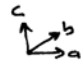
COE 561 Dr. El-Maleh

HW # 2

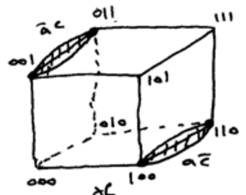
Q1 $f = ab + bc + ac$



$$\frac{\partial f}{\partial b} = f_b \oplus f_{\bar{b}}$$

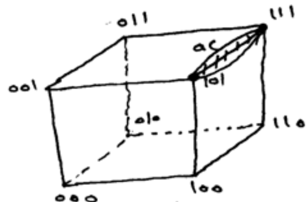
$f_b = a + c \quad ; \quad f_{\bar{b}} = ac$

$$(a+c) \oplus ac = (a+c)\,\overline{ac} + \overline{(a+c)}\,ac$$
$$= (a+c)(\bar{a}+\bar{c}) + \bar{a}\bar{c} \cdot ac$$
$$= a\bar{c} + \bar{a}c$$



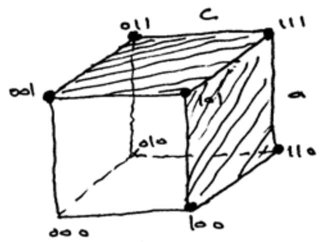
$\frac{\partial f}{\partial b}$

$C_b(f) = f_b \cdot f_{\bar{b}} = (a+c) \cdot ac$
$= ac$



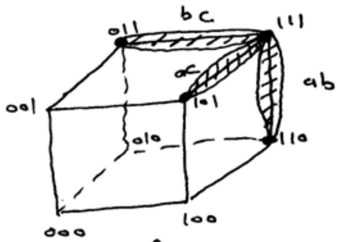
$C_b(f)$

$S_b(f) = f_b + f_{\bar{b}}$
$= (a+c) + ac$
$= a + c$



$S_b(f)$

Function representation :



f

Q2 $\quad f = ab + bc + ac$

$\phi_1 = a \qquad \phi_2 = \bar{a} b \qquad \phi_3 = \bar{a}\bar{b}$

$f = \phi_1 f_{\phi_1} + \phi_2 f_{\phi_2} + \phi_3 f_{\phi_3}$

$f \cdot \phi_1 \le f_{\phi_1} \le f + \bar{\phi}_1$

$ab + ac \le f_{\phi_1} \le \bar{a} + ab + bc + ac = \bar{a} + b + c$

$f \cdot \phi_2 \le f_{\phi_2} \le f + \bar{\phi}_2$

$\bar{a} bc \le f_{\phi_2} \le a + \bar{b} + c$

$f \cdot \phi_3 \le f_{\phi_3} \le f + \bar{\phi}_3$

$0 \le f_{\phi_3} \le a + b$

$\Rightarrow f = a(\bar{a} + b + c) + \bar{a}b(\bar{a}bc) + \bar{a}\bar{b}(0)$

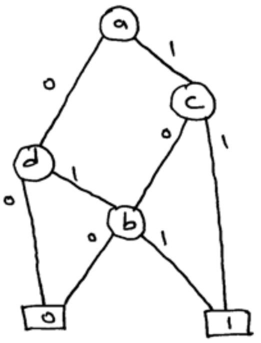
Note that there are several solutions as any of the cofactors can be used.
For example, the following cofactors can be used also: $f_{\phi_1} = b + c$; $f_{\phi_2} = c$; $f_{\phi_3} = 0$

$\Rightarrow f = a(b+c) + \bar{a}b(c) + \bar{a}\bar{b}(0)$

These cofactors are obtained by substituting the value of the basis in the function.

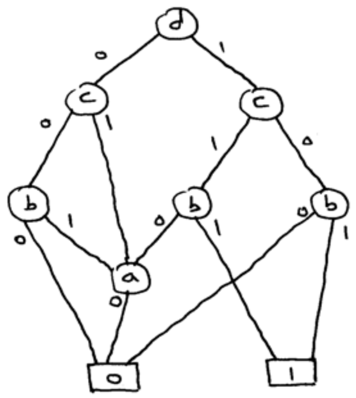
Q3 $f = ab + ac + bd$

One variable order that minimize the size of the ROBDD is $\{a, c, d, b\}$. The corresponding ROBDD is as follows:



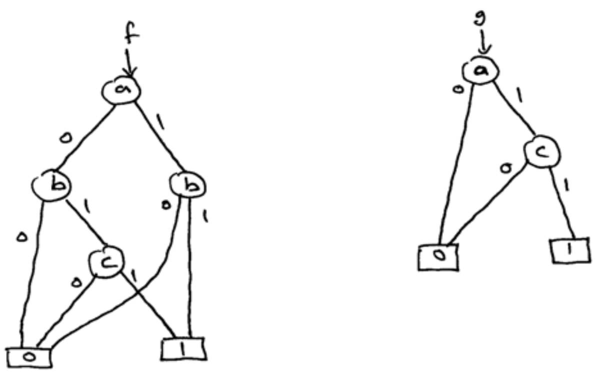
4 nodes

One variable order that maximizes the size of the ROBDD is $\{d, c, b, a\}$. The corresponding ROBDD is as follows:

