COE 561

Dr. El-Maleh

HW #3 Solution

Q1

$$F^{ON} = \bar{a}b\bar{c} + \bar{a}\bar{c}d + a\bar{b}d + c\bar{d}$$

$$F^{DC} = \bar{a}cd$$

(i) Positional-cube notation

		a	b	c	d
ON set	$\bar{a}b\bar{c}$	10	10	10	11
	$\bar{a}\bar{c}d$	10	11	10	01
	$a\bar{b}d$	01	10	11	01
	$c\bar{d}$	11	11	01	10
DC set	$\bar{a}cd$	10	11	01	01

(ii) Off-set Computation:

1. The sharp operator

$$F^{off} = U \# \{F^{ON} \cup F^{DC}\}$$

$= \{11\ 11\ 11\ 11 \# 10\ 10\ 10\ 11\} \cap \{11\ 11\ 11\ 11 \# 10\ 11\ 10\ 01\}$
$\cap \{11\ 11\ 11\ 11 \# 01\ 10\ 11\ 01\} \cap \{11\ 11\ 11\ 11 \# 11\ 11\ 01\ 10\}$
$\cap \{11\ 11\ 11\ 11 \# 10\ 11\ 01\ 01\}$

$$= \begin{Bmatrix} 01 & 11 & 11 & 11 \\ 11 & 01 & 11 & 11 \\ 11 & 11 & 01 & 11 \end{Bmatrix} \cap \begin{Bmatrix} 01 & 11 & 11 & 11 \\ 11 & 11 & 01 & 11 \\ 11 & 11 & 11 & 10 \end{Bmatrix} \cap \begin{Bmatrix} 10 & 11 & 11 & 11 \\ 11 & 01 & 11 & 11 \\ 11 & 11 & 11 & 10 \end{Bmatrix}$$

$$\cap \begin{Bmatrix} 11 & 11 & 10 & 11 \\ 11 & 11 & 11 & 01 \end{Bmatrix} \cap \begin{Bmatrix} 01 & 11 & 11 & 11 \\ 11 & 11 & 10 & 11 \\ 11 & 11 & 11 & 10 \end{Bmatrix}$$

$$= \left\{ \begin{matrix} 01 & 11 & 11 & 11 \\ 01 & 11 & 01 & 11 \\ 01 & 11 & 11 & 10 \\ 01 & 01 & 11 & 11 \\ 11 & 01 & 01 & 11 \\ 11 & 01 & 11 & 10 \\ 01 & 11 & 01 & 11 \\ 11 & 11 & 01 & 11 \\ 11 & 11 & 01 & 10 \end{matrix} \right\} \cap \left\{ \begin{matrix} 10 & 11 & 10 & 11 \\ 10 & 11 & 11 & 01 \\ 11 & 01 & 10 & 11 \\ 11 & 01 & 11 & 01 \\ 11 & 11 & 10 & 10 \end{matrix} \right\} \cap \left\{ \begin{matrix} 01 & 11 & 11 & 11 \\ 11 & 11 & 10 & 11 \\ 11 & 11 & 11 & 10 \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} 01 & 11 & 11 & 11 \\ 11 & 01 & 11 & 10 \\ 11 & 11 & 01 & 11 \end{matrix} \right\} \cap \left\{ \begin{matrix} 10 & 11 & 10 & 11 \\ 10 & 11 & 10 & 10 \\ 10 & 11 & 10 & 01 \\ 01 & 01 & 10 & 11 \\ 11 & 01 & 10 & 11 \\ 11 & 01 & 10 & 10 \\ 01 & 01 & 11 & 01 \\ 11 & 01 & 10 & 01 \\ 01 & 11 & 10 & 10 \\ 11 & 11 & 10 & 10 \\ 11 & 11 & 10 & 10 \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} 01 & 11 & 11 & 11 \\ 11 & 01 & 11 & 10 \\ 11 & 11 & 01 & 11 \end{matrix} \right\} \cap \left\{ \begin{matrix} 10 & 11 & 10 & 11 \\ 11 & 01 & 10 & 11 \\ 01 & 01 & 11 & 01 \\ 11 & 01 & 10 & 01 \\ 11 & 11 & 10 & 10 \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} 01 & 01 & 10 & 11 \\ 01 & 01 & 11 & 01 \\ 01 & 01 & 10 & 01 \\ 01 & 11 & 10 & 10 \\ 10 & 01 & 10 & 10 \\ 11 & 01 & 10 & 10 \end{matrix} \right\} = \left\{ \begin{matrix} 01 & 01 & 10 & 11 \\ 01 & 01 & 11 & 01 \\ 01 & 11 & 10 & 10 \\ 11 & 01 & 10 & 10 \end{matrix} \right\}$$

