

King Fahd University of Petroleum and Minerals
College of Computer Science and Engineering
Computer Engineering Department

COE 202: Digital Logic Design (3-0-3)

Term 112 (Spring 2012)

Final Exam

Monday May 28, 2012

7:00 p.m. – 9:30 p.m.

Time: 150 minutes, Total Pages: 13

Name: KEY **ID:** _____ **Section:** _____

Notes:

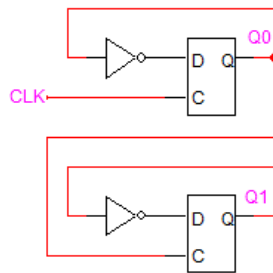
- Do not open the exam book until instructed
- Calculators are not allowed (basic, advanced, cell phones, etc.)
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

| Question | Maximum Points | Your Points |
|-----------------|-----------------------|--------------------|
| 1 | 21 | |
| 2 | 8 | |
| 3 | 10 | |
| 4 | 15 | |
| 5 | 29 | |
| 6 | 17 | |
| Total | 100 | |

Question 1.**(21 Points)**

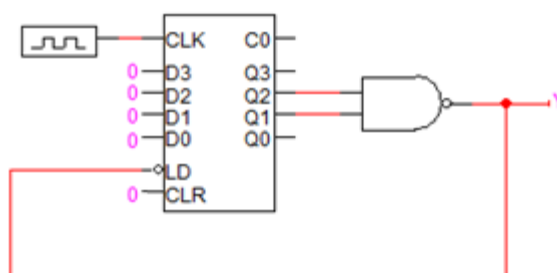
Answer the following questions by either **filling** the required spaces or **underlining** the correct answers:

- i. Given a synchronous sequential circuit with 20 states, the minimum number of flip-flops required to implement the circuit is 5 flip flops and the number of unused states is 12 states. **(3 points)**
- ii. Given a synchronous sequential circuit with 3 inputs, 3 flip-flops and 2 outputs, to implement the output and next state equations using a ROM requires a ROM with 64 locations storing 5 bits each. **(3 points)**
- iii. The following circuits implements a 2-bit (synchronous, asynchronous) down counter. **(1 point)**

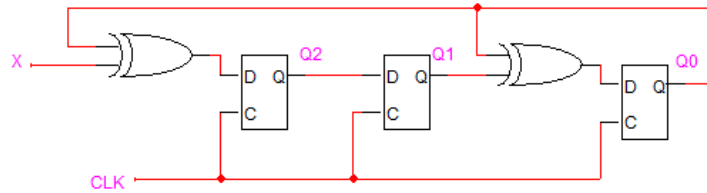


- iv. Given a 3-bit synchronous counter with outputs Q2, Q1 and Q0, assuming that the counter clock has a frequency of 16 MHz, then the frequency of Q0 is 8 MHz and the frequency of Q2 is 2 MHz. **(3 points)**

- v. For the circuit given below, assuming that its clock has a frequency of 14 MHz, then the frequency of Y is 2 MHz. **(2 points)**

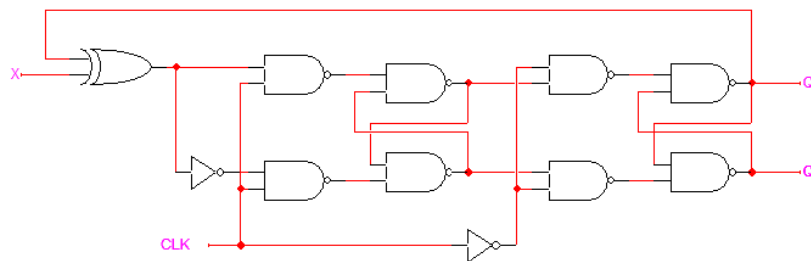


- vi. For the 3-bit register given below, given that $Q_2=0$, $Q_1=1$, $Q_0=1$, and $X=0$, the content of the register after one clock cycle will be $Q_2=1$, $Q_1=0$, $Q_0=0$. **(3 points)**