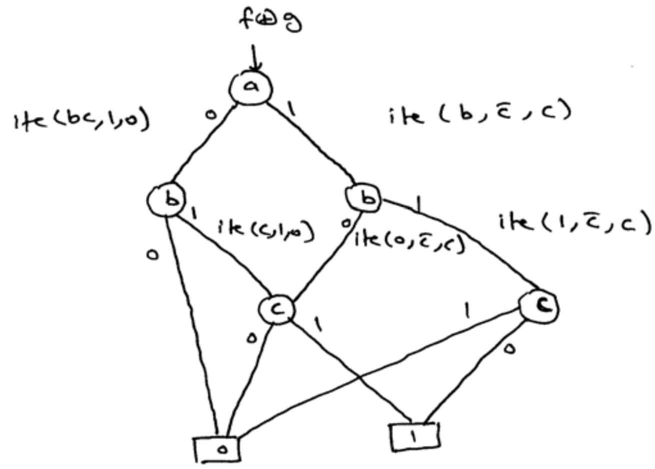


7 nodes

Q4 $\quad f = ab + bc \;;\quad g = ac$



$f \oplus g = ite(f, \bar{g}, g) = ite(ab+bc,\ \bar{a}+\bar{c},\ ac)$



Q5 $F^{ON} = a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}bc$

$F^{DC} = ab\bar{c}$

- Positional-cube notation:

	a	b	c
$a\bar{b}\bar{c}$	01	10	10
$\bar{a}b\bar{c}$	10	01	10
$\bar{a}bc$	10	01	01
$ab\bar{c}$	01	01	10

(i) $F^{off} = U \# \{F^{ON} \cup F^{DC}\}$

$$\{11\ 11\ 11 \# 01\ 10\ 10\} \cap \{11\ 11\ 11 \# 10\ 01\ 10\}$$
$$\cap \{11\ 11\ 11 \# 10\ 01\ 01\} \cap \{11\ 11\ 11 \# 01\ 01\ 10\}$$

$$= \begin{Bmatrix} 10 & 11 & 11 \\ 11 & 01 & 11 \\ 11 & 11 & 01 \end{Bmatrix} \cap \begin{Bmatrix} 01 & 11 & 11 \\ 11 & 10 & 11 \\ 11 & 11 & 01 \end{Bmatrix} \cap \begin{Bmatrix} 01 & 11 & 11 \\ 11 & 10 & 11 \\ 11 & 11 & 10 \end{Bmatrix} \cap \begin{Bmatrix} 10 & 11 & 11 \\ 11 & 10 & 11 \\ 11 & 11 & 01 \end{Bmatrix}$$

$$= \begin{Bmatrix} 10 & 10 & 11 \\ 10 & 11 & 01 \\ 01 & 01 & 11 \\ 11 & 01 & 01 \\ 01 & 11 & 01 \\ 11 & 10 & 01 \\ 11 & 11 & 01 \end{Bmatrix} \cap \begin{Bmatrix} 01 & 10 & 11 \\ 01 & 11 & 01 \\ 10 & 10 & 11 \\ 11 & 10 & 11 \\ 11 & 10 & 01 \\ 10 & 11 & 10 \\ 11 & 10 & 10 \end{Bmatrix} = \begin{Bmatrix} 10 & 10 & 11 \\ 01 & 01 & 11 \\ 11 & 11 & 01 \end{Bmatrix} \cap \begin{Bmatrix} 11 & 10 & 11 \\ 10 & 11 & 10 \\ 01 & 11 & 01 \end{Bmatrix}$$

$$= \begin{Bmatrix} 10 & 10 & 11 \\ 10 & 10 & 10 \\ 01 & 01 & 01 \\ 11 & 10 & 01 \\ 01 & 11 & 01 \end{Bmatrix} = \begin{Bmatrix} 10 & 10 & 11 \\ 11 & 10 & 01 \\ 01 & 11 & 01 \end{Bmatrix} = \bar{a}\bar{b} + \bar{b}c + ac$$

