HW #3

Q1. $X = AB E\bar{G} + ABFG + AB\bar{E}G + ACE\bar{G} + ACFG + AC\bar{E}G$
$+ DE\bar{G} + DFG + D\bar{E}G$

(i) Recursive Kernel Computation:

We assume that the variables are ordered in lexicographic order: $\{A, B, C, D, E, \bar{E}, F, G, \bar{G}\}$.

$i=1$: $\{A\}$

Cubes containing A : $\{ABE\bar{G}, ABFG, AB\bar{E}G, ACE\bar{G}, ACFG, AC\bar{E}G\} \geq 2$

$C = A$

Kernel found: $BE\bar{G} + BFG + B\bar{E}G + CE\bar{G} + CFG + C\bar{E}G$

Recursive call on the Kernel with $i = 2$ $\{B\}$

- $i=2$: $\{B\}$

  cubes containing B : $\{BE\bar{G}, BFG, B\bar{E}G\}$

  $C = B$

  Kernel found: $E\bar{G} + FG + \bar{E}G$

  Recursive call on the Kernel with $i=3$ $\{C\}$

  - No Kernels are found for literals $C, D, E, F, \bar{G}$
  - For literal G, the Kernel $F + \bar{E}$ is found

- $i=3$: $\{C\}$

  cubes containing C : $\{CE\bar{G}, CFG, C\bar{E}G\}$

  $C = C$

  Kernel found: $E\bar{G} + FG + \bar{E}G$

  Recursive call with $i = 4$ $\{D\}$

  - Kernel found $F + \bar{E}$

- $i=4$: $\{D\}$ No Kernels found
- $i=5$: $\{E\}$

  Cubes containing E : $\{BE\overline{G}, CE\overline{G}\}$, $C = E\overline{G}$

  Kernel found : $B + C$

- $i=6$: $\{\overline{E}\}$

  Cubes containing $\overline{E}$: $\{B\overline{E}G, C\overline{E}G\}$, $C = \overline{E}G$

  Kernel found : $B+C$

- $i=7$: $\{F\}$

  Cubes containing F : $\{BFG, CFG\}$, $C=FG$

  Kernel found : $B+C$

- $i=8$: $\{G\}$

  Cubes containing G : $\{BFG, B\overline{E}G, CFG, C\overline{E}G\}$, $C=G$

  Kernel found : $BF + B\overline{E} + CF + C\overline{E}$

  Recursive call with $i=9$ $\{\overline{G}\}$ will not produce any additional Kernels.

<u>$i=2$: $\{B\}$</u>

Cubes containing B : $\{ABE\overline{G}, ABFG, AB\overline{E}G\}$, $C = AB$

Since the cube contains literal A, no Kernel will be generated.

<u>$i=3$: $\{C\}$</u>

Cubes containing C : $\{ACE\overline{G}, ACFG, AC\overline{E}G\}$, $C = AC$

Since the cube contains literal A, no Kernel will be generated.

<u>$i=4$: $\{D\}$</u>

Cubes containing D : $\{DE\overline{G}, DFG, D\overline{E}G\}$, $C = D$

The Kernel found : $E\overline{G} + FG + \overline{E}G$

Recursive call with $i=5$ $\{E\}$. This will produce Kernel $F+\overline{E}$ when processing literal G.

$i = 5 : \{E\}$

Cubes containing E: $\{AB E\bar{G}, ACE\bar{G}, DE\bar{G}\}$, $C = E\bar{G}$

Kernel found : $AB + AC + D$

Recursive call on the Kernel with $i=6$ $\{\bar{E}\}$ will not produce any Kernel.

$i = 6 : \{\bar{E}\}$

Cubes containing $\bar{E}$: $\{AB\bar{E}G, AC\bar{E}G, D\bar{E}G\}$, $C = \bar{E}G$

Kernel found: $AB + AC + D$

Recursive call on the Kernel with $i=7$ $\{F\}$ will not produce any Kernel.

$i = 7 : \{F\}$

Cubes containing F: $\{ABFG, ACFG, DFG\}$, $C = FG$

Kernel found: $AB + AC + D$

Recursive call on the Kernel with $i=8$ $\{G\}$ will not produce any Kernel.

$i = 8 : \{G\}$

Cubes containing G: $\{ABFG, AB\bar{E}G, ACFG, AC\bar{E}G, DFG, D\bar{E}G\}$, $C = G$

Kernel found : $ABF + AB\bar{E} + ACF + AC\bar{E} + DF + D\bar{E}$

Recursive call on the Kernel with $i=9$ $\{\bar{G}\}$ will not produce any Kernel.

$i = 9 : \{\bar{G}\}$

Cubes containing $\bar{G}$: $\{ABE\bar{G}, ACE\bar{G}, DE\bar{G}\}$, $C = E\bar{G}$

Since the cube contains literal E, no Kernel will be generated.

