CoE 561 : 101

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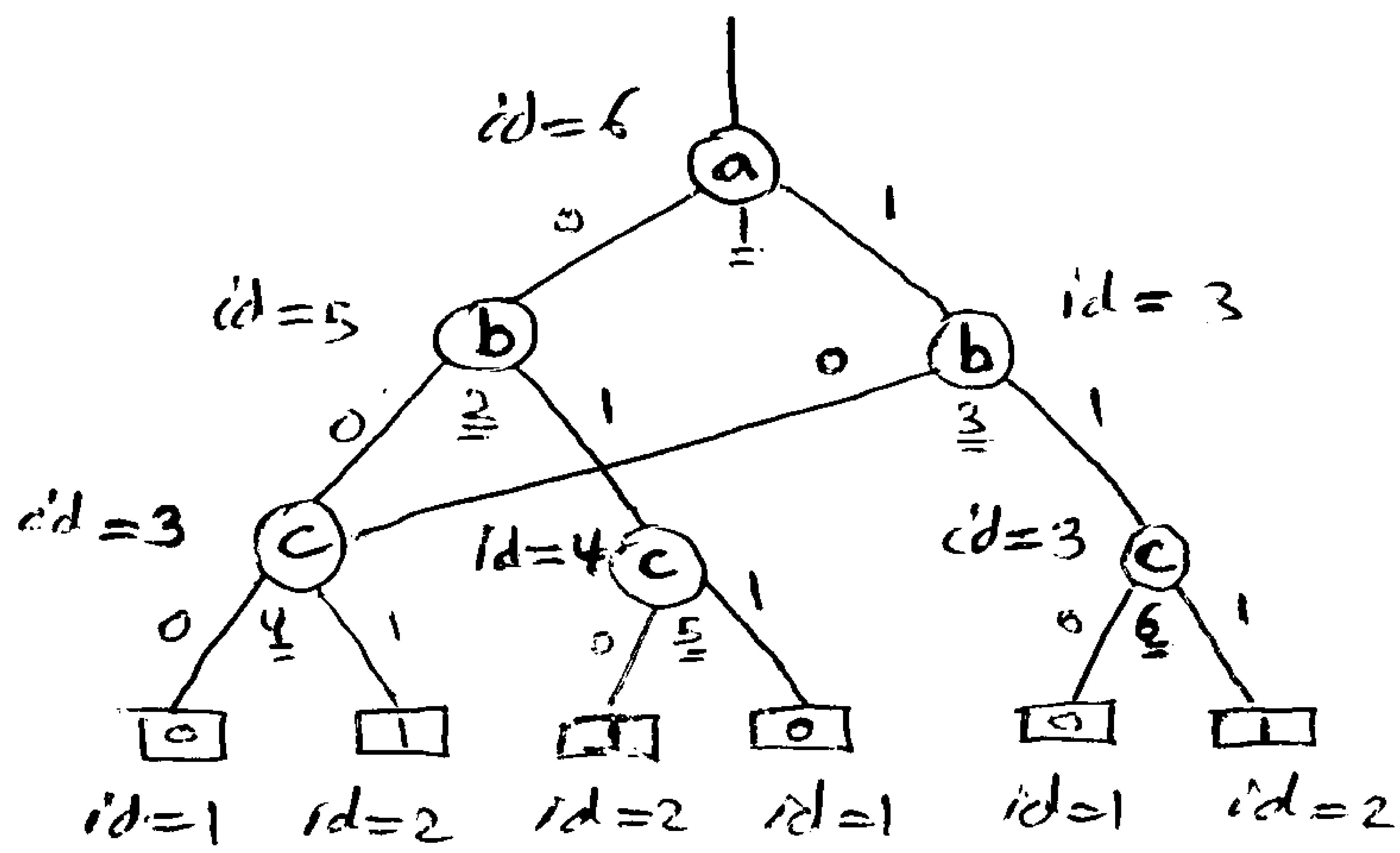
HW #1

Q1.



First, we set $id(v)=1$ for all leaf vertices with value 0 and $id(v)=2$ for all leaf vertices with value 1.

We initialize ROBDD with two leaf vertices for 0 and 1.

Then, we process vertices at level 3, i.e. nodes with index = c.