Another solution:

$$f^{off} = \{\{\{11\ 11\ 11 \# 01\ 10\ 10\} \# 10\ 01\ 10\} \# 10\ 01\ 01\} \# 01\ 01\ 10$$

$$11\ 11\ 11 \# 01\ 10\ 10 = \left\{\begin{matrix} 10 & 11 & 11 \\ 11 & 01 & 11 \\ 11 & 11 & 01 \end{matrix}\right\}$$

$$\left\{\begin{matrix} 10 & 11 & 11 \\ 11 & 01 & 11 \\ 11 & 11 & 01 \end{matrix}\right\} \# 10\ 01\ 10$$

$$= \left\{\begin{matrix} 10 & 10 & 11 \\ 10 & 11 & 01 \end{matrix}\right\} \cup \left\{\begin{matrix} 01 & 01 & 11 \\ 11 & 01 & 01 \end{matrix}\right\} \cup \left\{\begin{matrix} 01 & 11 & 01 \\ 11 & 10 & 01 \\ 11 & 11 & 01 \end{matrix}\right\}$$

$$= \left\{\begin{matrix} 11 & 11 & 01 \\ 10 & 10 & 11 \\ 01 & 01 & 11 \end{matrix}\right\}$$

$$\left\{\begin{matrix} 11 & 11 & 01 \\ 10 & 10 & 11 \\ 01 & 01 & 11 \end{matrix}\right\} \# 10\ 01\ 01$$

$$= \left\{\begin{matrix} 01 & 11 & 01 \\ 11 & 10 & 01 \end{matrix}\right\} \cup \left\{\begin{matrix} 10 & 10 & 11 \\ 10 & 10 & 10 \end{matrix}\right\} \cup \left\{\begin{matrix} 01 & 01 & 11 \\ 01 & 01 & 10 \end{matrix}\right\}$$

$$= \left\{\begin{matrix} 01 & 11 & 01 \\ 11 & 10 & 01 \\ 10 & 10 & 11 \\ 01 & 01 & 11 \end{matrix}\right\}$$

$$\left\{\begin{matrix} 01 & 11 & 01 \\ 11 & 10 & 01 \\ 10 & 01 & 11 \\ 01 & 01 & 11 \end{matrix}\right\} \# 01\ 01\ 10$$

$$= \left\{\begin{matrix} 01 & 10 & 01 \\ 01 & 11 & 01 \end{matrix}\right\} \cup \left\{\begin{matrix} 01 & 10 & 01 \\ 11 & 10 & 01 \\ 11 & 10 & 01 \end{matrix}\right\} \cup \left\{\begin{matrix} 10 & 10 & 11 \\ 10 & 10 & 11 \\ 10 & 10 & 01 \end{matrix}\right\} \cup \left\{\begin{matrix} 01 & 01 & 01 \end{matrix}\right\}$$

$$= \left\{\begin{matrix} 10 & 10 & 11 \\ 11 & 10 & 01 \\ 01 & 11 & 01 \end{matrix}\right\} = \bar{a}\bar{b} + \bar{b}c + ac$$

(ii) $F^{off} = U \# \{ F^{ON} \cup F^{DC} \}$

$$= \{11\ 11\ 11 \# 01\ 10\ 10\} \cap \{11\ 11\ 11 \# 10\ 01\ 10\}$$
$$\cap \{11\ 11\ 11 \# 10\ 01\ 01\} \cap \{11\ 11\ 11 \# 01\ 01\ 10\}$$

$$= \begin{Bmatrix} 10 & 11 & 11 \\ 01 & 01 & 11 \\ 01 & 10 & 01 \end{Bmatrix} \cap \begin{Bmatrix} 01 & 11 & 11 \\ 10 & 10 & 11 \\ 10 & 01 & 01 \end{Bmatrix} \cap \begin{Bmatrix} 01 & 11 & 11 \\ 10 & 10 & 11 \\ 10 & 01 & 10 \end{Bmatrix} \cap \begin{Bmatrix} 10 & 11 & 11 \\ 01 & 10 & 11 \\ 01 & 01 & 01 \end{Bmatrix}$$

$$= \begin{Bmatrix} 10 & 10 & 11 \\ 10 & 01 & 01 \\ 01 & 01 & 11 \\ 01 & 10 & 01 \end{Bmatrix} \cap \begin{Bmatrix} 01 & 10 & 11 \\ 01 & 01 & 01 \\ 10 & 10 & 11 \\ 10 & 01 & 10 \end{Bmatrix}$$

$$= \begin{Bmatrix} 10 & 10 & 11 \\ 01 & 01 & 01 \\ 01 & 10 & 01 \end{Bmatrix} = \bar{a}\bar{b} + abc + a\bar{b}c$$

(iii) f^{off} using recursive complementation

$$f^{ON} \cup f^{DC} = \begin{Bmatrix} 01 & 10 & 10 \\ 10 & 01 & 10 \\ 10 & 01 & 01 \\ 01 & 01 & 10 \end{Bmatrix}$$

- Select binate variable a

* Cofactor with respect to a $= \begin{Bmatrix} 11 & 10 & 10 \\ 11 & 01 & 10 \end{Bmatrix} \begin{matrix} \bar{b}\bar{c} \\ b\bar{c} \end{matrix}$

* Cofactor with respect to $\bar{a}$ $= \begin{Bmatrix} 11 & 01 & 10 \\ 11 & 01 & 01 \end{Bmatrix} \begin{matrix} b\bar{c} \\ bc \end{matrix}$

Then, we find the complement of $\begin{Bmatrix} 11 & 10 & 10 \\ 11 & 01 & 10 \end{Bmatrix}$