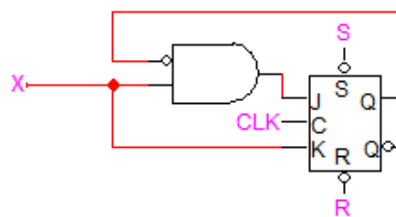


- vii. Given that $F_1=A'B'+AC$, $F_1'=A'B+AC'$ and $F_2=A'B+BC'+AB'$, $F_2'=A'B'+ABC$, to implement F_1 and F_2 using a PLA, the minimum number of AND gates needed is 3 AND gates (implementing F_1 and F_2'). **(2 points)**

- viii. The following circuit implements a (rising, falling) -edge triggered (D flip flop, T flip flop). **(3 points)**



- ix. Given the circuit below having a JK flip flop and assuming that the current state $Q=0$ and $X=1$, the next state will be 1. **(1 point)**



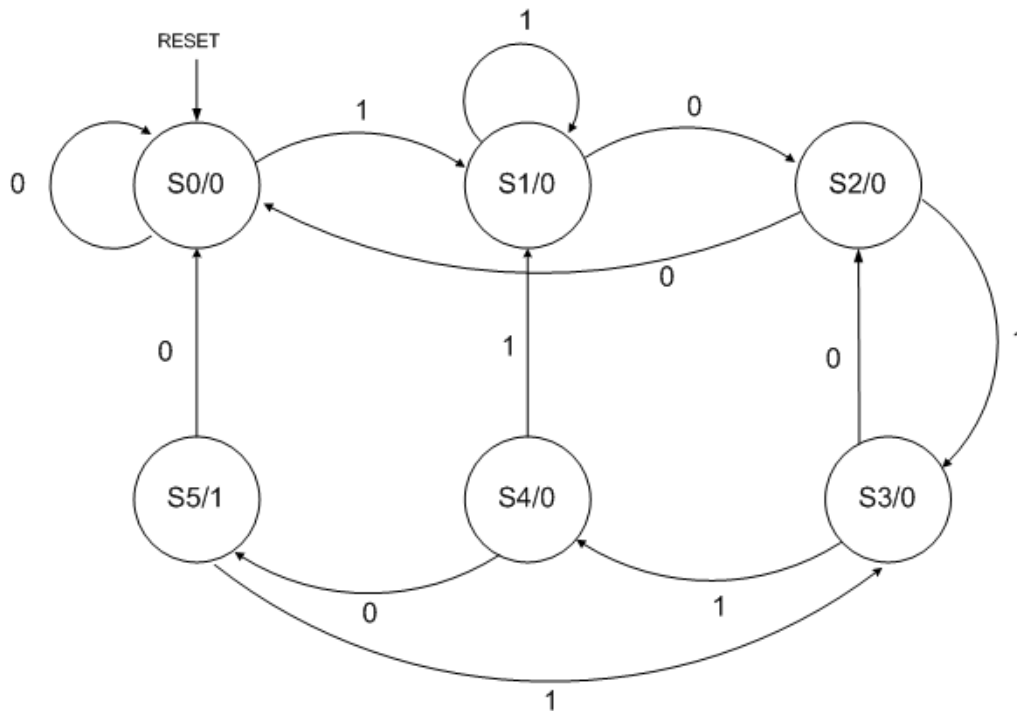
Question 2.

