***King Fahd University of Petroleum and Minerals***

***College of Computer Science and Engineering***

***Computer Engineering Department***

**COE 202: Digital Logic Design (3-0-3)**

**Term 142 (Spring 2014-2015)**

**Major Exam 1**

**Saturday February 28, 2015**

**Time: 90 minutes, Total Pages: 7**

**Name:\_KEY\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ID:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Section: \_\_\_\_\_\_\_**

**Notes:**

* Do not open the exam book until instructed
* **No Calculators are allowed** (*basic, advanced, cell phones, etc*.)
* Answer all questions
* All steps must be shown
* Any assumptions made must be clearly stated

|  |  |  |
| --- | --- | --- |
| **Question** | **Maximum Points** | **Your Points** |
| **1** | **25** |  |
| **2** | **20** |  |
| **3** | **10** |  |
| **Total** | **55** |  |

**Question 1. (25 points)**

1. Convert the following numbers from the given base to the other uncrossed bases listed in the table (if needed, express fractions up to 3 digits only). **(12 points)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Decimal** | **Binary** | **Octal** | **HEX** | **EXCESS-3 BCD** |
| **109.39** | **1101101.011** | **155.307** |  | **0100 0011 1100. 0110 1100** |
| **173.625** | **10101101.101** |  | **AD.A** |  |
| **231.281** |  | **347.22** | **E7.48** |  |

1. Perform the following arithmetic operations in the specified number system. **(8 points)**

|  |  |  |  |
| --- | --- | --- | --- |
| Octal Subtraction 4 5 1 2* 2 5 3 7

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 1 7 5 3  | Hexadecimal Addition F E A 3+ A F 9 D\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 1 A E 4 0 | Binary Subtraction 1 1 1 0 0 0 1 0- 1 0 1 1 1 1 1 1\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 0 0 1 0 0 0 1 1 | Binary Addition 11011011+01110111\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 101010010  |

1. Two number system with radixes $r\_{1}$ *and* $r\_{2}$*,* have the following two relations:
	1. $(69)\_{r\_{2}}= (100)\_{r\_{1}}$, *and*
	2. $(17)\_{r\_{2}}= (21)\_{r\_{1}}$

What are the values of $r\_{1}$ *and* $r\_{2}$ ? **(5 points)**

**Solution**

1. $6r\_{2}+9=r\_{1}^{2}$
2. $r\_{2}+7=2r\_{1}^{}+1$ 🡪 $r\_{2}=2r\_{1}^{}-6$ substituting in (a)

$r\_{1}^{2}-12r\_{1}^{}+27=0$ 🡪 $\left(r\_{1}-3\right)\left(r\_{1}-9\right)=0$ 🡪 **2 Solutions** $\left(r\_{1}=3\right) and \left(r\_{1}=9\right)$

1. $r\_{1}=3$ 🡪 $r\_{2}=0$ 🡪 Infeasible solution
2. $r\_{1}=9$ 🡪 $r\_{2}=12$ 🡪 Correct solution

**Question 2. (20 points)**

Use Boolean algebra to solve the following questions. Show clearly all your steps.

|  |
| --- |
| 1. Simplify each of the following Boolean functions to the specified number of literals in sum-of-products (SOP) representation:
2. $F1=x +\overline{x} y $ **(2 literals) (1 point)**

$ =\left(x +\overline{x}\right)\left(x+ y\right)=1 \left(x+y\right)=x+y $ 1. $F2=x y + \overline{x} z+y \overline{z} $ **(3 literals) (4 points)**

$$ =x y + \overline{x} z+y \overline{z}+y z \left(by consensus of x y and \overline{x} z\right) $$$$ =x y + \overline{x} z+y \left(\overline{z}+ z\right)= x y + \overline{x} z+y \left(by distributive law\right)$$$$ = \overline{x} z+y \left(by absorption xy is absrobed by y\right)$$1. $F3=x \overline{w} \overline{z}+ x \overline{w} \overline{y}+x w+x y z$ **(1 literal) (4 points)**