- we select binate variable b

  * cofactor w.r.t. b = $\{11\ 11\ 10\}$
  * cofactor w.r.t. $\bar{b}$ = $\{11\ 11\ 10\}$

So, the complement of $\begin{Bmatrix} 11 & 10 & 10 \\ 11 & 01 & 10 \end{Bmatrix}$ is $\begin{Bmatrix} 11 & 01 & 01 \\ 11 & 10 & 01 \end{Bmatrix}$

Next, we find the complement of $\begin{Bmatrix} 11 & 01 & 10 \\ 11 & 01 & 01 \end{Bmatrix}$

- we select binate variable c

  * cofactor w.r.t. c = $\{11\ 01\ 11\}$
  * cofactor w.r.t. $\bar{c}$ = $\{11\ 01\ 11\}$

So, the complement of $\begin{Bmatrix} 11 & 01 & 10 \\ 11 & 01 & 01 \end{Bmatrix}$ is $\begin{Bmatrix} 11 & 10 & 01 \\ 11 & 10 & 10 \end{Bmatrix}$

So, the complement of the function is

$$\begin{Bmatrix} 01 & 01 & 01 \\ 01 & 10 & 01 \end{Bmatrix} \cup \begin{Bmatrix} 10 & 10 & 01 \\ 10 & 10 & 10 \end{Bmatrix}$$

$$= \begin{Bmatrix} 01 & 01 & 01 \\ 01 & 10 & 01 \\ 10 & 10 & 01 \\ 10 & 10 & 10 \end{Bmatrix} = abc + a\bar{b}c + \bar{a}\bar{b}c + \bar{a}\bar{b}\bar{c} = ac + \bar{a}\bar{b}$$

Q6 $f = a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}bc$

To check if $bc \subseteq f$ we need to check if f_{bc} is a tautology

$$f = \begin{Bmatrix} 01 & 10 & 10 \\ 10 & 01 & 10 \\ 10 & 01 & 01 \end{Bmatrix} \qquad bc = \{11 \quad 01 \quad 01\}$$

$f_{bc} = \{10 \quad 11 \quad 11\}$

Not a tautology $\Rightarrow$ $bc \not\subseteq f$.

$\bar{a}b = \{10 \quad 01 \quad 11\}$

$$f_{\bar{a}b} = \begin{Bmatrix} 11 & 11 & 10 \\ 11 & 11 & 01 \end{Bmatrix}$$

Tautology as the cover depends on one variable and no column of 0's.

$\Rightarrow$ $\bar{a}b \subseteq f$.

Q7 $f = \bar{a}\bar{d} + \bar{a}b + a\bar{b} + a\bar{c}d$

(i) A cover in positional-cube notation

	a	b	c	d
$\bar{a}\bar{d}$	10	11	11	10
$\bar{a}b$	10	01	11	11
$a\bar{b}$	01	10	11	11
$a\bar{c}d$	01	11	10	01

(ii) Computation of prime implicants

Let us split the function based on the binate variable a.

$$f = \bar{a}(\bar{d} + b) + a(\bar{b} + \bar{c}d)$$

Since $f_{\bar{a}} = \bar{d} + b$ is unate, its prime implicants are $\{\bar{d}, b\}$.

Since $f_a = \bar{b} + \bar{c}d$ is unate, its prime implicants are $\{\bar{b}, \bar{c}d\}$.

we need to compute Consensus $\{(\bar{a}\bar{d}, \bar{a}b), (a\bar{b}, a\bar{c}d)\} = \{\bar{b}\bar{d}, b\bar{c}d\}$

$P(f) = \text{SCC}\{\bar{a}\bar{d}, \bar{a}b, a\bar{b}, a\bar{c}d, \bar{b}\bar{d}, b\bar{c}d\}$

$= \{\bar{a}\bar{d}, \bar{a}b, a\bar{b}, a\bar{c}d, \bar{b}\bar{d}, b\bar{c}d\}$

(iii) Since any cover contains the essential prime implicants, it is sufficient to check the prime implicants in the given cover.

$f = \bar{a}\bar{d} + \bar{a}b + a\bar{b} + a\bar{c}d$

* $\alpha = \bar{a}\bar{d}$

$G = \{\bar{a}b, a\bar{b}, a\bar{c}d\}$

$G \# \alpha = \{\bar{a}bd, a\bar{b}, a\bar{c}d\}$

$H = \text{Consensus}(G\#\alpha, \alpha)$

$= \{\bar{a}b, \bar{b}\bar{d}\}$

$H_\alpha = \{b, \bar{b}\}$ tautology

$\Rightarrow \alpha = \bar{a}\bar{d}$ is not an essential P.I.