$$= ab\bar{c} + abd + a\bar{c}\bar{d} + b\bar{c}\bar{d}$$

2 The Disjoint sharp operator:

$$
\begin{aligned}
F^{\#} &= \{11\ 11\ 11\ 11 \ \#\ 10\ 10\ 10\ 11\} \cap \{11\ 11\ 11\ 11 \ \#\ 10\ 11\ 10\ 01\} \\
&\cap \{11\ 11\ 11\ 11 \ \#\ 01\ 10\ 11\ 01\} \cap \{11\ 11\ 11\ 11 \ \#\ 11\ 11\ 01\ 10\} \\
&\cap \{11\ 11\ 11\ 11 \ \#\ 10\ 11\ 01\ 01\}
\end{aligned}
$$

$$
= \left\{\begin{array}{cccc} 01 & 11 & 11 & 11 \\ 10 & 01 & 11 & 11 \\ 10 & 10 & 01 & 11 \end{array}\right\} \cap \left\{\begin{array}{cccc} 01 & 11 & 11 & 11 \\ 10 & 11 & 01 & 11 \\ 10 & 11 & 10 & 10 \end{array}\right\} \cap \left\{\begin{array}{cccc} 10 & 11 & 11 & 11 \\ 01 & 01 & 11 & 11 \\ 01 & 10 & 11 & 10 \end{array}\right\}
$$

$$
\cap \left\{\begin{array}{cccc} 11 & 11 & 10 & 11 \\ 11 & 11 & 01 & 01 \end{array}\right\} \cap \left\{\begin{array}{cccc} 01 & 11 & 11 & 11 \\ 10 & 11 & 10 & 11 \\ 10 & 11 & 01 & 10 \end{array}\right\}
$$

$$
= \left\{\begin{array}{cccc} 01 & 11 & 11 & 11 \\ 10 & 01 & 01 & 11 \\ 10 & 01 & 10 & 10 \\ 10 & 10 & 01 & 11 \end{array}\right\} \cap \left\{\begin{array}{cccc} 10 & 11 & 10 & 11 \\ 10 & 11 & 01 & 01 \\ 01 & 01 & 10 & 11 \\ 01 & 01 & 01 & 01 \\ 01 & 10 & 10 & 10 \end{array}\right\} \cap \left\{\begin{array}{cccc} 01 & 11 & 11 & 11 \\ 10 & 11 & 10 & 11 \\ 10 & 11 & 01 & 10 \end{array}\right\}
$$

$$
= \left\{\begin{array}{cccc} 01 & 01 & 10 & 11 \\ 01 & 01 & 01 & 01 \\ 01 & 10 & 10 & 10 \\ 10 & 01 & 01 & 01 \\ 10 & 01 & 10 & 10 \\ 10 & 10 & 01 & 01 \end{array}\right\} \cap \left\{\begin{array}{cccc} 01 & 11 & 11 & 11 \\ 10 & 11 & 10 & 11 \\ 10 & 11 & 01 & 10 \end{array}\right\}
$$

$$
= \left\{\begin{array}{cccc} 01 & 01 & 10 & 11 \\ 01 & 01 & 01 & 01 \\ 01 & 10 & 10 & 10 \\ 10 & 01 & 10 & 10 \end{array}\right\} = a b \bar{c} + a b c d + a \bar{b} \bar{c} \bar{d} + \bar{a} b \bar{c} \bar{d}
$$

