King Fahd University of Petroleum and Minerals
College of Computer Science and Engineering
Computer Engineering Department

COE 202: Digital Logic Design (3-0-3)
Term 102 (Spring 2011)
Major Exam 1
Thursday March 17, 2011

Time: 90 minutes, Total Pages: 9

Name: Key **ID:**_____ **Section:** _____

Notes:

- Do not open the exam book until instructed
- **Calculators are not allowed** (*basic, advanced, cell phones, etc.*)
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

Question	Maximum Points	Your Points
1	25	
2	20	
3	10	
4	20	
Total	75	

Question 1. **(25 points)**

Fill in the Spaces: (Show all work needed to obtain your answer)

a. The expressions $A(B+CD)+\overline{B}\,C$ and $[A+(B\cdot(C+D))]\cdot\overline{B+C}$ are duals

b. For the logic function F(W, X, Y, Z), minterm $m_5 = X\overline{Y}Z$. F (True/False)

c. Counting the number of hours in one day in BCD requires a minimum of 8 (how many) bits.
24

d. The Boolean function $F(x,y)=\Sigma m(1,3)$ simplifies to one literal as Y.

$m_{01} + m_{11} = \overline{x}y + xy$

m_{1010} m_{1101} m_{0011} $= y(x+\overline{x})$

e. $F(A,B,C,D)=A\overline{B}C\overline{D}+AB\overline{C}D+\overline{A}\,\overline{B}CD$ is represented in the canonical shorthand form as
$F(A,B,C,D)=\Sigma m($ 3, 10, 13 $)$.
The complement $\overline{F}(A,B,C,D)=\Pi M($ 3, 10, 13 $)$

f. Assume some computer hardware that performs integer arithmetic in 5 bits. The largest decimal number that can be added to 12 without causing an incorrect result is 19.

(2^5-1) $31-12=19$

g. The decimal value of the largest 3-bit binary fraction is 0.875.

0.111 0.5 0.25 0.125

h. One factor that may limit gate fan out is propagation delay, current drive.

i. The largest 2-digit octal number has the decimal value 63.

$77)_8 = 7+7\times 8$

j. Using gates having propagation delay of 5 ns each, the input-to-output delay for a logic circuit that directly implements the logic function XYZ + WV will be 10 ns.

5 + 5

k. To implement the function $F(V, W, X, Y, Z) = \Pi(2,14,29)$ as a product of maxterms, we need ___3___ (how many) OR gates, each having ___5___(how many) inputs.

l. To represent any integer greater than 1, the binary system requires the largest number of digits among all number systems ___T___(True/False)

m. The Hi-Z logic state becomes relevant when connecting: ___iii___ (select one)

i) Inputs of logic gates together

ii) An output of a logic gate to inputs of other logic gates

iii) Outputs of logic gates together

Question 2. **(20 Points)**

a. Using up to 4-bit fractional accuracy, convert $(103.4375)_{10}$ to: **(8 Points)**

4 pts. i. Binary
2 pts. ii. Octal
2 pts. iii. Hexadecimal

i.

103	
51	1
25	1
12	1
6	0
3	0
1	1
0	1

$0.4375 \times 2 = 0.8750 \rightarrow 0$
$0.8750 \times 2 = 1.7500 \rightarrow 1$
$0.7500 \times 2 = 1.5000 \rightarrow 1$
$0.5000 \times 2 = 1.0000 \rightarrow 1$

$\Rightarrow (103.4375)_{10} = (1100111.0111)_2$

ii. $(\underline{001}\ \underline{100}\ \underline{111}.\underline{011}\ \underline{100})_2 = (147.34)_8$

iii. $(\underline{0110}\ \underline{0111}.\underline{0111})_2 = (67.7)_{16}$

b. Find the result of the following operations: **(9 Points)**

2 pts. i. $(37.4)_{16} + (59.7)_{16}$
3 pts. ii. $(37.4)_{8} + (59.7)_{16}$
2 pts. iii. $(101)_2 \times (110)_2$
2 pts. iv. $(111100)_2 - (100011)_2$

i.
$$\begin{array}{r} (37.4)_{16} \\ +(59.7)_{16} \\ \hline (90.B)_{16} \end{array}$$

ii. $(59.7)_{16} = (0101\ 1001.0111)_2$
$= (131.34)_8$

$$\Rightarrow \begin{array}{r} (37.4)_8 \\ +(131.34)_8 \\ \hline (170.74)_8 \end{array}$$

$(37.4)_8 = (011\ 111.\ 100)_2$
$= (1F.8)_{16}$

$$\Rightarrow \begin{array}{r} (1F.8)_{16} \\ +(59.7)_{16} \\ \hline (78.F)_{16} \end{array}$$

or in binary: $(1111000.1111)_2$

iii.
$$\begin{array}{r} (101)_2 \\ (110)_2 \\ \hline 000 \\ 101 \\ 101 \\ \hline (11110)_2 \end{array}$$

iv.
$$\begin{array}{r} (111100)_2 \\ -(100011)_2 \\ \hline (011001)_2 \end{array}$$

c. Determine the radix *r* for the following case: $(6A)_r = (86)_{14}$ **(3 Points)**

$$(6A)_r = (86)_{14} = 8 \times 14 + 6 = (118)_{10}$$

$$\Rightarrow 6r + 10 = 118$$

$$\Rightarrow 6r = 108$$

$$\Rightarrow \boxed{r = 18}$$

Question 3. **(10 Points)**

Prove the identity of each of the following Boolean functions using algebraic manipulation. Start with the left-hand side expression and derive from it the right-hand side expression.