* $\alpha = \bar{a}b$

$G = \{\bar{a}\bar{d}, a\bar{b}, a\bar{c}d\}$

$G \# \alpha = \{\bar{a}\bar{b}\bar{d}, a\bar{b}, a\bar{c}d\}$

$H = \text{Consensus}(G\#\alpha, \alpha)$

$= \{\bar{a}\bar{d}, b\bar{c}d\}$

$H_\alpha = \{\bar{d}, \bar{c}d\}$ not tautology

$\Rightarrow \alpha = \bar{a}b$ is an essential P.I.

– $\alpha = a\overline{b}$

$G = \{ \overline{a}\overline{d},\ \overline{a}b,\ a\overline{c}d \}$

$G \# \alpha = \{ \overline{a}\overline{d},\ \overline{a}b,\ a\overline{b}\,\overline{c}d \}$

$H = \{ \overline{b}\overline{d},\ a\overline{c}d \}$

$H\alpha = \{ \overline{d},\ \overline{c}d \}$ not tautology

$\Rightarrow \alpha = a\overline{b}$ is an essential P.I.

– $\alpha = a\overline{c}d$

$G = \{ \overline{a}\overline{d},\ \overline{a}b,\ a\overline{b} \}$

$G \# \alpha = \{ \overline{a}\overline{d},\ \overline{a}b,\ a\overline{b}c,\ a\overline{b}d \}$

$H = \{ b\overline{c}d,\ a\overline{b}d \}$

$H\alpha = \{ b,\ \overline{b} \}$ tautology

$\Rightarrow \alpha = a\overline{c}d$ is not an essential P.I.

Thus, f has two essential P.I.

$\{ \overline{a}b,\ a\overline{b} \}$.

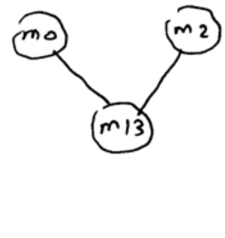
(iv) Minimum Cover based on Covering Problem formulation

	$\bar{a}\bar{d}$	$\bar{a}b$	$a\bar{b}$	$a\bar{c}d$	$\bar{b}\bar{d}$	$b\bar{c}d$
m0	1	0	0	0	1	0
m2	1	0	0	0	1	0
m4	1	1	0	0	0	0
m5	0	1	0	0	0	1
m6	1	1	0	0	0	0
m7	0	1	0	0	0	0
m8	0	0	1	0	1	0
m9	0	0	1	1	0	0
m10	0	0	1	0	1	0
m11	0	0	1	0	0	0
m13	0	0	0	1	0	1

We can see that column $\bar{a}b$ is essential because it is incident on row m7 which has a single 1. Similarly $a\bar{b}$ is essential as it is the only one covering m11. So, we select these two columns and remove the covered rows.

The reduced matrix becomes:

	$\bar{a}\bar{d}$	$a\bar{c}d$	$\bar{b}\bar{d}$	$b\bar{c}d$
m0	1	0	1	0
m2	1	0	1	0
m13	0	1	0	1



clique is 2. So, a lower bound is 2.

By running the Exact_Cover algorithm, $\bar{a}\bar{d}$ will be selected. Then, $a\bar{c}d$ will be selected. No better solution will be found.

Note that there are four minimum solutions with the same cost

$\{\bar{a}b, a\bar{b}, \bar{a}\bar{d}, a\bar{c}d\}$

or $\{\bar{a}b, a\bar{b}, \bar{a}\bar{d}, b\bar{c}d\}$

or $\{\bar{a}b, a\bar{b}, \bar{b}\bar{d}, a\bar{c}d\}$

or $\{\bar{a}b, a\bar{b}, \bar{b}\bar{d}, b\bar{c}d\}$

(v) Espresso-Exact finds the solution
$\{\bar{a}b, a\bar{b}, \bar{b}\bar{d}, b\bar{c}d\}$

(vi) Espresso also returns the same solution
$\{\bar{a}b, a\bar{b}, \bar{b}\bar{d}, b\bar{c}d\}$

Q8 $f(a,b,c,d) = \sum m(0,1,4,5,7,8,9,12,15)$

(i)

	a	b	c	d
$\bar{a}\bar{b}\bar{c}\bar{d}$	10	10	10	10
$\bar{a}\bar{b}\bar{c}d$	10	10	10	01
$\bar{a}b\bar{c}\bar{d}$	10	01	10	10
$\bar{a}b\bar{c}d$	10	01	10	01
$\bar{a}bcd$	10	01	01	01
$a\bar{b}\bar{c}\bar{d}$	01	10	10	10
$a\bar{b}\bar{c}d$	01	10	10	01
$ab\bar{c}\bar{d}$	01	01	10	10
$abcd$	01	01	01	01
	54	45	72	45

Then, we compute the implicant weights

$$\begin{bmatrix} 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 01 \\ 10 & 01 & 10 & 10 \\ 10 & 01 & 10 & 01 \\ 10 & 01 & 01 & 01 \\ 01 & 10 & 10 & 10 \\ 01 & 10 & 10 & 01 \\ 01 & 01 & 10 & 10 \\ 01 & 01 & 01 & 01 \end{bmatrix} * \begin{bmatrix} 5 \\ 4 \\ 4 \\ 5 \\ 7 \\ 2 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 20 \\ 21 \\ 21 \\ 22 \\ 17 \\ 19 \\ 20 \\ 20 \\ 16 \end{bmatrix}$$

So, the minterm $abcd$ = 01 01 01 01 is selected first for expansion.

