Question 1. **(25 points)**

a. Perform the following arithmetic operations in the specified number system. **(8 points)**

Binary Subtraction	**Hexadecimal Addition**	**Binary Addition**	**Octal Multiplication**
0 1 1 2 0 2 $\cancel{1}$000.$\cancel{10}$ −0111.01	2F3A +4BC2	1 1111 1 1 1001.11 +0110.01	1271 × 12
0001.01	7AFC Hex. Dec. A = 10 B = 11 C = 12 D = 13 E = 14 F = 15	10000.00	2562 +1271 15472

b. Give the binary, octal, and hexadecimal representations of the decimal value **130.** **(4 points)**

$\boxed{1}$ $(130)_{10} = 128 + 2$

$= 2^7 + 2^1$

$= (10000010)_2$

$\boxed{2}$ $(\underbrace{010}_{2}\ \underbrace{000}_{0}\ \underbrace{010}_{2})_2$

$\Rightarrow (130)_{10} = (202)_8$

$\boxed{3}$ $(\underbrace{1000}_{8}\ \underbrace{0010}_{2})_2$

$\Rightarrow (130)_{10} = (82)_{16}$

c. Suppose that we want to represent the decimal fraction 0.6. **(7 points)**

I. Find the 4-bit binary fraction that gives the closest value.
II. Find the 2-Digit Hexadecimal fraction that gives the closest value.
III. What do you observe?
IV. Without calculations, give the 4-digit Octal representation.

$\boxed{\text{I}}$

$$\left.\begin{aligned} 0.6 \times 2 &= \underline{1}.2 \\ 0.2 \times 2 &= \underline{0}.4 \\ 0.4 \times 2 &= \underline{0}.8 \\ 0.8 \times 2 &= \underline{1}.6 \end{aligned}\right\} \quad (0.6)_{10} \simeq (0.1001)_2$$

$\boxed{\text{II}}$

$$\left.\begin{aligned} 0.6 \times 16 &= \underline{9}.6 \\ 0.6 \times 16 &= \underline{9}.6 \end{aligned}\right\} \quad (0.6)_{10} \simeq (0.99)_{16}$$

$\boxed{\text{III}}$ - The pattern 1001 will repeat; i.e.,

$(0.6)_{10} \simeq 0.\underline{1001}\,\underline{1001}\,\underline{1001}\ldots\ldots$

- As for the hexadecimal representation, the digit $(9)_{16}$ will be repeated.

- Notice that $(1001)_2 = (9)_{16}$

$\boxed{\text{IV}}$ $(0.6)_{10} \simeq 0.\underbrace{100}_{4}\underbrace{110}_{6}\underbrace{011}_{3}\underbrace{001}_{1}$

$\simeq (0.4631)_8$

d. Write the sequence of hexadecimal numbers from 9F7 to A03. **(2 points)**

9F7	9FC	A01
9F8	9FD	A02
9F9	9FE	A03
9FA	9FF	
9FB	A00	

e. There are 26 letters in the English alphabet. Each letter has two possible shapes capital and small. Answer the following. **(4 points)**

I. If you are to assign a binary code to represent only these letters, what is the minimum number of bits needed for such encoding

II. Suggest an encoding that makes the conversion process between capital and small letters the easiest. In your suggested encoding, give the binary codes of 'A', 'B', Z', 'a', 'b', and 'z'.

$\boxed{\text{I}}$ Total number of letters $= 26 + 26 = \underline{\underline{52}}$

$\Rightarrow \boxed{2^6 = 64 > 52}$

Hence: You need exactly $\underline{\underline{6}}$ bits.

$\boxed{\text{II}}$ The "easiest" means an easy operation. Flipping a bit $(0 \rightleftarrows 1)$ is a very easy operation.

$$\text{'A'} = 000\ 000 \xrightarrow{\text{Flip the MSB}} \text{'a'} = 100\ 000$$

'B' = 000 001   'b' = 100 001

⋮

'Z' = 011 001   'z' = 111 001

Question 3. **(15 points)**

*Using **Algebraic manipulation***:

i. **(5 points)** Simplify the function $F(X,Y,Z) = \bar{X}\,\bar{Y}\,\bar{Z} + \bar{X}\,\bar{Y}\,Z + \bar{X}\,Y\,Z + X\,\bar{Y}\,Z + X\,Y\,Z$ to **three** literals.

