COE 202, Term 162

Digital Logic Design

HW# 2 Solution

Q.1. Prove the identity of each of the following Boolean functions using algebraic manipulation:

(i) A\` + AB + AC\` + AB\`C\` = A\` + B + C\`

(ii) (A + B + D)(A\` + C)(B + C + D) = (A + B + D)(A\` + C)

(iii) XZ + WY\`Z\` + W\`YZ\` + WX\`Z\` = XZ + WY\`Z\` + W\`XY + X\`YZ\`

Q.2. Simplify the following Boolean expressions to a minimum number of literals using algebraic manipulation:

(i) W\`X(Z\` + Y\`Z) + X(W + W\`YZ)

(ii) [(CD)\` + A]\` + A + CD + AB

(iii) (A + B\` + AB\`)(AB + A\`C + BC)

Q.3. Find the complement of the following Boolean functions and reduce them to a minimum number of literals:

(i) WX(Y\`Z + YZ\`) + W\`X\`(Y\`+Z)(Y+Z\`)

(ii) ABC + A\`CD

Q.4. Using DeMorgan\`s theorem, express the function F=A\`B\` + AB + B\`C

(i) With only OR and complement operations.

(ii) With only AND and complement operations.

Q.5. Show that the dual of the equivalence function F(A,B)=A\`B\` + AB is equal to its complement.

Q.6. A majority gate is a digital circuit whose output is equal to 1 if the majority of inputs are 1\`s. The output is 0 otherwise.

(i) By means of a truth table, find the Boolean function implemented by a 3-input majority gate.

(ii) Express the 3-input majority gate as a sum of minterms and a product of maxterms.

Q.7. Express the following functions in a sum of minterms and a product of maxterms :

(i) F(X,Y,Z) = (XY + Z)(Y + XZ)

(ii) F(A,B,C,D) = D(A\` + B) + B\`D

HW #2

Q1 (i) $\bar{A} + AB + A\bar{C} = \bar{A} + B + \bar{C}$

$$
\begin{aligned}
\bar{A} + AB + A\bar{C} &= (\bar{A}+A)\cdot(\bar{A}+B) + A\bar{C} && \text{by distributivity}\\
&= 1\cdot(\bar{A}+B) + A\bar{C} && \text{by complement}\\
&= \bar{A} + B + A\bar{C} && \text{by identity}\\
&= (\bar{A}+A)\cdot(\bar{A}+\bar{C}) + B && \text{by distributivity}\\
&= 1\cdot(\bar{A}+\bar{C}) + B && \text{by complement}\\
&= \bar{A}+\bar{C}+B && \text{by identity}\\
&= \bar{A}+B+\bar{C} && \text{by commutativ.}
\end{aligned}
$$

(ii) $(A+B+D)(\bar{A}+C)(B+C+D) = (A+B+D)(\bar{A}+C)$

We can prove that this identity is true by proving that its dual is true. The dual of this identity is

$ABD + \bar{A}C + BCD = ABD + \bar{A}C$

$ABD + \bar{A}C + BCD$ is equal to $ABD + \bar{A}C$ by the consensus theorem. Let $X = BD$, then

$AX + \bar{A}C + XC = AX + \bar{A}C$ by consensus

(iii) $XZ + W\bar{Y}\bar{Z} + \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} = XZ + W\bar{Y}\bar{Z} + \bar{W}XY + \bar{X}Y\bar{Z}$

$XZ + W\bar{Y}\bar{Z} + \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} = XZ + W\bar{Y}\bar{Z} + \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} + X\bar{W}Y$

This is because $XZ + \bar{W}Y\bar{Z} = X\underline{\underline{Z}} + \bar{W}Y\underline{\underline{\bar{Z}}} + X\bar{W}Y$ by consensus

$= XZ + W\bar{Y}\bar{Z} + \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} + X\bar{W}Y + \bar{X}Y\bar{Z}$

This is also by consensus since $\underline{\underline{\bar{W}}}Y\bar{Z} + \underline{\underline{W}}\bar{X}\bar{Z}$

$= \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} + \bar{X}Y\bar{Z}$

$= XZ + W\bar{Y}\bar{Z} + W\bar{X}\bar{Z} + X\bar{W}Y + \bar{X}Y\bar{Z}$

since $\underline{\underline{X}}\bar{W}Y + \underline{\underline{\bar{X}}}Y\bar{Z} + \bar{W}Y\bar{Z} = X\bar{W}Y + \bar{X}Y\bar{Z}$ by consensus

$= XZ + W\bar{Y}\bar{Z} + X\bar{W}Y + \bar{X}Y\bar{Z} = RHS$

since $W\underline{\underline{\bar{Y}}}\bar{Z} + W\bar{X}\bar{Z} + \bar{X}\underline{\underline{Y}}\bar{Z} = W\bar{Y}\bar{Z} + \bar{X}Y\bar{Z}$

$\underline{\underline{Q2}}$ (i) $\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ)$