Free set = {1, 3, 5, 7}

columns 3, 5, 7 cannot be raised as they have a distance 1 from the offset.

The implicant $\bar{a}bcd$ = 10 01 01 01 has a feasible supercube with $abcd$ = 01 01 01 01. So, column 1 is raised and we obtain the implicant bcd = 11 01 01 01. Since the free set is empty, we need to expand another implicant.

Next, we expand the implicant $a\bar{b}\bar{c}\bar{d}$ = 01 10 10 10
Free-set = {1, 4, 6, 8}. Column 6 cannot be raised because it has a distance 1 from off-set. The implicant $\bar{a}\bar{b}\bar{c}\bar{d}$ = 10 10 10 10 has a feasible supercube with $a\bar{b}\bar{c}\bar{d}$ = 01 10 10 10. So, column 1 is raised and we obtain the implicant 11 10 10 10 = $\bar{b}\bar{c}\bar{d}$. The implicant $\bar{a}b\bar{c}\bar{d}$ = 10 01 10 10 has a feasible supercube with the implicant $\bar{b}\bar{c}\bar{d}$ = 11 10 10 10. So, column 4 is raised and we obtain the implicant $\bar{c}\bar{d}$ = 11 11 10 10. We remove all the covered implicants from consideration. Column 8 cannot be raised because it has a distance 1 from off-set.

Next, we expand the implicant $a\bar{b}\bar{c}d$ = 01 10 10 01. Free-set = {2, 3, 5, 7}. Columns 3 and 5 cannot be raised because they have a distance 1 from off-set. The implicant $\bar{a}\bar{b}\bar{c}d$ = 10 10 10 01 has a feasible supercube with the implicant $a\bar{b}\bar{c}d$ = 01 10 10 01. So, column 2 is raised and we obtain the implicant $\bar{b}\bar{c}d$ = 11 10 10 01. Then, column 7 is raised and we obtain the implicant $\bar{b}\bar{c}$ = 11 10 10 11.

Finally, we expand the implicant $\bar{a}b\bar{c}d$ = 10 01 10 01. Free-set = {1, 4, 5, 8}. Column 1 cannot be raised as it has a distance 1 from off-set. Note that raising columns 4, 5, 8 has the same weight as it overlaps the same number of cubes. Let us raise column 4. We obtain the implicant $\bar{a}\bar{c}d$ = 10 11 10 01. Now, column 5 cannot be raised as it has distance 1 from off-set. We then raise column 8 and we obtain the implicant $\bar{a}\bar{c}$ = 10 11 10 11.
So, the obtained expanded cover is
{bcd, $\bar{c}\bar{d}$, $\bar{b}\bar{c}$, $\bar{a}\bar{c}$}.

Next, we apply the irredundancy check on the expanded cover $\{bcd, \bar{c}\bar{d}, \bar{b}\bar{c}, \bar{a}\bar{c}\}$.

It can be easily seen that all the implicant are relatively essential i.e. $E^r = \{bcd, \bar{c}\bar{d}, \bar{b}\bar{c}, \bar{a}\bar{c}\}$.
So, none of the implicants is redundant.

Next, we apply Reduce on the cover.
First, we compute the implicant weights

	a	b	c	d
bcd	11	01	01	01
$\bar{c}\bar{d}$	11	11	10	10
$\bar{b}\bar{c}$	11	10	10	11
$\bar{a}\bar{c}$	10	11	10	11
	4 3	3 3	3 1	3 3

$$\begin{bmatrix} 11 & 01 & 01 & 01 \\ 11 & 11 & 10 & 10 \\ 11 & 10 & 10 & 11 \\ 10 & 11 & 10 & 11 \end{bmatrix} * \begin{bmatrix} 4 \\ 3 \\ 3 \\ 3 \\ 3 \\ 1 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 \\ 19 \\ 19 \\ 19 \end{bmatrix}$$

Suppose that we select $\bar{a}\bar{c} = 10\ 11\ 10\ 11$ to reduce.

$Q = bcd + \bar{c}\bar{d} + \bar{b}\bar{c} = c\,(bd) + \bar{c}\,(\bar{b}+\bar{d})$

$\bar{Q} = c\,(\bar{b}+\bar{d}) + c\,(bd) = c\bar{b} + c\bar{d} + \bar{c}bd$

$\bar{Q}_{\bar{a}\bar{c}} = bd$

$\bar{a}\bar{c} \cap \text{supercube}(bd) = \bar{a}b\bar{c}d$

So, the implicant $\bar{a}\bar{c}$ is reduced to $\bar{a}b\bar{c}d$.