$= \bar{X}\,\bar{Y}\,\bar{Z} + \bar{X}\,\bar{Y}\,Z + \bar{X}\,Y\,Z + X\,Y\,Z + X\,\bar{Y}\,Z + X\,Y\,Z$

$= \bar{X}\,\bar{Y}\,(\bar{Z} + Z) + Y\,Z\,(\bar{X} + X) + XZ\,(\bar{Y} + Y)$

$= \bar{X}\,\bar{Y}\,(1) + Y\,Z\,(1) + XZ\,(1)$

$= \bar{X}\,\bar{Y} + Y\,Z + XZ$

$= \bar{X}\,\bar{Y} + Y\,Z + XZ + \bar{X}Z$ $\quad$ $(by\ conasensus\ between\ \bar{X}\,\bar{Y}\ and\ Y\,Z)$

$= \bar{X}\,\bar{Y} + Y\,Z + Z(X + \bar{X})$

$= \bar{X}\,\bar{Y} + Y\,Z + Z(1)$

$= \bar{X}\,\bar{Y} + Y\,Z + Z$

$= \bar{X}\,\bar{Y} + Z$ $\quad$ $(by\ absorption)$

ii. **(5 points)** Show that the function $F(A,B,C) = \bar{A}\,\bar{B} + A\,C + B\,\bar{C}$ is equal to the function $G(A,B,C) = A\,B + \bar{A}\,\bar{C} + \bar{B}\,C$ by starting from F and reaching to G.

$= \bar{A}\,\bar{B} + A\,C + B\,\bar{C} + \bar{B}C$ $\quad$ $(by\ consesnsus\ between\ \bar{A}\,\bar{B}\ and\ A\,C)$

$= \bar{A}\,\bar{B} + A\,C + B\,\bar{C} + \bar{B}C + \bar{A}\,\bar{C}$ $\quad$ $(by\ consesnsus\ between\ \bar{A}\,\bar{B}\ and\ B\,\bar{C})$

$= \bar{A}\,\bar{B} + A\,C + B\,\bar{C} + \bar{B}C + \bar{A}\,\bar{C} + A\,B$ $\quad$ $(by\ consesnsus\ between\ A\,C\ and\ B\,\bar{C})$

$= A\,C + B\,\bar{C} + \bar{B}C + \bar{A}\,\bar{C} + A\,B$ $\quad$ $(by\ consesnsus\ between\ \bar{B}C\ and\ \ \bar{A}\,\bar{C})$

$= B\,\bar{C} + \bar{B}C + \bar{A}\,\bar{C} + A\,B$ $\quad$ $(by\ consesnsus\ between\ \bar{B}C\ and\ \ A\,B)$

$= \bar{B}C + \bar{A}\,\bar{C} + A\,B$ $\quad$ $(by\ consesnsus\ between\ \bar{A}\,\bar{C} + A\,B)$

iii. **(5 points)** Provide a <u>simplified sum-of-product (SOP)</u> expression for the <u>**complement**</u> of the function: $F(A,B,C,D) = (A + \bar{B}\,C)\bar{D} + (\bar{A} + \bar{C})(B + D)$

$$\overline{F(A,B,C,D)} = \overline{[(A + \bar{B}\,C)\bar{D}] + [(\bar{A} + \bar{C})(B + D)]}$$

$$= \overline{[(A + \bar{B}\,C)\bar{D}]}.\overline{[(\bar{A} + \bar{C})(B + D)]}$$

$$= \left[\overline{(A + \bar{B}\,C)} + D\right].\left[\overline{(\bar{A} + \bar{C})} + \overline{(B + D)}\right]$$

$$= [\bar{A}\,.(B + \bar{C}) + D].[A\,C + \bar{B}\,\bar{D}]$$

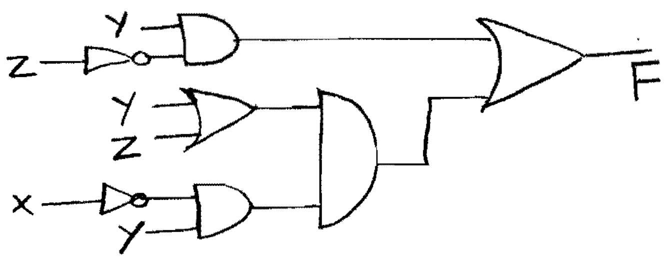
$$= \bar{A}\,.(B + \bar{C})\bar{B}\,\bar{D} + A\,C\,D$$