Kernel	Co-Kernel
$BE\bar{G} + BFG + B\bar{E}G + CE\bar{G} + CFG + C\bar{E}G$	A
$E\bar{G} + FG + \bar{E}G$	AB, AC, D
$F + \bar{E}$	ACG, DG, ABG
$B + C$	$AE\bar{G}$, $A\bar{E}G$, AFG
$BF + B\bar{E} + CF + C\bar{E}$	AG
$AB + AC + D$	$E\bar{G}$, $\bar{E}G$, FG
$ABF + AB\bar{E} + ACF + AC\bar{E} + DF + D\bar{E}$	G
X	1

(ii)

	Vari	A	B	C	D	E	$\bar{E}$	F	G	$\bar{G}$
Cube	R/C	1	2	3	4	5	6	7	8	9
$ABE\bar{G}$	1	1	1			1				1
$ABFG$	2	1	1					1	1	
$AB\bar{E}G$	3	1	1				1		1	
$ACE\bar{G}$	4	1		1		1				1
$ACFG$	5	1		1				1	1	
$AC\bar{E}G$	6	1		1			1		1	
$DE\bar{G}$	7				1	1				1
DFG	8				1			1	1	
$D\bar{E}G$	9				1		1		1	

Prime Rectangle	Cube	Kernel
$(\{1,2,3,4,5,6\},\{1\})$	A	$BE\bar{G} + BFG + B\bar{E}G + CEG + CFG + C\bar{E}G$
$(\{1,2,3\},\{1,2\})$	AB	$E\bar{G} + FG + \bar{E}G$
$(\{2,3\},\{1,2,8\})$	ABG	$\bar{E} + F$
$(\{4,5,6\},\{1,3\})$	AC	$E\bar{G} + FG + \bar{E}G$

Prime Rectangle	Cube	Kernel
$(\{5,6\},\{1,3,8\})$	ACG	$\bar{E}+F$
$(\{2,3,5,6\},\{1,8\})$	AG	$BF+B\bar{E}+CF+C\bar{E}$
$(\{2,5\},\{1,7,8\})$	AFG	$B+C$
$(\{3,6\},\{1,6,8\})$	$A\bar{E}G$	$B+C$
$(\{1,4\},\{1,5,9\})$	$AE\bar{G}$	$B+C$
$(\{7,8,9\},\{4\})$	D	$E\bar{G}+FG+\bar{E}G$
$(\{8,9\},\{4,8\})$	DG	$\bar{E}+F$
$(\{1,4,7\},\{5,9\})$	$E\bar{G}$	$AB+AC+D$
$(\{2,5,8\},\{7,8\})$	FG	$AB+AC+D$
$(\{3,6,9\},\{6,8\})$	$G\bar{E}$	$AB+AC+D$
$(\{2,3,5,6,8,9\},\{8\})$	G	$ABF+AB\bar{E}+ACF+AC\bar{E}+DF+D\bar{E}$

(iii) Quick factor of x based on first level-0 Kernel

$L(A) = 6 > 1$

$C = A$

Then, we call the procedure one-level-0-Kernel

on $\frac{x}{A} = BE\bar{G} + BFG + B\bar{E}G + CE\bar{G} + CFG + C\bar{E}G$

$L(B) = 3 > 1$

Then, we call the procedure one-level-0-Kernel

on $E\bar{G} + FG + \bar{E}G$

$L(G) = 2 > 1$

This will produce the first level-0 Kernel

$F + \bar{E}$.

$(h, r) = \text{Divide}(x,\ K = F+\bar{E})$

$h = ACG + ABG + DG$

$r = ABE\overline{G} + ACE\overline{G} + DE\overline{G}$

Then, we call Quick_factor on h and r.

For h, the first level-0 kernel found is $C+B$

$\Rightarrow (h_2, r_2) = \text{divide}(h, K_2 = C+B)$

$h_2 = AG$, $r_2 = DG$

Thus, h is factored to $(C+B)AG + DG$

For r, the first level-0 kernel found is $C+B$

$\Rightarrow (h_3, r_3) = \text{divide}(r, K_3 = C+B)$

$h_3 = AE\overline{G}$, $r_3 = DE\overline{G}$

Thus, r is factored to $(C+B)AE\overline{G} + DE\overline{G}$

Thus, x is factored to:

$(F+\overline{E})((C+B)AG + DG) + (C+B)AE\overline{G} + DE\overline{G}$

<u>16</u> literals

SIS produces a different and better result shown below because it uses an improved algorithm.