We next select to reduce the implicant $\bar{b}\bar{c}$.

$$Q = bcd + \bar{c}\bar{d} + \bar{a}b\bar{c}d = cd(b) + \bar{c}\bar{d}(1) + \bar{c}d(\bar{a}b) + c\bar{d}(0)$$

$$\bar{Q} = cd(\bar{b}) + \bar{c}\bar{d}(0) + \bar{c}d(a+\bar{b}) + c\bar{d}(1)$$
$$= cd\bar{b} + \bar{c}da + \bar{c}d\bar{b} + c\bar{d}$$

$$\bar{Q}_{\bar{b}\bar{c}} = da + d$$

$$\text{Supercube}(\bar{Q}_{\bar{b}\bar{c}}) = d$$

$$\bar{b}\bar{c} \cap \text{Supercube}(\bar{Q}_{\bar{b}\bar{c}}) = \bar{b}\bar{c}d$$

So, the implicant $\bar{b}\bar{c}$ is reduced to $\bar{b}\bar{c}d$.

We next select to reduce the implicant $\bar{c}\bar{d}$.

$$Q = bcd + \bar{a}b\bar{c}d + \bar{b}\bar{c}d = cd(b) + \bar{c}d(\bar{a}+\bar{b}) + c\bar{d}(0) + \bar{c}\bar{d}(0)$$

$$\bar{Q} = cd\bar{b} + \bar{c}dab + c\bar{d} + \bar{c}\bar{d}$$

$$\bar{Q}_{\bar{c}\bar{d}} = 1$$

$\bar{c}\bar{d} \cap \text{Supercube}(\bar{Q}_{\bar{c}\bar{d}}) = \bar{c}\bar{d}$.

So, the implicant cannot be reduced.

Finally, we select to reduce the implicant bcd.

$$Q = \bar{a}b\bar{c}d + \bar{b}\bar{c}d + \bar{c}\bar{d} = \bar{c}d(\bar{a}+\bar{b}) + \bar{c}\bar{d}(1) + c\bar{d}(0) + cd(0)$$

$$\bar{Q} = \bar{c}dab + c\bar{d} + cd$$

$\bar{Q}_{bcd} = 1$; $bcd \cap \text{Supercube}(\bar{Q}_{bcd}) = bcd$.

So, the implicant cannot be reduced.

The reduced cover is $\{\bar{a}b\bar{c}d, \bar{b}\bar{c}d, \bar{c}\bar{d}, bcd\}$.

Finally, we apply EXPAND on the reduced cover.

	a	b	c	d
$\bar a b \bar c d$	10	01	10	01
$\bar b \bar c d$	11	10	10	01
$\bar c \bar d$	11	11	10	10
$b c d$	11	01	01	01
	43	23	31	13

$$\begin{bmatrix} 10 & 01 & 10 & 01 \\ 11 & 10 & 10 & 01 \\ 11 & 11 & 10 & 10 \\ 11 & 01 & 01 & 01 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 3 \\ 3 \\ 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 13 \\ 15 \\ 16 \\ 14 \end{bmatrix}$$

We first expand the implicant $\bar a b \bar c d$.
Free-set $= \{2, 3, 6, 7\}$. Column 2 can't be raised since it has distance 1 from offset.
We can select to raise either column 3, 6, or 7.
Let us raise column 3. The implicant becomes $\bar a \bar c d$.
Now, column 6 can't be raised as it has distance 1 from offset. We then raise column 7 and we obtain the implicant $\bar a \bar c$. Note that none of the remaining 3 implicants is covered by the expanded implicant.

Next, we select to expand the implicant bcd.
Free-set $= \{3, 5, 7\}$. None of the columns can be raised as they have distance 1 with the offset.
So, the implicant bcd can't be expanded.

Then, we select to expand the implicant $\bar b \bar c d$.
Free-set $= \{4, 6, 7\}$. Columns 4 and 6 can't be raised as they have distance 1 from the offset.
So, we raise column 7 and we obtain the implicant $\bar b \bar c$.

Finally, we select to expand the implicant $\bar c \bar d$.
Free-set $= \{6, 8\}$. Both columns can't be raised, and the implicant can't be expanded.

So, the expanded cover is
$\{\bar a \bar c, \; bcd, \; \bar b \bar c, \; \bar c \bar d\}$.