(8 Points)

It is required to design a sequence detector that detects overlapped occurrences of the sequence **10110**. The circuit receives a serial input **X** and produces a serial output **Z**. The output **Z** will be 1 when the circuit detects the sequence **10110**. Assume the existence of a reset input to reset the machine to a reset state. Draw the state diagram of the circuit assuming a **Moore** model. *You are not required to derive the equations and the circuit.* The following is an example of some input and output streams:

Example:

| | | |
|--------|----------|---------------------------------|
| Input | X | 0 0 1 0 1 1 0 1 1 0 1 0 1 1 0 0 |
| Output | Z | 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 1 |



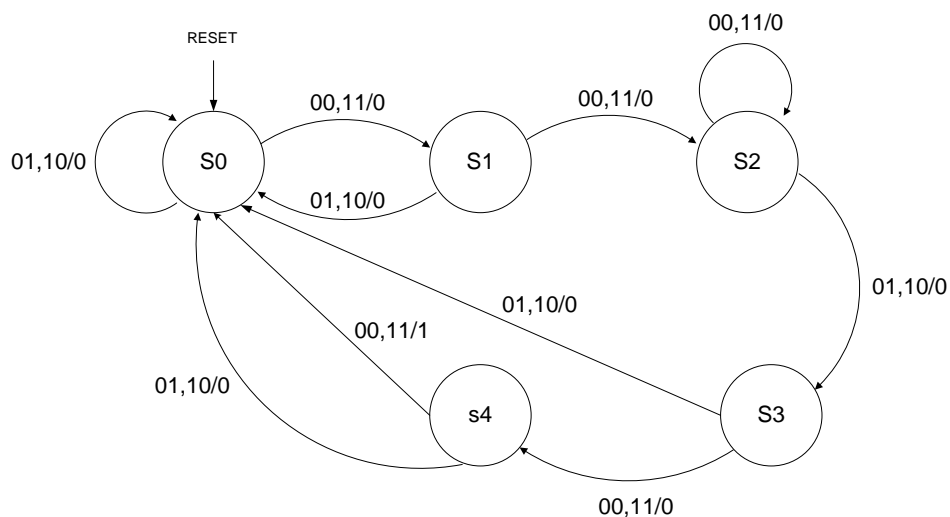
Question 3.

(10 Points)

It is required to design a sequential circuit that has two inputs X and Y and one output Z. The output equals "1" when it receives **two consecutive bits where $x=y$, followed by one bit where $x \neq y$, followed by two consecutive bits where $x=y$** . After receiving the required sequence, the circuit starts detecting these occurrences over again without overlap. The circuit has an additional reset input R which resets the circuit into the initial state. Draw the state diagram of the circuit assuming a Mealy model. *You are not required to derive the equations and the circuit.* The following is an example of some input and output streams:

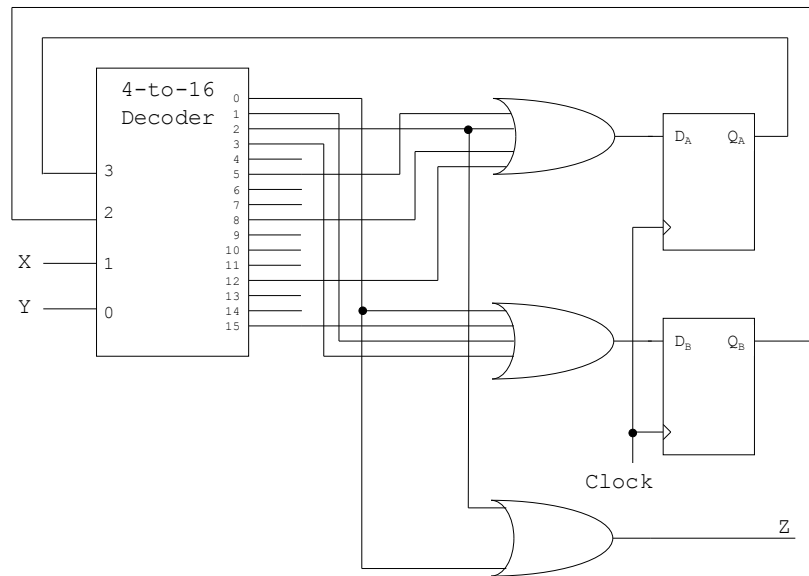
Example:

| | | | | | | | | | | | | | | |
|--------|---|---|-----|---|-----|---|-----|---|---|---|---|---|---|---|
| | | | x=y | | x≠y | | x=y | | | | | | | |
| Input | X | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| | Y | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| Output | Z | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |



Question 4.