$V = \{4, 5, 6\}$

None of the vertices is removed since $id(low(v)) \neq id(high(v))$.

We assign keys to all vertices $\in V$.

$Key(4) = (1,2)$, $Key(5) = (2,1)$, $Key(6) = (1,2)$

$old Key = (0,0)$

We next sort the vertices in V according to their keys. Thus, $V = \{4, 6, 5\}$.

$v = \{4\}$: since $Key(4) \neq oldKey$, $nextid = 3$, $id(4) = 3$, $oldKey = (1,2)$. we add $v = \{4\}$ to the ROBDD.

$v = \{6\}$: since $Key(6) = oldKey$, $id(6) = 3$.

$v = \{5\}$: since $Key(5) \neq oldKey$, $nextid = 4$, $id(5) = 4$, $oldKey = (2,1)$. we add $v = \{5\}$ to the ROBDD.

Next, we process vertices at level 2 with index = b.

$V = \{2, 3\}$

since $id(low(3)) = id(high(3)) = 3$, $id(3) = 3$ and vertex 3 is removed from V.

$Key(2) = (3,4)$, $oldKey = (0,0)$.

Since $Key(2) \neq oldKey$, $nextid = 5$, $id(2) = 5$, $oldKey = (3,4)$. We add $v = \{2\}$ to the ROBDD.

Finally, we process vertices at level 1 with index = a.