$x = (G(F+\overline{E}) + E\overline{G})(A(C+B)+D)$

The algorithm used by SIS is as follows:

```
QF2 (f) {
    if ( |f| ≤ 1 ) return f
    K = one_level_0_kernel(f)
    (h, r) = divide(f, K)
```

```
if ( |h| > 1 ) {
      k = cube-free(h)
} else
      k = one-literal-of(h)
}
(h, r) = divide (f, k)
return QF2(k) QF2(h) + QF2(r)
```

Thus, initially $k = F + \bar{E}$, $h = ACG + ABG + DG$

Since $|h| > 1$, then $k = AC + AB + D$

Then, $(h, r) = \text{divide}(x, AC + AB + D)$

$h = E\bar{G} + \bar{E}G + FG$ and $r = \phi$

Then, we apply QF2 on both k and h.

For $AC + AB + D$, level-0 kernel is $C + B$

$h_2 = A$, $r_2 = D$

Since $|h_2| = 1 \Rightarrow k_2 = A$

$(h_2, r_2) = \text{divide}(AC + AB + D, A)$

$\Rightarrow h_2 = C + B$, $r_2 = D$

Thus, we get the factor $A(C+B) + D$

For $E\bar{G} + \bar{E}G + FG$, level-0-kernel is $\bar{E} + F$

$h_3 = G$, $r_3 = E\bar{G}$

Since $|h_3| = 1 \Rightarrow k_3 = G$

$(h_3, r_3) = \text{divide}(E\bar{G} + \bar{E}G + FG, G)$

$\Rightarrow h_3 = \bar{E} + F$, $r_3 = E\bar{G}$

Thus, we get the factor $G(\bar{E} + F) + E\bar{G}$

$\Rightarrow$ The factor is $(A(C+B) + D)(G(\bar{E}+F) + E\bar{G})$

Q2. $X = ACDE + BCDE + \bar{A}\bar{B}FG + \bar{A}\bar{B}\bar{F}\bar{G} + ECF + E\bar{C}G$

$+ \bar{E}C\bar{F} + \bar{E}\bar{C}\bar{G}$ $\underline{\underline{28}}$ 14.

(1) <u>Double-cube divisors</u>

Double-cube divisor	Base	weight
$A + B$	CDE	$1*2-1-2+3+2 = 4$
$AD + F$	CE	$1*3-1-3+2 = 1$
$ACD + \bar{C}G$	E	$1*5-1-5+1 = 0$
$ADE + \bar{E}\bar{F}$	C	$1*5-1-5+1 = 0$
$BD + F$	CE	$1*3-1-3+2 = 1$
$BCD + \bar{C}G$	E	$1*5-1-5+1 = 0$
$BDE + \bar{E}\bar{F}$	C	$1*5-1-5+1 = 0$
$FG + \bar{F}\bar{G}$	$\bar{A}\bar{B}$	$1*4-1-4+2 = 1$
$\bar{A}\bar{B}G + EC$	F	$1*5-1-5+1 = 0$
$\bar{A}\bar{B}F + E\bar{C}$	G	$1*5-1-5+1 = 0$
$\bar{A}\bar{B}\bar{G} + \bar{E}C$	$\bar{F}$	$1*5-1-5+1 = 0$
$\bar{A}\bar{B}\bar{F} + \bar{E}\bar{C}$	$\bar{G}$	$1*5-1-5+1 = 0$
$CF + \bar{C}G$	E	$2*4-2-4+1+1 = 4$
$EF + \bar{E}\bar{F}$	C	$1*4-1-4+1 = 0$
$EG + \bar{E}\bar{G}$	$\bar{C}$	$1*4-1-4+1 = 0$
$C\bar{F} + \bar{C}\bar{G}$	$\bar{E}$	$2*4-2-4+1+1 = 4$

(ii) Fast Extraction

From part (i), we can see that $W_{dmax} = 4$ and either of the double cube divisors $(A+B)$, $(CF+\bar{C}G)$ or $(C\bar{F}+\bar{C}\bar{G})$ can be selected.
The highest weight single cube divisor is $S = CE$ with a weigh $W_{smax} = 1$
Thus, a double cube divisor is selected and we extract $A+B$ and the resulting network is:

$$[1] = A+B$$
$$x = [1]CDE + \overline{[1]}FG + \overline{[1]}\bar{F}\bar{G} + ECF + E\bar{C}G + \bar{E}C\bar{F} + \bar{E}\bar{C}\bar{G}$$

$\underline{\underline{24}}$ lit.

Double cube divisors are updated in the same way and the double cube divisor $CF+\bar{C}G$ will be extracted as it has the highest weight of 4. Thus, the resulting network is:

$$[1] = A+B$$
$$[2] = CF+\bar{C}G$$
$$x = [1]CDE + \overline{[1]}FG + \overline{[1]}\bar{F}\bar{G} + [2]E + \overline{[2]}\bar{E}$$

$\underline{\underline{20}}$ lit.

Since none of the remaining divisors has a positive weight, none of them will be extracted.

The same result is produced by SIS.