Consider the following sequential circuit:



- a. Derive simplified Boolean expressions for the D_A and D_B inputs of the flip flops and the external output Z . **(6 points)**

Considering the decoder and using K-maps we get the following:

$$D_A = \sum m(2,5,8,12) = \bar{Q}_A \cdot \bar{Q}_B \cdot X \cdot \bar{Y} + \bar{Q}_A \cdot Q_B \cdot \bar{X} \cdot Y + Q_A \cdot \bar{X} \cdot \bar{Y}$$

$$D_B = \sum m(0,1,3,15) = \bar{Q}_A \cdot \bar{Q}_B \cdot \bar{X} + \bar{Q}_A \cdot \bar{Q}_B \cdot Y + Q_A \cdot Q_B \cdot X \cdot Y$$

$$Z = \sum m(0,2) = \bar{Q}_A \cdot \bar{Q}_B \cdot \bar{X} \cdot \bar{Y} + \bar{Q}_A \cdot \bar{Q}_B \cdot X \cdot \bar{Y} = \bar{Q}_A \cdot \bar{Q}_B \cdot \bar{Y}$$

- b. Is the circuit type *Mealy* or *Moore*? Justify your answer.

(1 point)

Since $Z = \bar{Q}_A \cdot \bar{Q}_B \cdot Y = F(\text{Present States}, \text{External Inputs}) \Rightarrow$ **Mealy**

- c. Provide a state table showing {present state and external inputs} and {next state and external output}. **(8 points)**

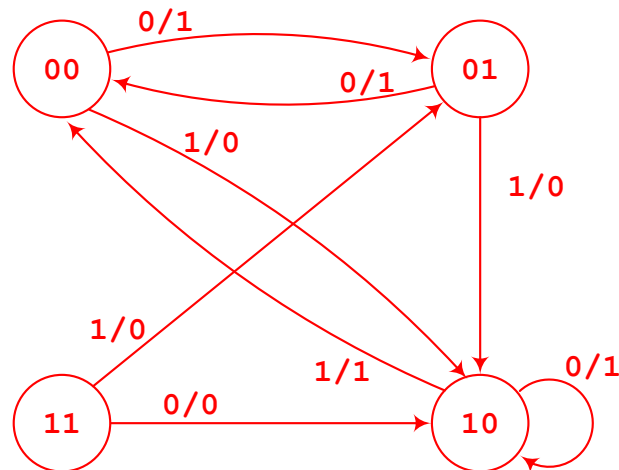
| Q_A | Q_B | X | Y | Q_A^+ | Q_B^+ | Z |
|-------|-------|-----|-----|---------|---------|-----|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 0 |

Question 5.**(29 Points)**

Consider the following state table:

| Q_A | Q_B | X | Q_A^+ | Q_B^+ | Z |
|-------|-------|-----|---------|---------|-----|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 |

1. Draw the state diagram corresponding to the given state table.

(4 points)2. If falling edge-triggered D-FF(s) and simple gates (i.e., AND, OR, NOT gates) are to be used to implement the given state table, provide the following:

a. The simplified Boolean expressions of all FF inputs.

(4 points)

Using K-maps we get the following:

$$D_A = \bar{Q}_A \cdot X + Q_A \cdot \bar{X}$$

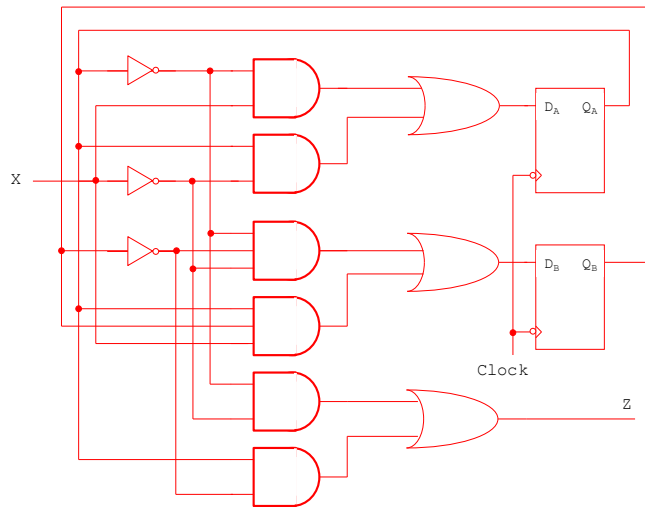
$$D_B = \bar{Q}_A \cdot \bar{Q}_B \cdot \bar{X} + Q_A \cdot Q_B \cdot X$$

b. The simplified Boolean expression of the output.

(2 points)

$$Z = \bar{Q}_A \cdot \bar{X} + Q_A \cdot \bar{Q}_B$$

- c. The logic diagram of the circuit. *Label your circuit properly.* (2 points)



3. If the same previous state table is implemented using a single ROM and a single register, provide the following:

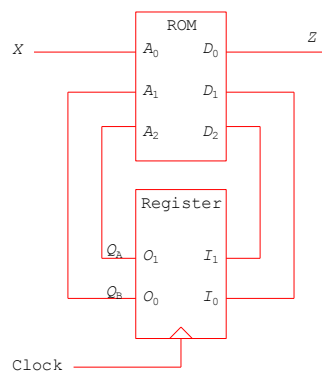
- a. The size of the required ROM, and its total capacity in bits. (3 points)

$8 \times 3 = 24 \text{ bits}$

- b. The complete ROM table. (2 points)

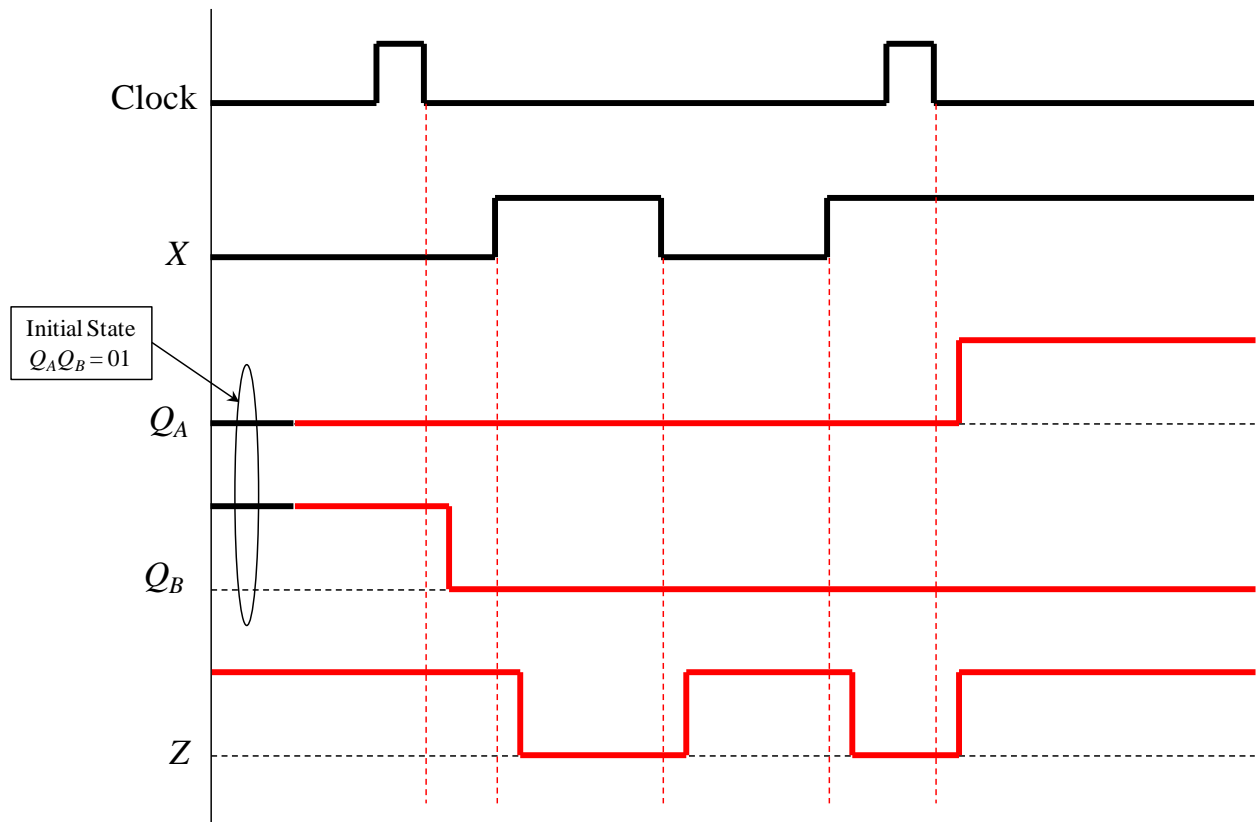
| A_2 | A_1 | A_0 | D_2 | D_1 | D_0 |
|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 |