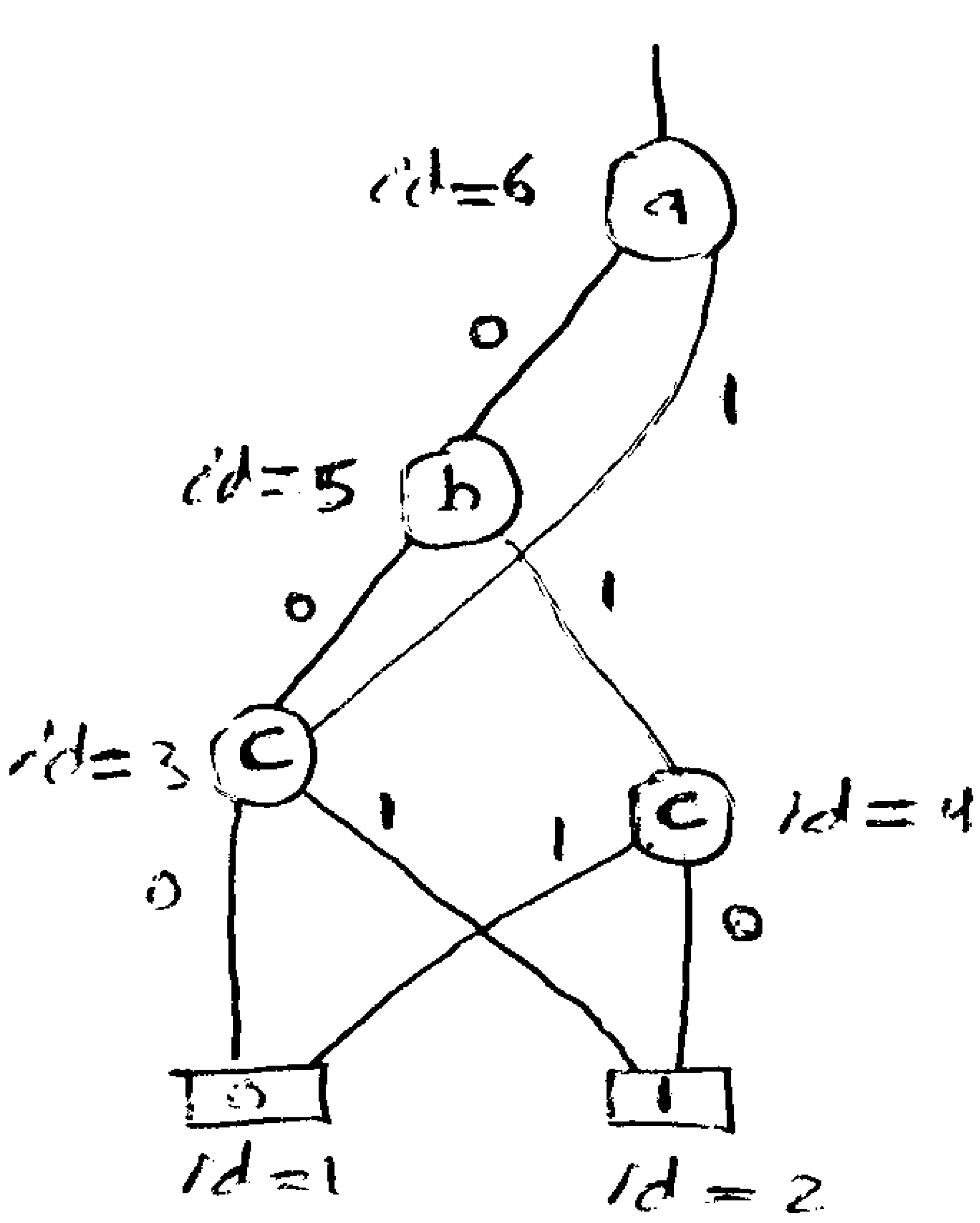
$V = \{1\}$

since $id(low(1)) \neq id(high(1))$, the vertex is not removed. $Key(1) = (5,3)$, $oldKey = (0,0)$.