(ii)

Running EXPAND on the initial cover:

```
# espresso -Dexpand -t -v  hw2q8.pla
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb f
.dc
# READ        Time was 0.00 sec, cost is c=9(9) in=36 out=9 tot=45
# COMPL       Time was 0.00 sec, cost is c=3(3) in=8 out=3 tot=11
# PLA is hw2q8.pla with 4 inputs and 1 outputs
# ON-set cost is  c=9(9) in=36 out=9 tot=45
# OFF-set cost is c=3(3) in=8 out=3 tot=11
# DC-set cost is  c=0(0) in=0 out=0 tot=0
EXPAND: 1111 1 (covered 1)
EXPAND: 1000 1 (covered 3)
EXPAND: 1001 1 (covered 1)
EXPAND: 0101 1 (covered 0)
# EXPAND      Time was 0.00 sec, cost is c=4(0) in=9 out=4 tot=13
# READ        1 call(s) for 0.00 sec ( 0.0%)
# COMPL       1 call(s) for 0.00 sec ( 0.0%)
# EXPAND      1 call(s) for 0.00 sec ( 0.0%)
# expand      Time was 0.00 sec, cost is c=4(0) in=9 out=4 tot=13
.i 4
.o 1
.ilb a b c d
.p 4
-111 1
--00 1
-00- 1
0-0- 1
.e
# WRITE       Time was 0.00 sec, cost is c=4(0) in=9 out=4 tot=13
```

Running IRREDUNDANT on the expanded cover:

```
# espresso -Dirred -t -v  exp
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb f
.dc
# READ        Time was 0.00 sec, cost is c=4(4) in=9 out=4 tot=13
# COMPL       Time was 0.00 sec, cost is c=0(0) in=0 out=0 tot=0
# PLA is exp with 4 inputs and 1 outputs
# ON-set cost is  c=4(4) in=9 out=4 tot=13
# OFF-set cost is c=0(0) in=0 out=0 tot=0
```

```
# DC-set cost is  c=0(0) in=0 out=0 tot=0
# IRRED: F=4 E=4 R=0 Rt=0 Rp=0 Rc=0 Final=4 Bound=0
# IRRED        Time was 0.00 sec, cost is c=4(4) in=9 out=4 tot=13
# READ         1 call(s) for 0.00 sec ( 0.0%)
# COMPL        1 call(s) for 0.00 sec ( 0.0%)
# IRRED        1 call(s) for 0.00 sec ( 0.0%)
# irred Time was 0.00 sec, cost is c=4(4) in=9 out=4 tot=13
.i 4
.o 1
.ilb a b c d
.p 4
-111 1
--00 1
-00- 1
0-0- 1
.e
# WRITE        Time was 0.00 sec, cost is c=4(4) in=9 out=4 tot=13
```

Running REDUCE on the irredundant cover:

```
# espresso -Dreduce -t -v  irred
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb f
.dc
# READ         Time was 0.00 sec, cost is c=4(4) in=9 out=4 tot=13
# COMPL        Time was 0.00 sec, cost is c=0(0) in=0 out=0 tot=0
# PLA is irred with 4 inputs and 1 outputs
# ON-set cost is  c=4(4) in=9 out=4 tot=13
# OFF-set cost is c=0(0) in=0 out=0 tot=0
# DC-set cost is  c=0(0) in=0 out=0 tot=0
REDUCE: 0-0- 1 to 0101 1 0.00 sec
REDUCE: -00- 1 to -001 1 0.00 sec
# REDUCE       Time was 0.00 sec, cost is c=4(2) in=12 out=4 tot=16
# READ         1 call(s) for 0.00 sec ( 0.0%)
# COMPL        1 call(s) for 0.00 sec ( 0.0%)
# REDUCE       1 call(s) for 0.00 sec ( 0.0%)
# reduce       Time was 0.00 sec, cost is c=4(2) in=12 out=4 tot=16
.i 4
.o 1
.ilb a b c d
.p 4
0101 1
-001 1
--00 1
-111 1
```

```
.e
# WRITE        Time was 0.00 sec, cost is c=4(2) in=12 out=4 tot=16
```

Running EXPAND on the reduced cover:

```
# espresso -Dexpand -t -v  red
# UC Berkeley, Espresso Version #2.3, Release date 01/31/88
.olb f
.dc
# READD        Time was 0.00 sec, cost is c=4(4) in=12 out=4 tot=16
# COMPL        Time was 0.00 sec, cost is c=3(2) in=8 out=3 tot=11
# PLA is red with 4 inputs and 1 outputs
# ON-set cost is  c=4(4) in=12 out=4 tot=16
# OFF-set cost is c=3(2) in=8 out=3 tot=11
# DC-set cost is  c=0(0) in=0 out=0 tot=0
EXPAND: 0101 1 (covered 0)
EXPAND: -111 1 (covered 0)
EXPAND: -001 1 (covered 0)
EXPAND: --00 1 (covered 0)
# EXPAND       Time was 0.00 sec, cost is c=4(0) in=9 out=4 tot=13
# READ         1 call(s) for 0.00 sec ( 0.0%)
# COMPL        1 call(s) for 0.00 sec ( 0.0%)
# EXPAND       1 call(s) for 0.00 sec ( 0.0%)
# expand       Time was 0.00 sec, cost is c=4(0) in=9 out=4 tot=13
.i 4
.o 1
.ilb a b c d
.p 4
0-0- 1
-111 1
-00- 1
--00 1
.e
# WRITE        Time was 0.00 sec, cost is c=4(0) in=9 out=4 tot=13
```