$$ =x \left(\overline{w} \overline{z}+ \overline{w} \overline{y}+ w+ y z\right) (by distributive law)$$$$ =x \left( \overline{z}+ \overline{w} \overline{y}+ w+ y z\right) (by simplification as w+\overline{w} \overline{z}=w+ \overline{z} )$$$$ =x \left( \overline{z}+ \overline{y}+ w+ y z\right) \left(by simplification as w+\overline{w} \overline{y}=w+ \overline{y} \right) $$$$ =x \left( \overline{z}+ \overline{y}+ w+ z\right) \left(by simplification as \overline{y}+y z= \overline{y}+z\right)$$$$ =x \left(1\right)=x \left(since \overline{z}+ z=1\right)$$1. $F4=\overbar{\left(x+\overline{y}\right)} \overbar{\left(x y+\overline{x} z\right)}$ **(3 literals) (4 points)**

 $ = \overline{x} y \left(\overline{x}+ \overline{y}\right)\left(x+ \overline{z}\right) (by Demorgan^{'}slaw) $$$ = \overline{x} y \left(x+ \overline{z}\right) (by disibutive law) $$$$ = \overline{x} y \overline{z} (by disibutive law)$$ |
| 1. Given the Boolean function $F\left(X,Y,Z\right)=\left(Y+\overbar{Z}\right)\left(\overbar{X}+Y\right)$: **(5 points)**
2. Express F as a **product-of-maxterms**, $F=\prod\_{}^{}M$.

$F=\prod\_{}^{}M(1, 4, 5)$ 1. Find the ***algebraic* sum-of-minterms** expression for *F*.

 $ F=∑m(0, 2, 3, 6, 7)$$$ =\overline{X} \overline{Y} \overline{Z}+ \overline{X} Y \overline{Z}+\overline{X} Y Z+X Y \overline{Z}+X Y Z$$1. Given the following Boolean function expressed using sum-of-products representation. $F\left(X,Y,Z\right)=X Y+ \overbar{X} Z$, express F as a product-of-sums (NOT as product-of-maxterms) representation. **(2 points)**

$ \overbar{F}=\left(\overbar{X}+\overbar{Y}\right)\left(X+ \overbar{Z}\right)$ (by Demonrgan\s law) $= \overbar{X} \overbar{Z}+X \overbar{Y}+ \overbar{Y} \overbar{Z}$ (by distributive law) $= \overbar{X} \overbar{Z}+X \overbar{Y} $ (by consensus) $F=\left(X+ Z\right)\left(\overbar{X}+Y\right) (By taking the complement of \overbar{F} using Demongan^{'}slaw)$ |
|  |

**Question 3. (10 points)**

1. Without simplification, write the Boolean algebra equation that represents F: **(2 points)**

F = EC’+ E’AB

1.
2. Fill the table based on the Logic diagram **(3 points)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Gate** | **Delay (*ns*)** | **Fanin** | **Driving Load** |
| G1 | 2 |  | 1 |
| G2 | 1 |  | 2 |
| G3 | 3 | 4 |  |
| G4 | 2 | 2 | 1 |
| G5 | 2 | 3 |  |

1. What is the worst-case delay? **(1 point)**

2+2+2=6ns

1. What is the worst-case delay path? **(1 point)**

G1, G4, G5

* 1. You are required to mark the *VIL, VIH, VOL, VOH* parameters on the following diagram given that the values of these parameters are *selected* from the set **{0.5*v*, 1.0*v*, 3.5*v*, 4.2*v*}**\***.** **(2 points)**

\*(*Voltage values are given in ascending order, i.e. not necessarily in the same order of the VIL, VIH, VOL, VOH parameters*)

|  |  |  |  |
| --- | --- | --- | --- |
| **Input voltages** |  | **Output voltages** |  |
|  |  |  |  |
|  | *4v* |  | *VOH=4.2v* |
|  | *VIH=3.5v* |  |  |
|  | *3v* |  |  |
|  |  |  |  |
|  | *2v* |  |  |
|  |  |  |  |
|  | *VIL=1.0v* |  |  |
|  |  |  | *VOL=0.5v* |
|  | *0v* |  |  |

* 1. Calculate the Noise Margin for logic 1 (NM1)? **(1 point)**

NM1= *VOH – VIH* = 4.2 – 3.5 = 0.7 v