Since $Key(1) \neq oldKey$, $nextid = 6$, $id(1) = 6$.

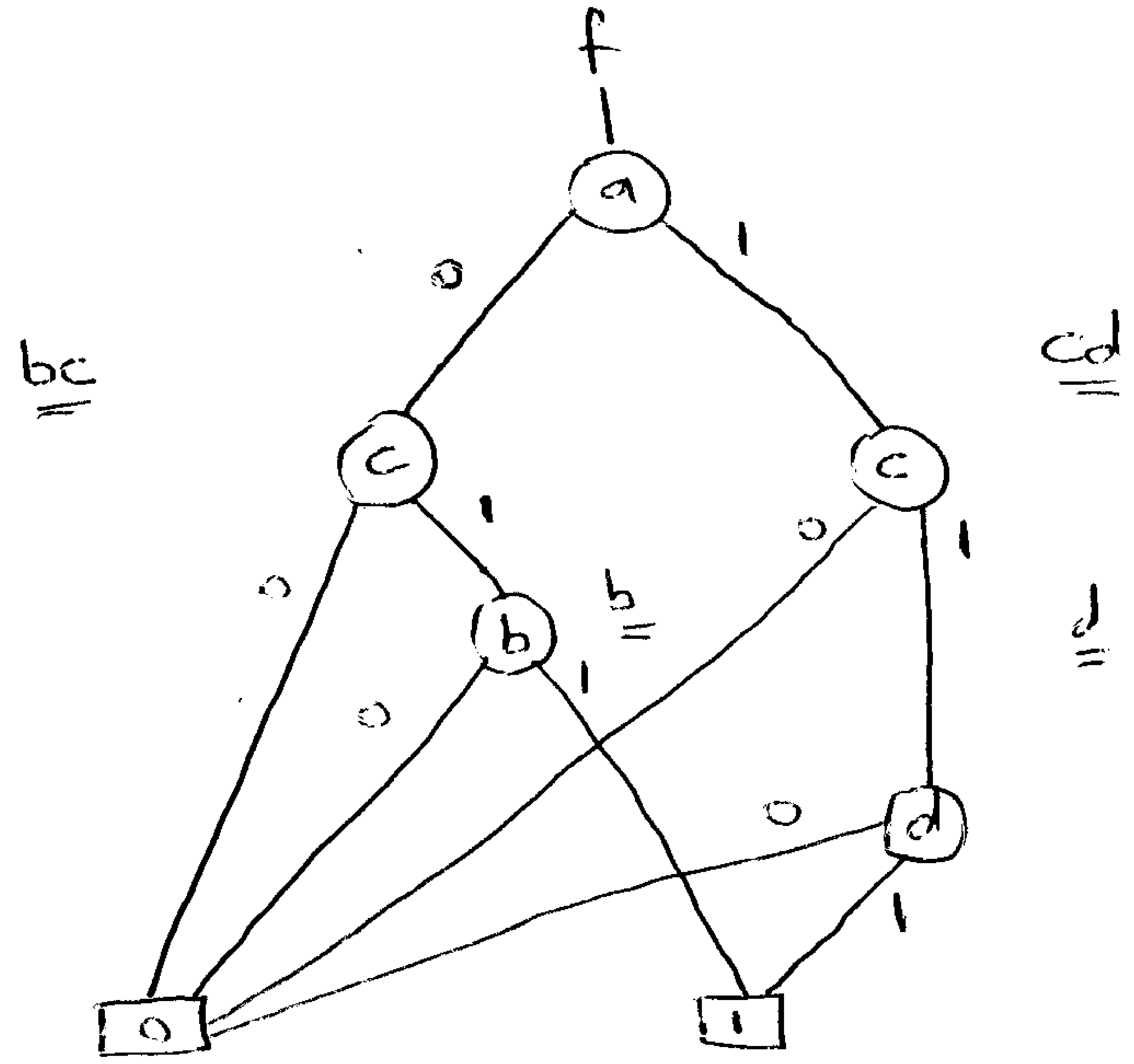
We add $v = \{1\}$ to the ROBDD.