$$= \bar{A}\,\bar{B}\bar{C}\,\bar{D} + A\,C\,D$$

Question 3. **(10 points)**

Consider the logic function $F(X,Y,Z) = Y\overline{Z} + (Y+Z)(\overline{X}Y)$

a. Give the logic diagram for the function F as given above (without simplifications)



b. Express F(X,Y,Z) as a sum of products $F = Y\bar{Z} + \bar{X}YY + \bar{X}YZ$

$= Y\bar{Z} + \bar{X}Y + \bar{X}YZ$

(Repeat)

c. Express F as a sum of minterms in the form $F = \Sigma m(\ldots)$

$Y\bar{Z} = \bar{X}Y\bar{Z} + XY\bar{Z}$, $\bar{X}Y = \bar{X}YZ + \bar{X}Y\bar{Z}$ (Repeat)

010 110 011

$= \Sigma m(2, 3, 6)$

d. Fill in the column for F in the truth table shown

	X	Y	Z	F(X,Y,Z)	G(X,Y,Z)
0	0	0	0	0	1
1	0	0	1	0	1
2	0	1	0	1	0
3	0	1	1	1	1
4	1	0	0	0	1
5	1	0	1	0	0
6	1	1	0	1	1
7	1	1	1	0	0

e. For the function G(X,Y,Z) in the truth table above:

- Express G as a product of Maxterms in the form $G = \Pi M(\ldots)$

  $G = \Pi M(2, 5, 7)$

  010 101 111

  xyz

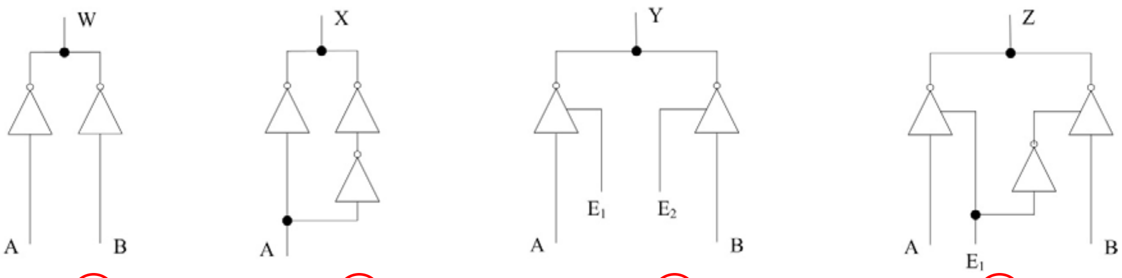
- Express G as an *algebraic* product of Maxterms, G(X,Y,Z) = $(X + \bar{Y} + Z)(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})$

- $\overline{G}(X,Y,Z) = \Pi M($ 0, 1, 3, 4, 6 $)$

Question 4. **(10 points)**

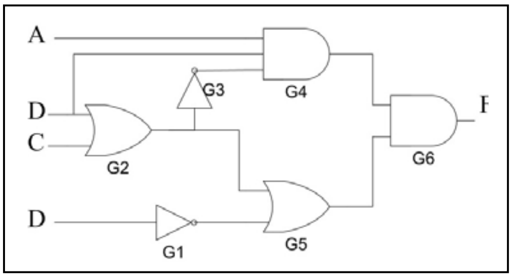
a. For each of the following four circuits, indicate whether the circuit is valid or not by circling (T) when valid or (F) when Not *where* A, B, E_1, and E_2 are independent of one another and may assume any binary values.

b. For each valid circuit, give the Boolean expression of its output.



Valid: T F	Valid: T F	Valid: T F	Valid: T F
W = ________	X = ________	Y = ________	$Z = E\,\bar{A} + \bar{E}\,\bar{B}$

c. Given the propagation delay of the basic gates; INVERTERs 1ns each, AND gates 3ns each and OR gates 5ns each answer the following:



i. What is the critical path delay (longest propagation delay along a path in this circuit)? What is the maximum frequency at which the circuit may be operated?

Critical Path Delay = G2+ G5 + G6 = 13 ns
MAX Freq = $1/(13\text{x}10^{-9}) = \approx 77\text{MHz}$

ii. The largest gate fan-in is 3 of gate G4

iii. The gate that is driving the largest load is gate ____G2____