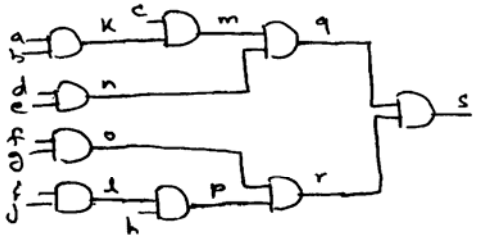
COE 561 Dr. Aiman El-Maleh

HW#4

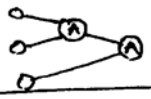
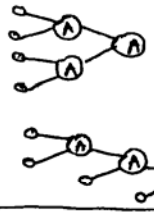
Q1 Library cells:

AND2 with cost = 2
AND3 with cost = 3
AND4 with cost = 4

(i)



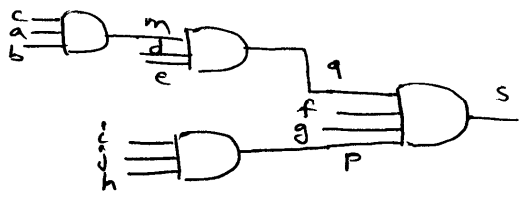
The library has the following cells and pattern graphs:

ID	Cell	Pattern Graph	cost
t1	AND2		2
t2	AND3		3
t3	AND4		4

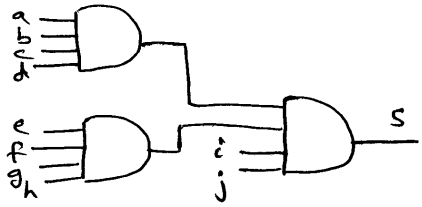
The following table shows the possible matches for each vertex and the corresponding cover cost:

vertex	Match	Gate	Cost
K	t1	AND2 (a,b)	2
n	t1	AND2 (d,e)	2
o	t1	AND2 (f,g)	2
l	t1	AND2 (i,j)	2
m	t1	AND2 (c,K)	2+2 = 4
	t2	AND3 (c,a,b)	3
p	t1	AND2 (l,h)	2 + 2 = 4
	t2	AND3 (i,j,h)	3
q	t1	AND2 (m,n)	2 + 3 + 2 = 7
		AND3 (c,K,n)	3 + 2 + 2 = 7
	t2	AND3 (m,d,e)	3 + 3 = 6
	t2	AND4 (c,K,d,e)	4 + 2 = 6
	t3	AND4 (c,a,b,n)	4 + 2 = 6
	t3		
r	t1	AND2 (o,p)	2 + 2 + 3 = 7
	t2	AND3 (f,g,p)	3 + 3 = 6
		AND3 (o,l,h)	3 + 2 + 2 = 7
	t2	AND4 (f,g,l,h)	4 + 2 = 6
	t3	AND4 (o,i,j,h)	4 + 2 = 6
	t3		
s	t1	AND2 (q,r)	2 + 6 + 6 = 14
		AND3 (m,n,r)	3 + 3 + 2 + 6 = 14
	t2	AND3 (q,o,p)	3 + 6 + 2 + 3 = 14
	t2	AND4 (m,n,o,p)	4 + 3 + 2 + 2 + 3 = 14
	t3	AND4 (c,K,n,r)	4 + 2 + 2 + 6 = 14
	t3	AND4 (q,o,l,h)	4 + 6 + 2 + 2 = 14
	t3	AND4 (q,f,g,p)	4 + 6 + 3 = 13
	t3	AND4 (m,d,e,r)	4 + 3 + 6 = 13

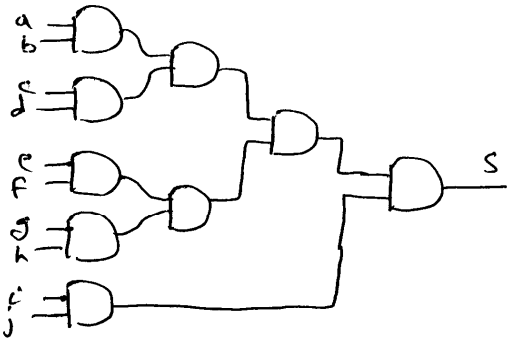
An optimum mapping of the given decomposition has a cost of 13. There are several minimum coverings. One minimum cover is:



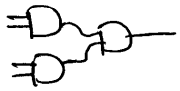
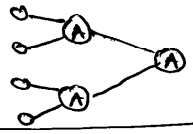
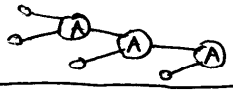
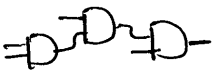
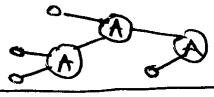
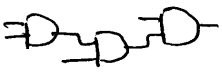
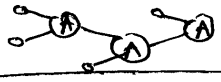
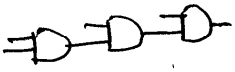
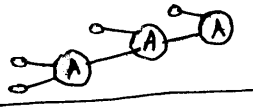
(ii) No, this is not the best implementation possible. The following implementation has a cost of 12:



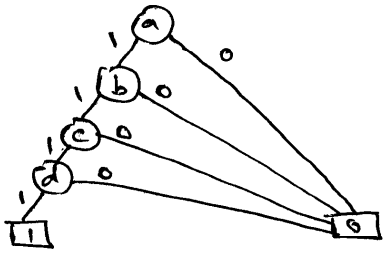
The following decomposition leads to the minimum solution of cost 12



(iii) The pattern trees for the 4-input AND gate cell that need to be stored in the library assuming 2-input AND gates as base functions:

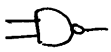
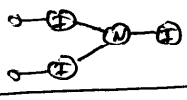
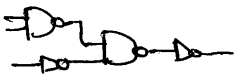
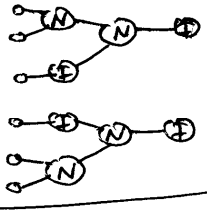
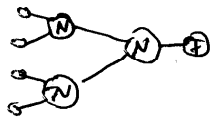
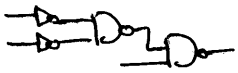
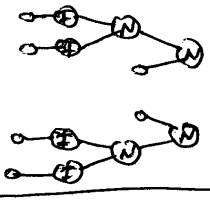
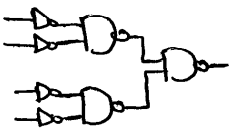
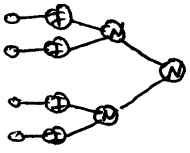
Gate representation	Pattern tree
	
	
	
	
	

(iv) Assume that the AND4 cell is $f = abcd$ since all the four variables are symmetric $C_4 = \{a, b, c, d\} \Rightarrow |C_4| = 1$, only one ROBDD needs to be stored as shown below:

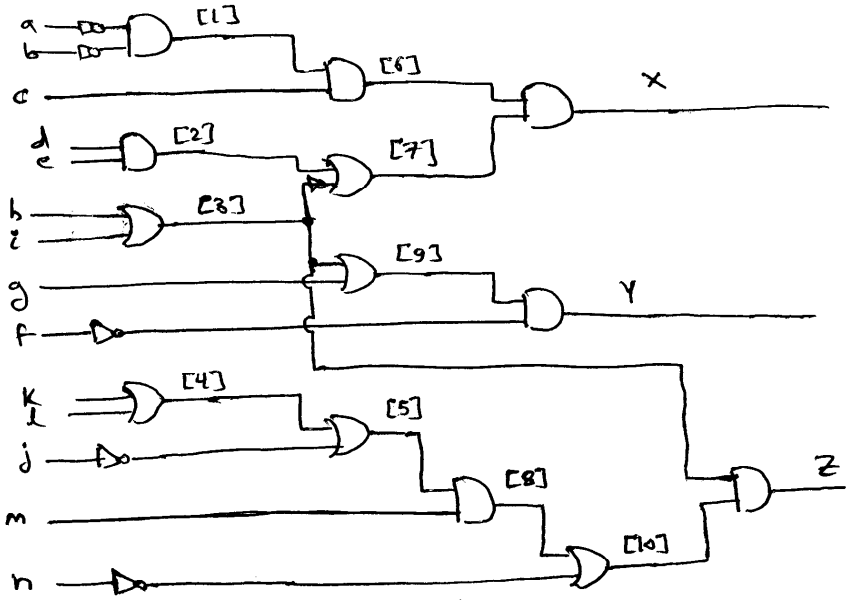


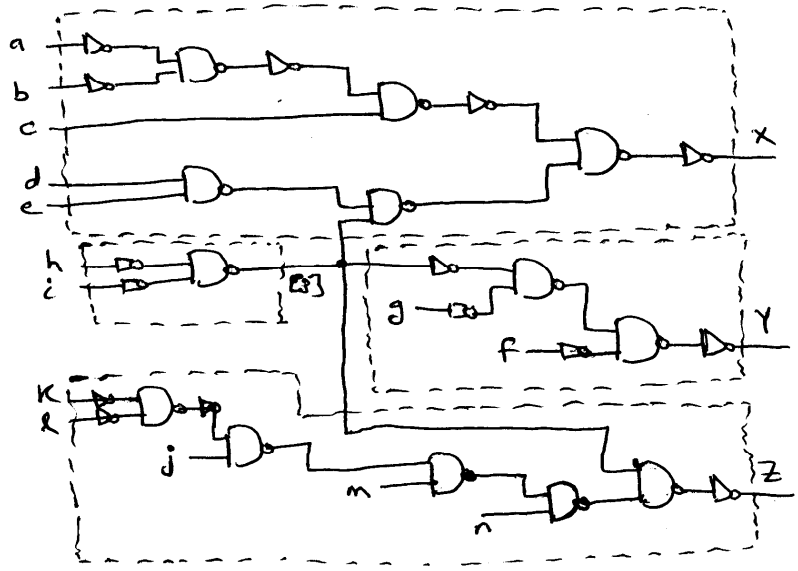
Q2

(c) Pattern trees of the library cells using NAND2 and INV as base functions

ID	cell	gate implementation	Pattern trees	cost
t_1	INV			2
t_2	NAND2			3.5
t_3	NOR2			3
t_4	AOI21			4
t_5	AOI22			5
t_6	OAI21			4
t_7	OAI22			5

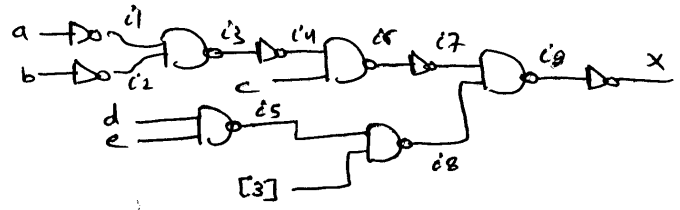
(ii) Decomposition of network using NAND2 and INV cells





(iii) As shown in the previous page, the decomposed network is partitioned into 4 subject graphs, each one will be mapped separately.

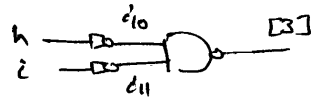
(iv) Minimum cover



vertex	match	Gate	cost
i_1	t_1	INV(a)	2
i_2	t_1	INV(b)	2
i_3	t_2	NAND2(i_1,i_2)	3.5+2+2 = 7.5
i_4	t_1	INV(i_3)	2 + 7.5 = 9.5
	t_3	NOR2(a,b)	3
i_5	t_2	Nand2(d,e)	3.5
i_6	t_2	Nand2(i_4,c)	3.5 + 3 = 6.5
i_7	t_1	INV(i_6)	2 + 6.5 = 8.5
i_8	t_2	Nand2(i_5,[3])	3.5 + 3.5 = 7
i_9	t_2	Nand2(i_7,i_8)	3.5 + 8.5 + 7 = 19
x	t_4	AOI21(i_6,i_5,[3])	4 + 6.5 + 3.5 = 14
	t_1	INV(i_9)	2 + 19 = 21

So, the minimum cost is 14 for this subject graph.

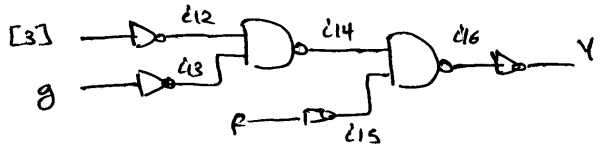
Next, we consider the second subject graph



vertex	match	Gate	Cost
i10	t1	INV(h)	2
i11	t1	INV(i)	2
[3]	t2	NAND2(i10, i11)	3.5 + 2 + 2 = 7.5

So, the minimum cost for this subject graph is 7.5.

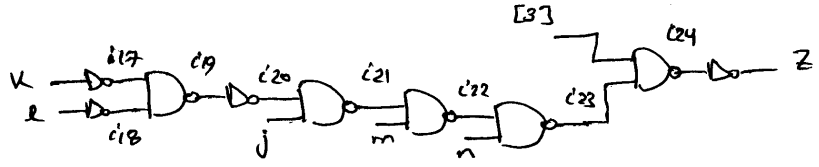
Then, we consider the third subject graph



vertex	match	Gate	Cost
i12	t1	INV([3])	2
i13	t1	INV(g)	2
i14	t2	NAND2(i12, i13)	3.5 + 2 + 2 = 7.5
i15	t1	INV(f)	2
i16	t2	NAND2(i14, i15)	3.5 + 7.5 + 2 = 13
	t6	OAI21([3], g, i15)	4 + 2 = 6
Y	t1	INV(i16)	2 + 6 = 8
	t4	AOI21(i12, i13, f)	4 + 2 + 2 = 8

So, the minimum cover has a cost of 8.