Thus, the formed ROBDD is:

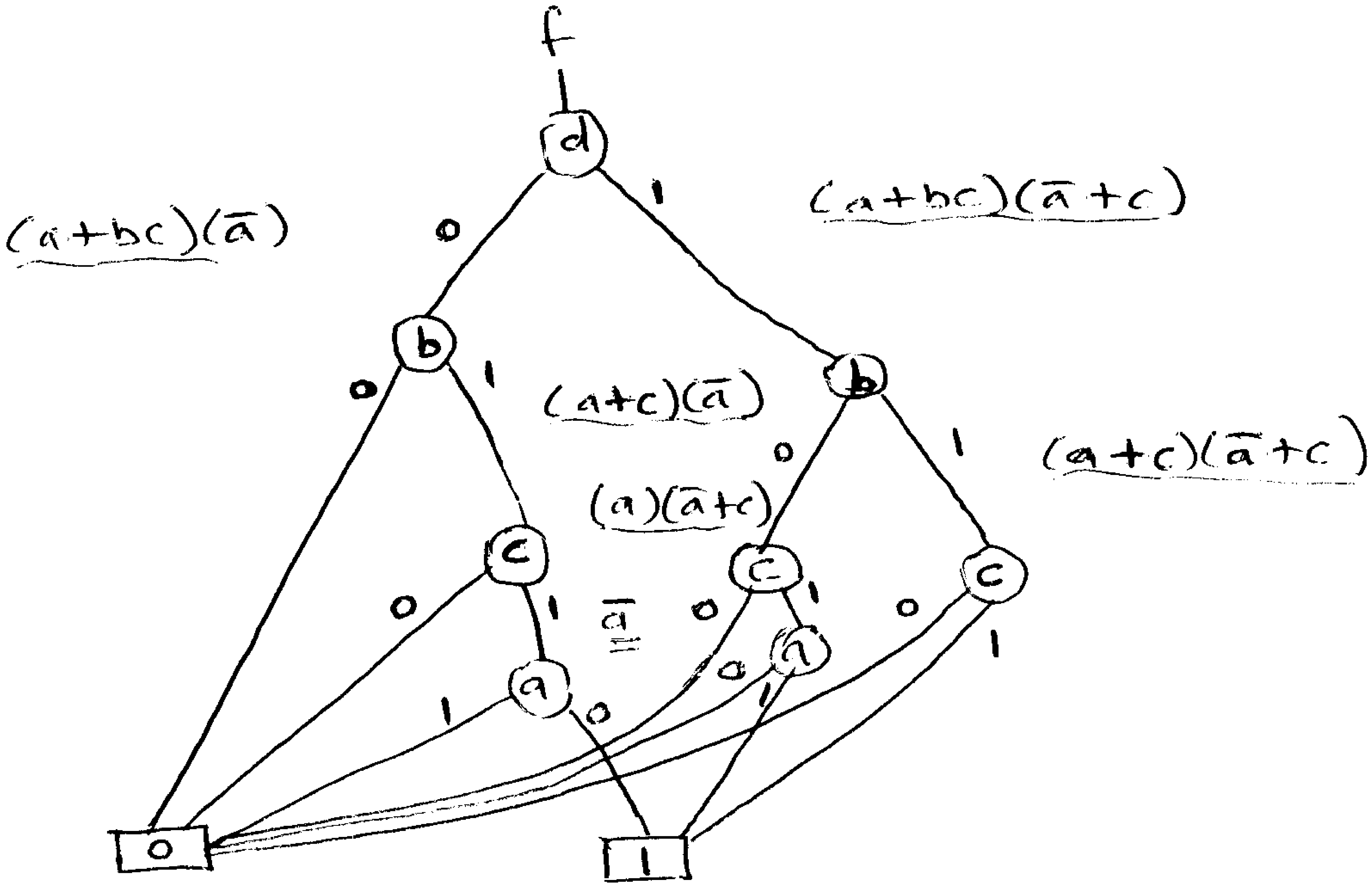


Q2. $f = (a + bc)(\bar{a} + cd)$

(i) ROBDD with variable order $\{a, c, b, d\}$



(ii) ROBDD with variable order $\{d, b, c, a\}$



(iii) We can see that the variable order $\pi(i)$ produces a smaller size ROBDD than (ii). As a general heuristic, we should choose the variable that eliminates the largest number of terms in our expression.

Q3. $f = (a+bc)(\bar{a}+cd)$, $g = (a+b)(c+d)$

(i) $f \oplus g$ based on orthonormal basis expansion

We can use any orthonormal basis but we will choose the orthonormal basis $\{\bar{a}\bar{c}, \bar{a}c, a\bar{c}, ac\}$ as they will simplify the cofactors.

$$f = \bar{a}\bar{c}\,[0] + \bar{a}c\,[b] + a\bar{c}\,[0] + ac\,[d]$$

$$g = \bar{a}\bar{c}\,[bd] + \bar{a}c\,[b] + a\bar{c}\,[d] + ac\,[1]$$

$$f \oplus g = \bar{a}\bar{c}\,[0 \oplus bd] + \bar{a}c\,[b \oplus b] + a\bar{c}\,[0 \oplus d] + ac\,[d \oplus 1]$$

$$= \bar{a}\bar{c}\,[bd] + \bar{a}c\,[0] + a\bar{c}\,[d] + ac\,[\bar{d}]$$

$$= \bar{a}\bar{c}bd + a\bar{c}d + ac\bar{d}$$

(ii) ITE diagram for the function $f \cdot g$

$f \cdot g = ITE(f, g, 0)$ with variable order $\{a, b, c, d\}$

$= ITE((a+bc)(\bar{a}+cd), (a+b)(c+d), 0)$

- $x = a$

  $t = ITE(cd, c+d, 0)$

  - $x = b$

    $t = ITE(cd, c+d, 0)$

    - $x = c$

      $t = ITE(d, 1, 0) = d$ (trivial case)
      we assign $id = 3 \Rightarrow t = 3$

      $e = ITE(0, d, 0) = 0$ (trivial case)
      $\Rightarrow e = 1$
      since $t \neq e$, we add the entry $(c, 3, 1)$
      in the unique table with $id = 4$

    $\Rightarrow t = 4$
    we add an entry in the computed table
    with $\{(cd, c+d, 0), 4\}$.

    $e = ITE(cd, c+d, 0)$
    since this has already been computed,
    $e = 4$

    since $t = e = 4 \Rightarrow t = 4$

  $e = ITE(bc, b(c+d), 0)$

- $x = b$

  $t = ITE(c, c+d, 0)$

- $x = c$

$t = ITE(1, 1, 0) = 1 \Rightarrow t = 2$

$e = ITE(0, d, 0) = 0 \Rightarrow e = 1$

since $t \neq e$, we add $(c, 2, 1)$ to the unique table with id = 5.

$\Rightarrow t = 5$

we update the computed table with

$\{(c, c+d, 0), 5\}$

$e = ITE(0, 0, 0) = 0 \Rightarrow e = 1$

since $t \neq e$, we add an entry $(b, 5, 1)$ to the unique table with id = 6.

$\Rightarrow e = 6$. We add $\{(bc, b(c+d), 0), 6\}$ to comp. table.

since $t \neq e$, we add the entry $(a, 4, 6)$ with id = 7.