3. Recursive complementation procedure:

$$f^{ON} \cup f^{DC} = \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 10 & 10 & 10 & 11 \\ 10 & 11 & 10 & 01 \\ 01 & 10 & 11 & 01 \\ 11 & 11 & 01 & 10 \\ 10 & 11 & 01 & 01 \end{bmatrix} \end{array} \begin{array}{l} \\ \bar{a}\bar{b}\bar{c} \\ \bar{a}\bar{c}d \\ a\bar{b}d \\ c\bar{d} \\ \bar{a}cd \end{array}$$

- Select binate variable $a = [01 \quad 11 \quad 11 \quad 11]$

* cofactor with respect to $a = \begin{bmatrix} 11 & 10 & 11 & 01 \\ 11 & 11 & 01 & 10 \end{bmatrix} \begin{array}{l} \bar{b}d \\ c\bar{d} \end{array}$

* cofactor with respect to $\bar{a} = [10 \quad 11 \quad 11 \quad 11]$

$$= \begin{bmatrix} 11 & 10 & 10 & 11 \\ 11 & 11 & 10 & 01 \\ 11 & 11 & 01 & 10 \\ 11 & 11 & 01 & 01 \end{bmatrix} \begin{array}{l} \bar{b}\bar{c} \\ \bar{c}d \\ c\bar{d} \\ cd \end{array}$$

Then, we compute the complement of $\begin{bmatrix} 11 & 10 & 11 & 01 \\ 11 & 11 & 01 & 10 \end{bmatrix}$

- we select the binate variable d

* cofactor with respect to $d = [11 \quad 10 \quad 11 \quad 11]\ \bar{b}$

* cofactor with respect to $\bar{d} = [11 \quad 11 \quad 01 \quad 11]\ c$

So, the complement of $\begin{bmatrix} 11 & 10 & 11 & 01 \\ 11 & 11 & 01 & 10 \end{bmatrix}$ is

$$\begin{bmatrix} 11 & 01 & 11 & 01 \\ 11 & 11 & 10 & 10 \end{bmatrix} \begin{array}{l} bd \\ \bar{c}\bar{d} \end{array}$$

Next, we compute the complement of

$$\left\{\begin{matrix} 11 & 10 & 10 & 11 \\ 11 & 11 & 10 & 01 \\ 11 & 11 & 01 & 10 \\ 11 & 11 & 01 & 01 \end{matrix}\right\} \begin{matrix} \bar{b}\bar{c} \\ \bar{c}d \\ c\bar{d} \\ cd \end{matrix}$$

We select the binate variable c

* cofactor with respect to $c = \left\{\begin{matrix} 11 & 11 & 11 & 10 \\ 11 & 11 & 11 & 01 \end{matrix}\right\} \begin{matrix} \bar{d} \\ d \end{matrix}$

  The complement here is void because it depends on a single variable.

* cofactor with respect to $\bar{c} = \left\{\begin{matrix} 11 & 10 & 11 & 11 \\ 11 & 11 & 11 & 01 \end{matrix}\right\} \begin{matrix} \bar{b} \\ d \end{matrix}$

  Since the cofactor is unate $\bar{b} + d$

  its complement is $b\bar{d} + 0 = b\bar{d}$

So, the complement of $\bar{b}\bar{c} + \bar{c}d + c\bar{d} + cd$

$= \bar{c}\, b\bar{d}$

Thus, the complement of the whole function is $a\,[\,bd + \bar{c}\bar{d}\,] + \bar{a}\,[\,b\bar{c}\bar{d}\,]$

$= abd + a\bar{c}\bar{d} + \bar{a}b\bar{c}\bar{d}$

(iii) To check whether the cube $\bar{b}d$ is contained in f, we need to check if $f_{\bar{b}d}$ is tautology

$$f = \left\{ \begin{array}{cccc} 10 & 10 & 10 & 11 \\ 10 & 11 & 10 & 01 \\ 01 & 10 & 11 & 01 \\ 11 & 11 & 01 & 10 \\ 10 & 11 & 01 & 01 \end{array} \right\} \qquad \bar{b}d = \{ 11 \quad 10 \quad 11 \quad 01 \}$$

$$f_{\bar{b}d} = \begin{array}{l} \left\{ \begin{array}{cccc} 10 & 11 & 10 & 11 \\ 10 & 11 & 10 & 11 \\ 01 & 11 & 11 & 11 \\ 10 & 11 & 01 & 11 \end{array} \right\} \end{array} \begin{array}{l} \bar{a}\bar{c} \\ \bar{a}\bar{c} \\ a \\ \bar{a}c \end{array}$$

we split the cofactor $f_{\bar{b}d}$ using the binate variable a.