Q3.

$$x = AC + B$$
$$Y = x + \bar{A}B$$
$$Z = Y\bar{C}$$

(i) $SDC_x = x \oplus (AC+B) = \bar{x}(AC+B) + x(\bar{A}+\bar{C})\bar{B}$

$= \bar{x}AC + \bar{x}B + x\bar{A}\bar{B} + x\bar{C}\bar{B}$

(ii) <u>Simplification of Y</u>

AB \ Cx	00	01	11	10
00	0	X	X	0
01	X	1	1	X
11	X	1	1	X
10	0	X	1	X

$Y = x$

(iii) $ODC_Y = C$

(iv)

AB \ Cx	00	01	11	10
00	0	X	X	X
01	X	1	X	X
11	X	1	X	X
10	0	X	X	X

We can see that there are two possible solutions either $Y = x$ or $Y = B$

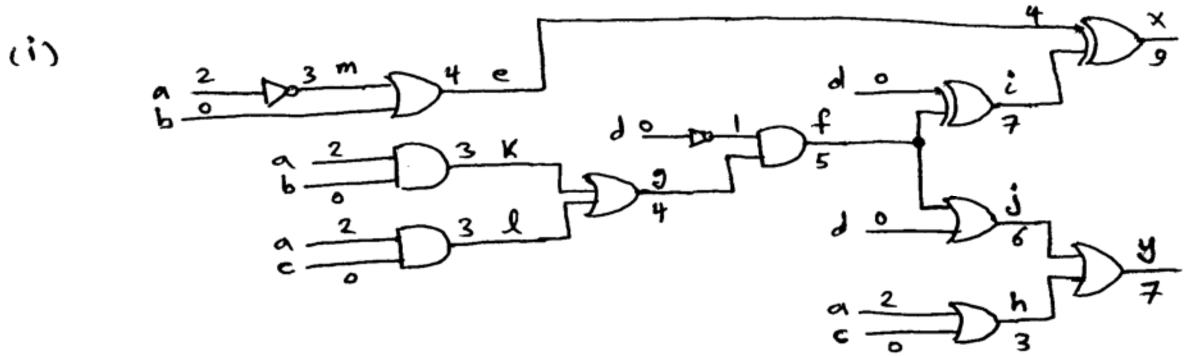
Selecting $Y = B$ will result in eliminating x.

(v) The result produced by SIS is:

$$x = AC + B$$
$$Y = x$$
$$Z = Y\bar{C}$$

Q4.

(i)



The data ready times are shown in the figure.
The maximum propagation delay is 9. To compute the slack for each node, we assume that the required time is equal to 9 for both x and y.

$s_x = 9 - 9 = 0$

$\bar{t}_e = 9 - 2 = 7$; $s_e = 7 - 4 = 3$

$\bar{t}_i = 9 - 2 = 7$; $s_i = 7 - 7 = 0$

$\bar{t}_y = 9$; $s_y = 9 - 7 = 2$

$\bar{t}_j = 9 - 1 = 8$; $s_j = 8 - 6 = 2$

$\bar{t}_h = 9 - 1 = 8$; $s_h = 8 - 3 = 5$

$\bar{t}_f = \min\{7-2, 8-1\} = 5$; $s_f = 5 - 5 = 0$

$\bar{t}_g = 5 - 1 = 4$; $s_g = 4 - 4 = 0$

$\bar{t}_k = 4 - 1 = 3$; $s_k = 3 - 3 = 0$

$\bar{t}_l = 4 - 1 = 3$; $s_l = 3 - 3 = 0$

$\bar{t}_m = 7 - 1 = 6$; $s_m = 6 - 3 = 3$

$\bar{t}_a = \min\{6-1, 3-1, 3-1, 8-1\} = 2$; $s_a = 2 - 2 = 0$

$\bar{t}_b = \min\{7-1, 3-1\} = 2$; $s_b = 2 - 0 = 2$

$\bar{t}_c = \min\{3-1, 8-1\} = 2$; $s_c = 2 - 0 = 2$

$\bar{t}_d = \min\{7-2, 8-1, 5-2\} = 3$; $s_d = 3 - 0 = 3$

(ii) The topological critical path is the one with slack = 0.

So, we have two critical paths:

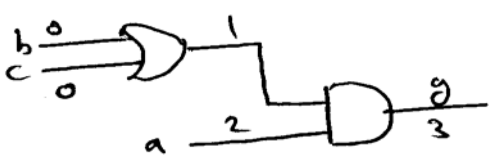
$\{a, k, g, f, i, x\}$ and

$\{a, l, g, f, i, x\}$

The maximum propagation delay is 9.

(iii) $g = ab + ac$

To improve the delay of g, it can be implemented as $g = a(b+c)$



This makes the delay of g to be reduced from 4 to 3, and the delay of the circuit to be reduced from 9 to 8.