- c. The block diagram of this implementation (**You must CLEARLY LABEL each signal and each component inputs and outputs**). (3 points)



4. Consider the same previous state table which is repeated here for your convenience. Assume that the initial state of the circuit implementation of the given state table is $(Q_A Q_B = 01)$, draw the waveforms of Q_A , Q_B , and Z in response to the shown applied input X . Assume that the circuit uses falling edge-triggered D-FF(s). (9 points)

| Q_A | Q_B | X | Q_A^+ | Q_B^+ | Z |
|-------|-------|-----|---------|---------|-----|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 |



Question 6.**(17 Points)**

A mod-16 *binary* counter consists of 4 D-Flip Flops with inputs (D_0, D_1, D_2, D_3) and *corresponding* outputs (Q_0, Q_1, Q_2, Q_3). The input equations of the counter are given by:

$$D_0 = \overline{Q_0}, \quad D_1 = Q_1 \oplus Q_0, \quad D_2 = Q_2 \oplus Q_1 Q_0, \quad \text{and} \quad D_3 = Q_3 \oplus Q_2 Q_1 Q_0$$

The counter also has a carry-out signal $C_{out} = Q_3 Q_2 Q_1 Q_0$

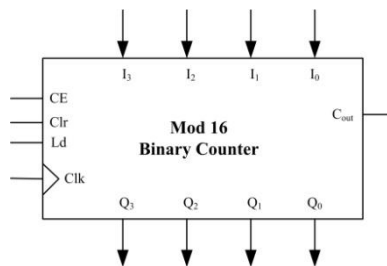
(I) Show, through drawing a logic diagram, how can the above design be modified to include:

- A count enable input “CE” such that if $CE = 0$ counting is disabled (i.e. count does not change with incoming clock pulses) while if $CE = 1$ counting is enabled. **(2 points)**
- Show how can the design in (a) be modified to include a *synchronous* clear input “CLR”, which clears the counter on the next active clock edge when $CLR=1$. **(2 points)**

- Show how can the design in (b) be modified to include a load input “LD” with additional 4 Data-In inputs I_3, I_2, I_1 and I_0 such that the counter operation is in accordance with the shown function table. **(2 points)**