* cofactor with respect to a

  $\{ 11 \quad 11 \quad 11 \quad 11 \} \Rightarrow$ tautology

* cofactor with respect to $\bar{a}$

  $\left\{ \begin{array}{cccc} 11 & 11 & 10 & 11 \\ 11 & 11 & 01 & 11 \end{array} \right\} \Rightarrow$ tautology

So, the cofactor $f_{\bar{b}d}$ is a tautology and the cube $\bar{b}d$ is contained in f.

(iv) Prime implicants computation

$$f = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{c}d + ab\bar{d} + c\bar{d} + \bar{a}cd$$

Let us split the function based on the variable a.

$$f = \bar{a}\{\bar{b}\bar{c} + \bar{c}d + c\bar{d} + cd\} + a[\bar{b}d + c\bar{d}]$$

we need to find the prime implicants of $f_{\bar{a}}$ and f_a.

To find the prime implicants of f_a, we split f_a based on the variable d.

$f_{ad} = \bar{b}$, $f_{a\bar{d}} = c$

$\Rightarrow$ P.I. of f_a = scc $\{d\bar{b}, \bar{d}c, \bar{b}c\}$

$= \{d\bar{b}, c\bar{d}, \bar{b}c\}$

To find the P.I. of $f_{\bar{a}}$, we split $f_{\bar{a}}$ based on the binary variable c.

$f_{\bar{a}c} = \bar{d} + d = 1 \Rightarrow$ its P.I.'s are $\{c\}$

$f_{\bar{a}\bar{c}} = \bar{b} + d \Rightarrow$ its PIs are $\{\bar{c}\bar{b}, \bar{c}d\}$

So, the P.I.'s of $f_{\bar{a}}$ are

scc $\{c, \bar{b}\bar{c}, \bar{c}d, \bar{b}, d\} = \{\bar{b}, c, d\}$

Thus, the prime implicants of f are

scc $\{a\bar{b}d, ac\bar{d}, a\bar{b}c, \bar{a}c, \bar{a}\bar{b}, \bar{a}d,$
$\bar{b}cd, c\bar{d}, \bar{b}c, \bar{b}d, \bar{b}c\bar{d}, \bar{b}c, \bar{b}d, \bar{b}cd\}$

$= \{\bar{a}c, \bar{a}\bar{b}, \bar{a}d, c\bar{d}, \bar{b}c, \bar{b}d\}$.

(v) Essential prime implicants

$$G = f^{on} \cup f^{dc} = \bar{a}b\bar{c} + \bar{a}\bar{c}d + a\bar{b}d + c\bar{d} + \bar{a}cd$$

* $\alpha = \bar{a}c$

$$\{f^{on} \cup f^{dc}\} \# \alpha = \{\bar{a}b\bar{c}, \bar{a}\bar{c}d, a\bar{b}d, ac\bar{d}\}$$

$$H = \text{Consensus}(G \# \alpha, \alpha)$$

$$= \{\bar{a}b, \bar{a}d, \bar{b}cd, c\bar{d}\}$$

$$H_\alpha = \{\bar{b}, d, \bar{b}d, \bar{d}\} \Rightarrow \text{tautology}$$

$\Rightarrow \alpha = \bar{a}c$ is <u>not</u> an essential P.I.

* $\alpha = \bar{a}\bar{b}$

$$G \# \alpha = \{\bar{a}b\bar{c}d, a\bar{b}d, bc\bar{d}, ac\bar{d}, \bar{a}bcd\}$$

$$H = \{\bar{a}\bar{c}d, \bar{b}d, \bar{a}c\bar{d}, \bar{b}c\bar{d}, \bar{a}cd\}$$

$$H_\alpha = \{\bar{c}d, d, c\bar{d}, c\bar{d}, cd\}$$

$$= \{d, c\} \Rightarrow \text{Not tautology}$$

$\Rightarrow \alpha = \bar{a}\bar{b}$ is an essential P.I.

