

## ● **Queuing Models (2)**

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## **Today's lecture**

- More examples on queuing systems

## Single Server Queue

- Table 7.6 on page 160 summarizes the formulas for a single server queues

## Example (1)

If an M/M/1 queue has arrivals at a rate of two per minute and serves at a rate of four per minute, how many customers are found in the system on average?  
How many customers are found in service on average?

## Example (1)

Solution:

$$= \lambda / \mu = \rho \quad *T_s = 2 * 0.25 = 0.5$$

Average number in system =  $q$

$$= \rho / (1 - \rho) = 1$$

Average number in service =  $q - w$

$$= 1 - 0.5 = 0.5$$

## Example (2)

What is the utilization of an M/M/1 queue that has four customers waiting on average?

## Example (2)

Solution:

$$w = \frac{2}{1 - \rho} = 4$$
$$2 + 4 - 4 = 0$$

$$= 0.236$$

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## Example (3)

Messages arrive at random to be sent across a communication link with a data rate of 9600 bps. The link is 70% utilized, and the average message length is 1000 octets. Determine the average waiting time for constant-length messages and for exponentially distributed length messages?

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## Example (3)

Solution:

$$T_s = 0.833 \text{ sec}; \quad \rho = 0.7$$

Constant-length messages:

$$T_w = T_s / (2 - \rho) = 0.972 \text{ sec}$$

Exponentially-distributed:

$$T_w = T_s / (1 - \rho) = 1.944 \text{ sec}$$

## Example (4)

Consider a single queue with a constant service time of four sec and a Poisson input with mean rate of 0.20 items per second.

- a. Find the mean and standard deviation of queue size?
- b. Find the mean and standard deviation of queuing time?

## Example (4)

Solution:

$$= / = *Ts = 0.2 *4 = 0.8$$

a.  $q = 2.4$

$$q = 2.4$$

b.  $Tq = 12$

$$Tq = 9.24$$