$= \bar{W}X(\bar{Z} + \bar{Y}) + X(W + YZ)$

$= \bar{W}X\bar{Z} + \bar{W}X\bar{Y} + XW + XYZ$

$= X(W + \bar{W}\bar{Z}) + \bar{W}X\bar{Y} + XYZ$

$= X(W + \bar{Z}) + \bar{W}X\bar{Y} + XYZ$

$= XW + X\bar{Z} + \bar{W}X\bar{Y} + XYZ$

$= X(W + \bar{W}\bar{Y}) + X(\bar{Z} + YZ)$

$= X(W + \bar{Y}) + X(\bar{Z} + Y)$

$= XW + X\bar{Y} + X\bar{Z} + XY$

$= X(\bar{Y} + Y) + XW + X\bar{Z}$

$= X + XW + X\bar{Z}$

$= X$ one literal

(ii) $[(CD)' + A]' + A + CD + AB$

$= CD\bar{A} + A + CD + AB$

$= CD + A + AB$

$= CD + A$ three literals

(iii) $(A+\bar{B}+A\bar{B})(AB+\bar{A}C+BC)$

$= AB + ABC + \bar{A}\bar{B}C$

$= AB + \bar{A}\bar{B}C$ 5 literals

Q3

(i) $wx(\bar{Y}Z+Y\bar{Z}) + \bar{w}\bar{x}(\bar{Y}+Z)(Y+\bar{Z})$

$= wx\bar{Y}Z + wxY\bar{Z} + \bar{w}\bar{x}(\bar{Y}\bar{Z}+ZY)$

$= wx\bar{Y}Z + wxY\bar{Z} + \bar{w}\bar{x}\bar{Y}\bar{Z} + \bar{w}\bar{x}ZY$

It can be seen that this expression can not be simplified

$= wx(\bar{Y}Z+Y\bar{Z}) + \bar{w}\bar{x}(\bar{Y}\bar{Z}+YZ)$

The complement of this function is:

$= [wx(\bar{Y}Z+Y\bar{Z}) + \bar{w}\bar{x}(\bar{Y}\bar{Z}+YZ)]'$

$= [(wx)' + (\bar{Y}Z+Y\bar{Z})'][(\bar{w}\bar{x})' + (\bar{Y}\bar{Z}+YZ)']$

$= [\bar{w}+\bar{x} + (Y+\bar{Z})(\bar{Y}+Z)][w+x+(Y+Z)(\bar{Y}+\bar{Z})]$

$= [\bar{w}+\bar{x}+yz+\bar{y}\bar{z}][w+x+Y\bar{Z}+\bar{Y}Z]$ 12 literals

(ii) $ABC + \bar{A}CD$

$= [AB+\bar{A}D]\cdot C$

The complement of this function is

$= \{[AB+\bar{A}D]\cdot C\}'$

$= [(\bar{A}+\bar{B})(A+\bar{D})] + \bar{C}$

$= \bar{A}\bar{D} + A\bar{B} + \bar{C}$ 5 literals

Q4 $F = \bar{A}\bar{B} + AB + \bar{B}C$

(i) with only OR and complement operations

$$F = (\bar{A}\bar{B})'' + (AB)'' + (\bar{B}C)''$$
$$= (A+B)' + (\bar{A}+\bar{B})' + (B+\bar{C})'$$

(ii) with only AND and complement operations

$$F = [\bar{A}\bar{B} + AB + \bar{B}C]''$$
$$= [(\bar{A}\bar{B})' \cdot (AB)' \cdot (\bar{B}C)']'$$

Q5 $F = \bar{A}\bar{B} + AB$

The dual of the function $F^d = (\bar{A}+\bar{B})(A+B)$

The complement of the function $\bar{F} = [\bar{A}\bar{B} + AB]'$

$= (A+B)\cdot(\bar{A}+\bar{B})$

Thus, $\bar{F} = F^d$

Q6 (i) 3-input majority gate

A	B	C	M
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(ii) $M(A,B,C) = \Sigma(m3, m5, m6, m7)$
$= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$M(A,B,C) = \Pi(M0, M1, M2, M4)$
$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)$

Q7 (i) $F(X,Y,Z) = (XY+Z)(Y+XZ)$

$= XY + XYZ + YZ + XZ$

$= XY + XZ + YZ$

$= XY(Z+\bar{Z}) + XZ(Y+\bar{Y}) + YZ(X+\bar{X})$

$= XYZ + XY\bar{Z} + XYZ + X\bar{Y}Z + XYZ + \bar{X}YZ$

$= XYZ + XY\bar{Z} + X\bar{Y}Z + \bar{X}YZ$

$= \sum(m3, m5, m6, m7)$

$= \prod(M_0, M1, M2, M4)$

$= (X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+Y+Z)$

(ii) $F(A,B,C,D) = D(\bar{A}+B) + \bar{B}D$

$= \bar{A}D + BD + \bar{B}D$

$= \bar{A}D(C+\bar{C}) + BD(A+\bar{A}) + \bar{B}D(A+\bar{A})$

$= \bar{A}BD + \bar{A}\bar{B}D + ABD + \bar{A}BD + A\bar{B}D + \bar{A}\bar{B}D$

$= \bar{A}BD + \bar{A}\bar{B}D + ABD + A\bar{B}D$

$= \bar{A}BD(C+\bar{C}) + \bar{A}\bar{B}D(C+\bar{C}) +$
$ABD(C+\bar{C}) + A\bar{B}D(C+\bar{C})$

$= \bar{A}BCD + \bar{A}B\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D$
$+ ABCD + AB\bar{C}D + A\bar{B}CD + A\bar{B}\bar{C}D$

$= \sum(m1, m3, m5, m7, m9, m11, m13, m15)$

$= \prod(M_0, M2, M4, M6, M8, M_{10}, M12, M14)$

$= (A+B+C+D)(A+B+\bar{C}+D)(A+\bar{B}+C+D)$
$(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+D)$
$(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)$