* $\alpha = \bar{a}d$

$$G \# \alpha = \{\bar{a}b\bar{c}\bar{d}, a\bar{b}d, c\bar{d}\}$$

$$H = \{\bar{a}b\bar{c}, \bar{b}d, \bar{a}c\}$$

$$H_\alpha = \{b\bar{c}, \bar{b}, c\} = \{\bar{b}, c\} \Rightarrow \text{Not tautology}$$

$\Rightarrow \alpha = \bar{a}d$ is an essential P.I.

* $\alpha = c\bar{d}$

$G \# \alpha = \{ \bar{a}\bar{b}\bar{c}, \bar{a}\bar{c}d, abd, \bar{a}cd \}$

$H = \{ \bar{a}b\bar{d}, a\bar{b}c, \bar{a}c \}$

$H\alpha = \{ \bar{a}\bar{b}, a\bar{b}, \bar{a} \} = \{ \bar{b}, \bar{a} \} \Rightarrow$ Not tautology

$\Rightarrow \alpha = c\bar{d}$ is an essential P.I.

* $\alpha = \bar{b}c$

$G \# \alpha = \{ \bar{a}b\bar{c}, \bar{a}\bar{c}d, a\bar{b}\bar{c}d, c\bar{d}, \bar{a}bcd \}$

$H = \{ \bar{a}\bar{b}, \bar{a}\bar{b}d, a\bar{b}d, \bar{b}\bar{d}, \bar{a}cd \}$

$H\alpha = \{ \bar{a}, \bar{a}d, ad, \bar{d}, \bar{a}d \}$

$= \{ \bar{a}, a, \bar{d} \} \Rightarrow$ Tautology

$\Rightarrow \alpha = \bar{b}c$ is <u>not</u> an essential P.I.

* $\alpha = \bar{b}d$

$G \# \alpha = \{ \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}b\bar{c}d, c\bar{d}, \bar{a}bcd \}$

$H = \{ \bar{a}\bar{b}\bar{c}, \bar{a}\bar{c}d, \bar{b}c, \bar{a}cd \}$

$H\alpha = \{ \bar{a}\bar{c}, \bar{a}\bar{c}, c, \bar{a}c \}$

$= \{ \bar{a}, c \} \Rightarrow$ Not tautology

$\Rightarrow \alpha = \bar{b}d$ is an essential P.I.

(vi) <u>Minimum Cover:</u>

We formulate the problem as a covering problem as shown below:

	$\bar{a}c$	$\overset{*}{\bar{a}\bar{b}}$	$\overset{*}{\bar{a}d}$	$\overset{*}{c\bar{d}}$	$\bar{b}c$	$\overset{*}{\bar{b}d}$
$\bar{a}\bar{b}\bar{c}\bar{d}$ ✓	0	1	0	0	0	0
$\bar{a}\bar{b}\bar{c}d$ ✓	0	1	1	0	0	1
$\bar{a}\bar{b}c\bar{d}$ ✓	1	1	0	1	1	0
$\bar{a}b\bar{c}d$ ✓	0	0	1	0	0	0
$\bar{a}bc\bar{d}$ ✓	1	0	0	1	0	0
$abc\bar{d}$ ✓	0	0	0	1	0	0
$a\bar{b}\bar{c}d$ ✓	0	0	0	0	0	1
$a\bar{b}cd$ ✓	0	0	0	0	1	1
$a\bar{b}c\bar{d}$ ✓	0	0	0	1	1	0

As, we can see from the above matrix, there are four essential P.I's : $\bar{a}\bar{b}$, $\bar{a}d$, $c\bar{d}$, $\bar{b}d$.

Selecting those four essential prime implicants covers all the minterms. So, a minimum cover is $F = \bar{a}\bar{b} + \bar{a}d + c\bar{d} + \bar{b}d$.

(vii) The same cover is found by Espresso-Exact.

(viii) The same cover is found by Espresso.