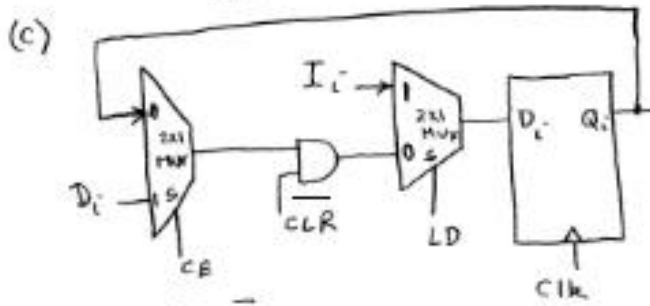
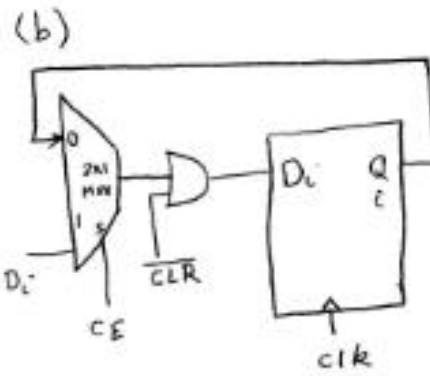
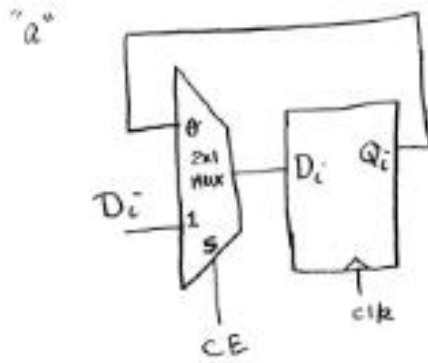
| Counter Function Table | | | | |
|------------------------|----|-----|-----|---|
| LD | CE | CLR | Clk | Function |
| 0 | 0 | 0 | X | Counting Disabled (No change in current count) |
| 0 | 1 | 0 | ↑ | Counting enabled |
| 0 | X | 1 | ↑ | Current count ← 0000 |
| 1 | X | X | ↑ | Current count ← $I_3 I_2 I_1 I_0$ |

(II) Using the mod-16 counter in (c), show how to turn this counter into a **mod-10 counter** with the same input control signals (CE, CLR, and LD) and a *proper* carry-out signal C_{out} . You may show your logic modifications using the symbol shown below. **(3 points)**



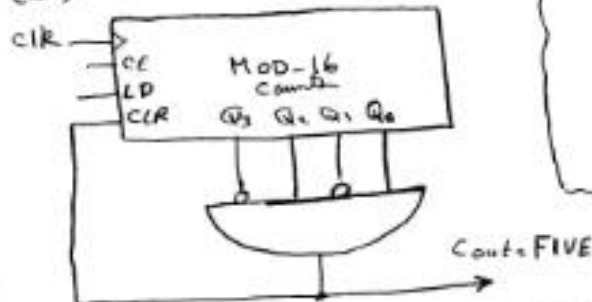
(III) Using the mod-16 counter in (c), show how to turn this counter into a **mod-6 counter** with the same input control signals (CE, CLR, and LD) and a *proper* carry-out signal C_{out} . You may show your logic modifications using the symbol shown above. **(3 points)**

(IV) Given a 1 Hz clock, show how the counters designed in (II) and (III) can be used to build a stop watch that counts seconds (0 to 59). The counter should have a C_{out} signal that may be used for a minutes counter. The outputs of the mod-10 and mod 6 counters will drive two 7-segment displays to be able to read the number of passed seconds in decimal (00 to 59). **(5 points)**

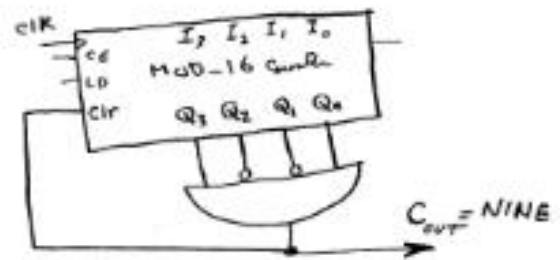


$$D_i = \begin{cases} D_0 = \bar{Q}_0 \\ D_1 = Q_1 \oplus Q_0 \\ D_2 = Q_2 \oplus Q_0 \oplus Q_1 \\ D_3 = Q_3 \oplus Q_0 \oplus Q_1 \oplus Q_2 \end{cases}$$

(III) MOD-6



(II) MOD-10



(IV)