(i) $\bar{a}\,\bar{c} + a\,d + b\,\bar{c}\,d = \bar{a}\,\bar{c} + a\,d$

A`C` + A D + B C` D = A`C` + A D + B C` D + C` D (by consensus between A`C` + A D)
= A`C` + A D + C` D (by absorption of B C` D in C` D)
= A`C` + A D (by consensus between A`C` + A D)

Another Solution:
A`C` + A D + B C` D = A`C` + A D + B C` D (A + A`) = A`C` + A D + A B C` D + A` B C` D
= A`C` + A D (by absorption of A B C` D in AD and absorption of A` B C` D in A` C`)

(ii) $\overline{(\bar{a}\,[\bar{c} + d] + c\,[\bar{b} + \bar{d}] + \bar{c}\,\bar{d})} = a\,d\,(b + \bar{c})$

= (a + c d`) (c` + b d) (c +d) (by Demogan's Law)

= (a c` + a b d) (c + d) (by distributive law)

= (a c` d + a b c d + a b d) (by distributive law)

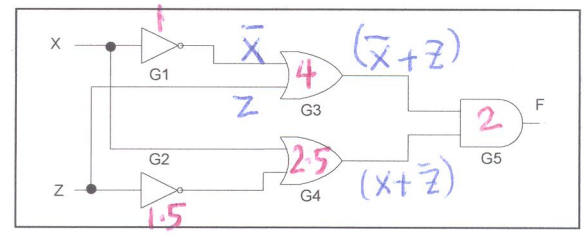
= a c` d + a b d (by absorption of a b c d in a b d)

= a d (c` + b) (by distributive law)

Question 4. **(20 Points)**

a. For the circuit shown, the propagation delay (in nano-seconds) for each gate is listed in the table below. **(5 Points)**

Gate	Propagation Delay (in Nano[1]-Seconds)
G1	1 ns
G2	1.5 ns
G3	4 ns
G4	2.5 ns
G5	2 ns



(i) What is the Boolean expression of the output function F

$F = (\bar{X}+Z)(X+\bar{Z})$ (2)

(ii) What is the worst case path delay for this circuit

Path 1 $G1 - G3 - G5 = 7$ ns

w.c. path delay = 7 ns (3)

Path 2 $G2 - G4 - G6 = 6$ ns

b. Express the following function in the sum of minterms form $\{\Sigma(m_i)\}$ and the product of Maxterms form $\{\Pi(M_i)\}$ **(6 Points)**

$F(A,B,C,D) = A'C(B'+D) = \bar{A}C\bar{B} + \bar{A}CD$ (m_3)

$= \bar{A}C\bar{B}(D+\bar{D}) + \bar{A}CD(B+\bar{B}) = \underset{m_2}{\bar{A}\bar{B}C\bar{D}} + \bar{A}\bar{B}CD + \underbrace{\bar{A}\bar{B}CD}_{m_3} + \underbrace{\bar{A}BCD}_{m_7}$

(3) $= \Sigma m(2,3,7)$

(3) $= \Pi M(0,1,4,5,6,8,9,10,11,12,13,14,15)$

0-1, 4-6, 8-15

[1] Nano = 10^{-9}

c. For the Boolean function ***F***(A, B, C, D) = $\sum$(m0, m2, m5, m7, m11, m15,) **(5 Points)**

i. Write the corresponding algebraic Boolean expression ***F(A,B,C,D)*** (without simplification)

$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}CD + ABCD$

2

ii. What is the product of Maxterms form of *F* {**Π** (M_i)}?

1

$F = \pi M(1,3,4,6,8,9,10,12,13,14)$

iii. Write the corresponding product of Maxterms Boolean expression ***F(A,B,C,D)*** (without simplification)

$F = (A+B+C+\bar{D})(A+B+\bar{C}+\bar{D})(A+\bar{B}+C+D)(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+D)$

2 $(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+D)(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$

d. Using Boolean Algebra, put the sum of minterms function ***F*(x, y, z)** into its simplest form; where ***F*(x, y, z)**= $x'y'z' + xy'z + x'yz' + x'y'z$ **(4 Points)**

$= \bar{x}\bar{z}(\bar{y}+y) + \bar{y}z(x+\bar{x})$

4 $\quad\quad 1 \quad\quad\quad 1$

$= \bar{x}\bar{z} + \bar{y}z$