Q2

(i) Expand

		a	b	c	d	weight
m0	$\bar{a}\bar{b}\bar{c}\bar{d}$	10	10	10	10	23
✓ m1	$\bar{a}\bar{b}\bar{c}d$	10	10	10	01	24
✓ m2	$\bar{a}\bar{b}c\bar{d}$	10	10	01	10	26
✓ m3	$\bar{a}\bar{b}cd$	10	10	01	01	27
✓ m5	$\bar{a}b\bar{c}d$	10	01	10	01	21
✓ m6	$\bar{a}bc\bar{d}$	10	01	01	10	23
✓ m7	$\bar{a}bcd$	10	01	01	01	24
✓ m9	$a\bar{b}\bar{c}d$	01	10	10	01	21
✓ m10	$a\bar{b}c\bar{d}$	01	10	01	10	23
✓ m11	$a\bar{b}cd$	01	10	01	01	24
✓ m14	$abc\bar{d}$	01	01	01	10	20
	weight	7 4	7 4	4 7	5 6	

Next, we compute the implicant weights

$$\begin{bmatrix} 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 01 \\ 10 & 10 & 01 & 10 \\ 10 & 10 & 01 & 01 \\ 10 & 01 & 10 & 01 \\ 10 & 01 & 01 & 10 \\ 10 & 01 & 01 & 01 \\ 01 & 10 & 10 & 01 \\ 01 & 10 & 01 & 10 \\ 01 & 10 & 01 & 01 \\ 01 & 01 & 01 & 10 \end{bmatrix} * \begin{bmatrix} 7 \\ 4 \\ 7 \\ 4 \\ 4 \\ 7 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 23 \\ 24 \\ 26 \\ 27 \\ 21 \\ 23 \\ 24 \\ 21 \\ 23 \\ 24 \\ 20 \end{bmatrix}$$

we first expand the implicant with the minimum weight i.e. $ab c\bar{d}$.

Free set = $\{1, 3, 5, 8\}$

off-set is :

	a	b	c	d
$\bar{a} b \bar{c} \bar{d}$	10	01	10	10
$a b \bar{c} \bar{d}$	01	01	10	10
$a b \bar{c} d$	01	01	10	01
$a b c d$	01	01	01	01
$a \bar{b} \bar{c} \bar{d}$	01	10	10	10

- No column can always be raised
- Column 5 & 8 cannot be raised.

So, remaining free set = $\{1, 3\}$

overexpanded cube = $c\bar{d}$

we only need to check the minterms $\bar{a}\bar{b} c\bar{d}$, $\bar{a} b c\bar{d}$, and $a\bar{b} c\bar{d}$ for detection of feasibly covered cubes.

$sc(\bar{a}\bar{b} c\bar{d}, abc\bar{d}) = c\bar{d}$ feasible

$sc(\bar{a} b c\bar{d}, abc\bar{d}) = bc\bar{d}$ feasible

$sc(a\bar{b} c\bar{d}, abc\bar{d}) = ac\bar{d}$ feasible

So, α is expanded with $\bar{a}\bar{b} c\bar{d}$ resulting in the expanded cube $\underline{\underline{c\bar{d}}}$.

So, we remove all the cubes covered by the expanded cube, i.e. m2, m6, m10, and m14.

Next, we expand either m5 or m9 since they have the same weight. Let us expand $m5 = \bar{a} b \bar{c} d$.

Free list = $\{2, 3, 6, 7\}$

Columns 2 and 7 cannot be raised.

So, the free list is $\{3, 6\}$.

Overexpanded cube is $\bar{a} d$.

We only need to check the minterms $\bar{a}\bar{b}\bar{c}d$, $\bar{a}\bar{b}cd$, and $\bar{a}bcd$ for detection of feasibly covered cubes.

$sc(\bar{a}\bar{b}\bar{c}d, \bar{a}b\bar{c}d) = \bar{a}\bar{c}d$ feasible

$sc(\bar{a}\bar{b}cd, \bar{a}b\bar{c}d) = \bar{a}d$ feasible

$sc(\bar{a}bcd, \bar{a}b\bar{c}d) = \bar{a}bd$ feasible

So, the cube $\bar{a}d$ is selected and we remove m1, m3, m5 and m7

We then expand $m9 = a\bar{b}\bar{c}d$

Free set = $\{1, 4, 6, 7\}$

Columns 4 and 7 cannot be raised.

So, the free-set is $\{1, 6\}$.

Overexpanded cube = $\bar{b}\bar{c}$

Only minterm $a\bar{b}cd$ need to be checked for feasibility.