Finally, we find a minimum cover for the last subject graph



vertex	match	Gate	cost
i17	t1	INV(K)	2
i18	t1	INV (l)	2
i19	t2	Nand2 (i17, i18)	3.5 + 2 + 2 = 7.5
i20	t1	INV (i19)	2 + 7.5 = 9.5
	t3	NOR2 (K, l)	3
i21	t2	Nand2 (i20, j)	3.5 + 3 = 6.5
i22	t2	Nand2 (i21, m)	3.5 + 6.5 = 10
i23	t2	Nand2 (i22, n)	3.5 + 10 = 13.5
i24	t2	Nand2 ([3], i23)	3.5 + 13.5 = 17
Z	t1	INV (i24)	2 + 17 = 19

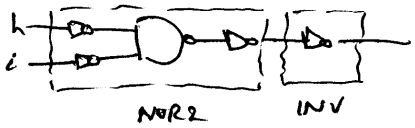
So, the minimum cost for mapping this graph is 19.

The overall cost of mapping the whole network is 14 + 7.5 + 8 + 19 = 48.5

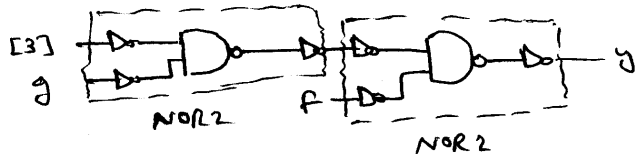
(v) We will consider each subject graph for possible optimization by inserting a cascade of inverters

For the first subject graph, no possible reductions in the area cost.

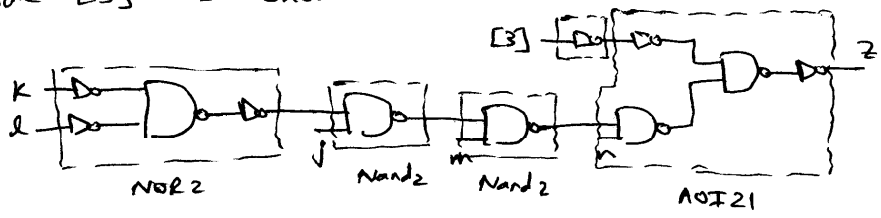
However, for the second subject graph inserting two inverters at the output will reduce the cost from 7+5 to 5 as shown below:



For the third subject graph, inserting two inverters at 014 reduces the cost from 8 to 6 as shown below:



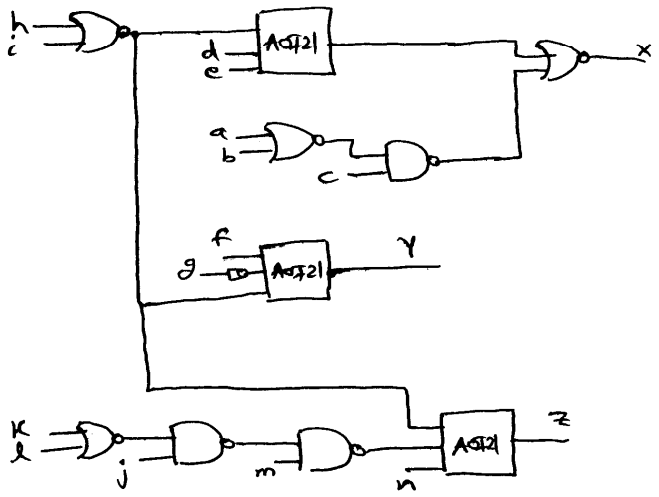
The cost of the fourth graph can be reduced from 19 to 16 by inserting inverter pairs at node [3] as shown below: INV



Thus, the total cost of the reduced solution is

$14 + 5 + 6 + 16 = \underline{\underline{41}}$

(v) using the command map -s -m 0 , s/s produces the following solution with a cost of 36.5



Note that this solution is obtained by taking advantage of inverter optimization across the whole circuit and not just the local subject graphs.