - 1 = ?$

**A.** The answer is **1 borrow 1**.

**Explanation:** We perform the operation in 2 steps:

- $1 - 1 = 0$
- We then *subtract* **1** from the above result, i.e.  $0 - 1$  which is **1 borrow 1**.

**Q.** What is  $0 - 1 - 1 = ?$

**A.** The answer is **0 borrow 1**.

**Explanation:** We perform the operation in 2 steps:

- $0 - 1 = 1$  borrow 1
- We then *subtract* 1 from the above result, which yields 0 borrow 1.

**Subtraction Example**

|       |   |   |   |   |   |   |   |
|-------|---|---|---|---|---|---|---|
| Col # | 5 | 4 | 3 | 2 | 1 | 0 |   |
|       | - | 0 | 1 | 1 | 1 | 1 | Borrows                                   |
|       | 1 | 0 | 1 | 1 | 0 | 0 |   |
|       | 1 | 0 | 0 | 1 | 1 | 1 | -   |
|       | 0 | 0 | 0 | 1 | 0 | 1 | Result of Binary Subtraction (Difference) |

### Binary Multiplication (example)

|                     |   |   |   |   |   |
|---------------------|---|---|---|---|---|
| <b>Multiplicand</b> | 1 | 0 | 1 | 1 |   |
| <b>Multiplier</b>   |   | 1 | 0 | 1 | x |
|                     |   | 1 | 0 | 1 | 1 |
|                     | 0 | 0 | 0 | 0 | + |
| 1                   | 0 | 1 | 1 |   | + |
| 1                   | 1 | 0 | 1 | 1 | 1 |

## Arith. With Bases Other Than 10

**Example:** Base 5  $\rightarrow$  Digit Set= {0, 1, 2, 3, 4}

$$\begin{aligned}(2)_5 + (3)_5 &= (5)_{10} \\ &= (?)_5 \\ &= (10)_5\end{aligned}$$

### Addition Table

| + | 0 | 1  | 2  | 3  | 4  |
|---|---|----|----|----|----|
| 0 | 0 |    |    |    |    |
| 1 | 1 | 2  |    |    |    |
| 2 | 2 | 3  | 4  |    |    |
| 3 | 3 | 4  | 10 | 11 |    |
| 4 | 4 | 10 | 11 | 12 | 13 |

$$=5 = 0 \times 5^0 + 1 \times 5^1$$

$$=6 = 1 \times 5^0 + 1 \times 5^1$$

$$=8 = 3 \times 5^0 + 1 \times 5^1$$

### Multiplication Table

| * | 0 | 1 | 2  | 3  | 4  |
|---|---|---|----|----|----|
| 0 | 0 |   |    |    |    |
| 1 | 0 | 1 |    |    |    |
| 2 | 0 | 2 | 4  |    |    |
| 3 | 0 | 3 | 11 | 14 |    |
| 4 | 0 | 4 | 13 | 22 | 31 |

$$=6 = 1 \times 5^0 + 1 \times 5^1$$

$$=9 = 4 \times 5^0 + 1 \times 5^1$$

$$=16 = 1 \times 5^0 + 3 \times 5^1$$