$sc(a\bar{b}cd, a\bar{b}\bar{c}d) = a\bar{b}d$ feasible.

So, free list = $\{1\}$. Then, minterm m11 & m9 removed.

Next, we check for raising column 1 and the expanded cube $\bar{b}d$ is found feasible.

Finally, we expand the cube $m_0 = \bar{a}\bar{b}\bar{c}\bar{d}$

Free - list $= \{2, 4, 6, 8\}$

columns 2 & 4 cannot be raised.

So, the free list is $\{6, 8\}$.

Overexpanded cube $= \bar{a}\bar{b}$

There are no feasibly covered cubes.

We raise column 8 as it overlaps the maximum number of cubes.

So, expanded cube is $\bar{a}\bar{b}\bar{c}$. The free list is $\{6\}$. Then, we raise column 6 and it is found feasible and we get the expanded cube $\bar{a}\bar{b}$.

Thus, the obtained expanded cover is

$\{c\bar{d}, \bar{a}d, \bar{b}d, \bar{a}\bar{b}\}$

Irredundant Check

The expanded cover is $\{c\bar{d}, \bar{a}d, \bar{b}d, \bar{a}\bar{b}\}$

$E' = \{c\bar{d}, \bar{a}d, \bar{b}d, \bar{a}\bar{b}\}$ as each implicant covers at least one minterm not covered by other implicants. So, all the implicants are irredundant and the cover does not change

Reduce

	a	b	c	d	weight
$c\bar{d}$	11	11	01	10	18
$\bar{a}d$	10	11	11	01	20
$\bar{b}d$	11	10	11	01	20
$\bar{a}\bar{b}$	10	10	11	11	20
	4 2	4 2	3 4	2 3	

We next compute the implicant's weight

$$\begin{bmatrix} 11 & 11 & 01 & 10 \\ 10 & 11 & 11 & 01 \\ 11 & 10 & 11 & 01 \\ 10 & 10 & 11 & 11 \end{bmatrix} * \begin{bmatrix} 4 \\ 2 \\ 4 \\ 2 \\ 3 \\ 4 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 18 \\ 20 \\ 20 \\ 20 \end{bmatrix}$$

We start by reducing the implicant with the largest weight. We will choose to reduce $\bar{a}\bar{b}$ first.

$\alpha = \overline{a}\,b$

$Q = \{ F \cup F^{DC} \} - \alpha = \{ c\overline{d}, \overline{a}d, bd \} = \overline{d}(c) + d(\overline{a} + b)$

$\overline{Q} = \overline{d}\,\overline{c} + dab$

$\overline{Q}_{\alpha} = \overline{c}\,\overline{d}$

$\overline{a}b \cap sc(\overline{c}\,\overline{d}) = \overline{a}b\overline{c}\overline{d}$

So, $\overline{a}b$ is reduced to $\overline{a}b\overline{c}\overline{d}$.

Next, assume that we select $\overline{b}d$

$Q = \{ c\overline{d}, \overline{a}d, \overline{a}b\overline{c}\overline{d} \} = \overline{d}(c + \overline{a}b\overline{c}) + d(\overline{a})$
$= \overline{d}(c + \overline{a}b) + d(\overline{a})$

$\overline{Q} = da + \overline{d}\,\overline{c}a + \overline{d}\,\overline{c}\,\overline{b}$

$\overline{Q}_{\overline{b}d} = a$

$\Rightarrow \overline{b}d \cap sc(a) = a\overline{b}d$

We then attempt to reduce $\overline{a}d$.

$Q = \{ c\overline{d}, \overline{a}b\overline{c}\overline{d}, a\overline{b}d \} = d(a\overline{b}) + \overline{d}(c + \overline{a}b\overline{c})$

$= d(a\overline{b}) + \overline{d}(c + \overline{a}b)$

$\overline{Q} = d\overline{a} + db + \overline{d}\,\overline{c}a + \overline{d}\,\overline{c}\,\overline{b}$

$\overline{Q}_{\overline{a}d} = 1$

$\Rightarrow \overline{a}d \cap sc(1) = \overline{a}d$

$\Rightarrow \overline{a}d$ cannot be reduced.