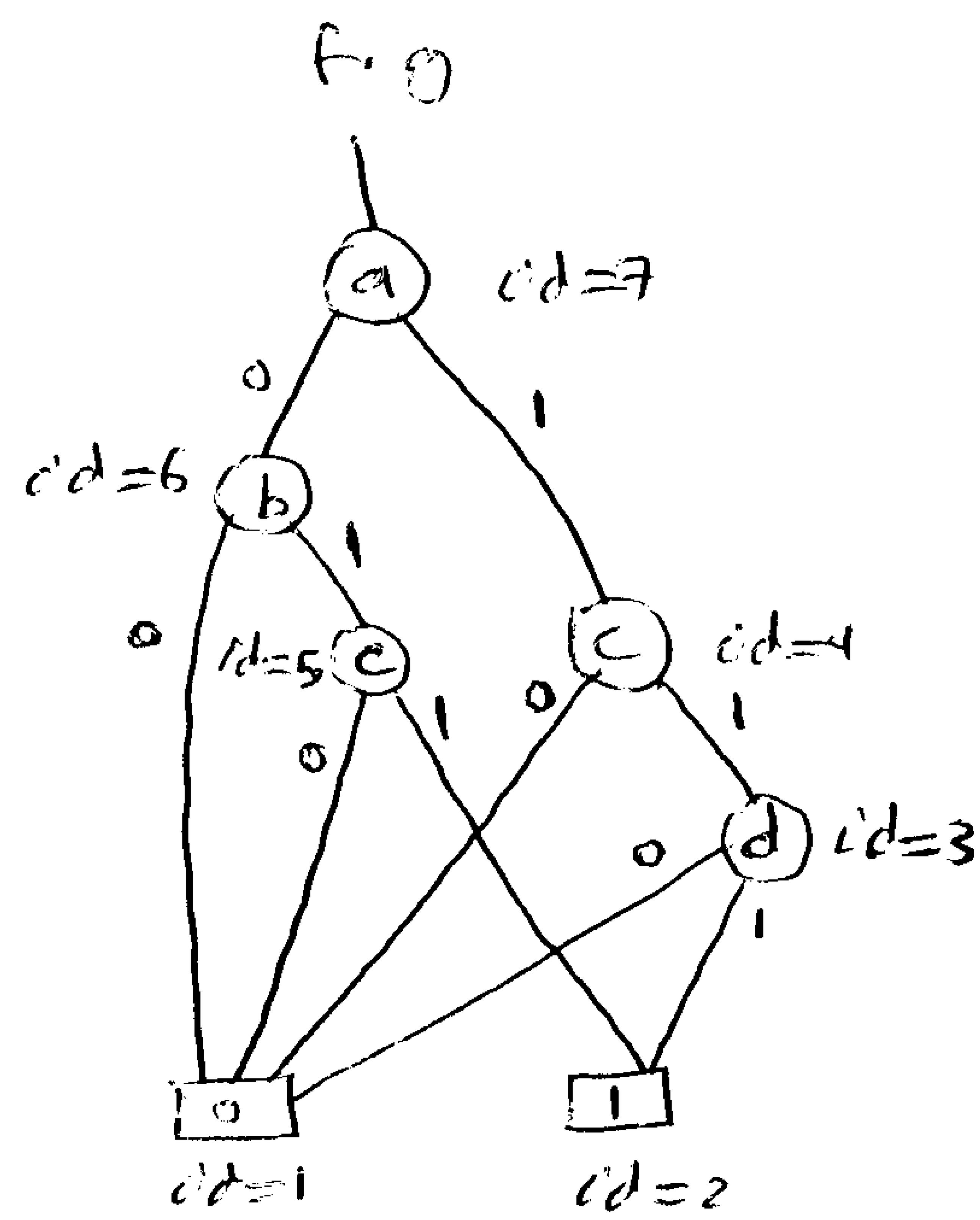
Unique Table:

id	var	H	L
3	d	2	1
4	c	3	1
5	c	2	1
6	b	5	1
7	a	4	6

Computed Table:

f	g	h	id
cd	c+d	0	4
c	c+d	0	5
bc	b(c+d)	0	6

ROBDD



Q4. The matrix to be covered:

	c1	c2	c3	c4	c5	c6	c7	c8
r1	1	0	0	0	1	1	0	0
r2	1	0	0	1	0	0	1	0
r3	1	0	1	0	0	0	0	1
r4	0	1	0	0	1	0	0	1
r5	0	1	0	1	0	0	1	0
r6	0	1	1	0	0	1	0	0

There are no essential columns & no row dominance. c4 dominates c7 $\Rightarrow$ c7 is removed. Next, we select c1 and call exact-cover with $X = (1,0,0,0,0,0,0,0)$ and $b = (1,1,1,1,1,1,1,1)$ and the matrix:

	c2	c3	c4	c5	c6	c8
r4	1	0	0	1	0	1
r5	1	0	1	0	0	0
r6	1	1	0	0	1	0

Since c2 dominates all other columns, they get removed and c2 becomes essential and r5 selected. Since matrix has no rows, then $X = (1,1,0,0,0,0,0,0)$ and $b = (1,1,0,0,0,0,0,0)$.

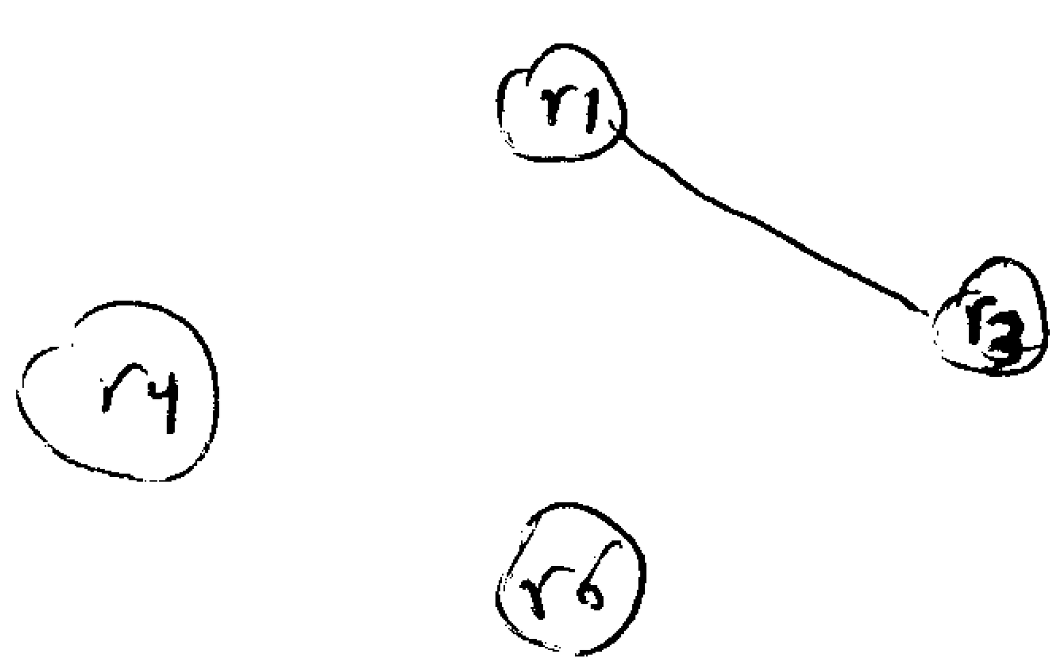
Next, exact-cover is called with c1 not selected with $x = (0,0,0,0,0,0,0,0)$ and $b = (1,1,0,0,0,0,0,0)$ and the matrix:

	c2	c3	c4	c5	c6	c8
r1	0	0	0	1	1	0
r2	0	0	1	0	0	0
r3	0	1	0	0	0	1
r4	1	0	0	1	0	1
r5	1	0	1	0	0	0
r6	1	1	0	0	1	0

c4 is essential and is selected resulting in the reduced matrix.

	c2	c3	c5	c6	c8
r1	0	0	1	1	0
r3	0	1	0	0	1
r4	1	0	1	0	1
r6	1	1	0	1	0

There is no row dominance or column dominance
Thus, we compute a lower bound



largest clique = 2
$\Rightarrow$ current estimate is
$1 + 2 = 3 \geq |b| = 2$

$\Rightarrow$ returned solution is $(1,1,0,0,0,0,0,0)$
Since returned solution is not $< |b|$, the final returned solution is $(1,1,0,0,0,0,0,0)$.

Q5. $F = \bar{A}\bar{C} + \bar{C}D + A\bar{D} + BC + \bar{A}\bar{B} + AC$

since all variables are binate, we can select any variabe.

$$F = \bar{A}\,[\bar{C} + \bar{C}D + BC + \bar{B}]$$
$$+ A\,[\bar{C}D + \bar{D} + BC + C]$$

we need to show that both $F_{\bar{A}}$ and F_A are tautology.

$$F_{\bar{A}} = [\bar{C} + \bar{C}D + BC + \bar{B}]$$

since D is positive unate, it is sufficient to show that $F_{\bar{A}\bar{D}}$ is tautology.

$$F_{\bar{A}\bar{D}} = [\bar{C} + BC + \bar{B}]$$

we can expand on any variable,

$$F_{\bar{A}\bar{D}\bar{C}} = 1$$
$$F_{\bar{A}\bar{D}C} = [B + \bar{B}] = 1$$

$\Rightarrow F_{\bar{A}\bar{D}}$ is Tautology $\Rightarrow F_{\bar{A}}$ is Tautology.

$$F_A = [\bar{C}D + \bar{D} + BC + C]$$

since F_A is positive unate with respect to B, it is sufficient to show that $F_{A\bar{B}}$ is tautology.

$$F_{A\bar{B}} = [\bar{C}D + \bar{D} + C]$$

$$F_{A\bar{B}\bar{C}} = [D + \bar{D}] = 1$$
$$F_{A\bar{B}C} = 1$$

$\Rightarrow F_{A\bar{B}}$ is Tautology $\Rightarrow F_A$ is Tautology.

$\Rightarrow F$ is Tautology.

Q6. $F = \bar{A}\bar{B} + AB + B\bar{C} + AC + CD + B\bar{D}$

(i) We will expand on unate variables whenever possible.

$$F = \bar{A}\,[\bar{B} + B\bar{C} + CD + B\bar{D}]$$
$$+ A\,[B + B\bar{C} + C + CD + B\bar{D}]$$

$$= \bar{A}\,[\bar{B}[1] + B\,[\bar{C} + CD + \bar{D}]]$$
$$+ A\,[1 + \bar{B}\,[C + CD]]$$

$$= \bar{A}\,[\bar{B}\,[1] + B\,[\bar{C}[1] + C\,[D+\bar{D}]]]$$
$$+ A\,[1 + \bar{B}\,[1 + \bar{C}\,[0]]]$$

$$= \bar{A}\,[\bar{B}\,[1] + B\,[\bar{C}[1] + C\,[1]]]$$
$$+ A\,[1 + \bar{B}\,[1 + \bar{C}\,[0]]]$$

$$\Rightarrow \bar{F} = \bar{A}\,[\bar{B}\,[0] + B\,[\bar{C}[0] + C\,[0]]]$$
$$+ A\,[0 + \bar{B}\,[0 + \bar{C}\,[1]]]$$

$$= A\bar{B}\bar{C}$$

(ii) $F = \bar{A}\,[\,\bar{B} + B\bar{C} + CD + B\bar{D}\,]$
$+ A\,[\,B + B\bar{C} + C + CD + B\bar{D}\,]$

$= \bar{A}\,[\,\bar{B}\,[1] + B\,[\bar{C} + CD + \bar{D}\,]\,]$
$+ A\,[\,\bar{B}\,[C + CD] + B\,[1]\,]$

$= \bar{A}\,[\,\bar{B}\,[1] + B\,[\,\bar{C}\,[1] + C\,[1]\,]\,]$
$+ A\,[\,\bar{B}\,[C + CD] + B\,[1]\,]$

prime implicants of $f_{\bar{A}B} = scc\{\bar{C}, C, 1\} = 1$
prime implicants of $f_{\bar{A}\bar{B}} = 1$
$\Rightarrow$ prime implicants of $f_{\bar{A}} = scc\{B, \bar{B}, 1\} = 1$

prime implicants of $f_{AB} = 1$
prime implicants of $f_{A\bar{B}} = scc\{C, CD\} = \{C\}$
$\Rightarrow$ prime implicants of $f_A = scc\{B, \bar{B}C, C\} = \{B, C\}$

$\Rightarrow$ Prime implicants of $f = scc\{\bar{A}, AB, AC, B, C\}$
$= \{\bar{A}, B, C\}$