Finally, we consider reducing the cube $c\bar{d}$.

$$Q = \{\bar{a}\bar{b}\bar{c}\bar{d},\ a\bar{b}d,\ \bar{a}d\}$$

$$= \bar{d}(\bar{a}\bar{b}\bar{c}) + d(\bar{a} + a\bar{b})$$

$$= \bar{d}(\bar{a}\bar{b}\bar{c}) + d(\bar{a} + \bar{b})$$

$$\bar{Q} = \bar{d}a + \bar{d}b + \bar{d}c + dab$$

$$\bar{Q}_{c\bar{d}} = 1$$

$$\Rightarrow c\bar{d} \cap SC(1) = c\bar{d}$$

Thus, $c\bar{d}$ cannot be reduced.

So, the reduced cover is $\{c\bar{d},\ \bar{a}d,\ a\bar{b}d,\ \bar{a}\bar{b}\bar{c}\bar{d}\}$.

Expand 2:

	a	b	c	d	weight
$\bar{a}\bar{b}\bar{c}\bar{d}$	10	10	10	10	12
$a\bar{b}d$	01	10	11	01	14
$\bar{a}d$	10	11	11	01	17
$c\bar{d}$	11	11	01	10	16
	3 2	4 2	3 3	2 2	

$$\begin{bmatrix} 10 & 10 & 10 & 10 \\ 01 & 10 & 11 & 01 \\ 10 & 11 & 11 & 01 \\ 11 & 11 & 01 & 10 \end{bmatrix} * \begin{bmatrix} 3 \\ 2 \\ 4 \\ 2 \\ 3 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 14 \\ 17 \\ 16 \end{bmatrix}$$

We select the cube $\bar{a}\bar{b}\bar{c}\bar{d}$ to expand first.

Free list $= \{2, 4, 6, 8\}$

Columns 2 & 4 cannot be raised.

Overexpanded -cube is $\bar{a}\bar{b}$.

None of the cubes is feasibly covered.

So, we can choose to raise column 5 or 8

we raise column 5 and we get $\bar{a}\bar{b}\bar{d}$,

Then, we raise column 8 and we get $\bar{a}b$.

So, $\bar{a}\bar{b}\bar{c}\bar{d}$ is expanded to $\bar{a}\bar{b}$.

Next, we expand the cube $a\bar{b}d$.

Free set $= \{1, 4, 7\}$

Columns 4 & 7 cannot be raised.

So, column 1 is raised and the cube is expanded to $\bar{b}d$.

Then, we expand the cube $c\bar{d}$.

Free list $= \{5, 8\}$.

Columns 5 & 8 cannot be raised. So, the cube cannot be expanded.

Finally, we expand the cube $\bar{a}d$.

Free-list $= \{2, 7\}$

Columns 2 & 7 cannot be raised. So, the cube cannot be expanded.

So, the expanded cover is $\{\bar{a}\bar{b}, \bar{b}d, c\bar{d}, \bar{a}d\}$.

(vii) Verify your results after each step performed by running espresso tool.

<u>The initial cover is:</u>

```
.i 4
.o 1
.ilb a b c d
.olb y
.p 11
0000 1
0001 1
0010 1
0011 1
0101 1
0110 1
0111 1
1001 1
1010 1
1011 1
1110 1
.dc
.e
```

<u>Running EXPAND on the Initial cover:</u>

```
54 sunfire7> espresso -D expand two.pla
.olb y
.dc
.i 4
.o 1
.ilb a b c d
.p 4
--10 1
-0-1 1
0--1 1
00-- 1
.e
55 sunfire7>
```

So the expanded cover is {cd`, b`d, a`d, a`b`} which is the same as we obtained.

<u>Running IRREDUNDANT check on the expanded cover</u>:

```
62 sunfire7> espresso -Dexpand two.pla > two_expanded.pla
63 sunfire7> espresso -Dirred two_expanded.pla
.olb y
.dc
.i 4
.o 1
.ilb a b c d
.p 4
--10 1
-0-1 1
0--1 1
00-- 1
.e
```

So the resulted cover is {cd`, b`d, a`d, a`b`} which is the same as we obtained.

Running REDUCE on the irredundant checked cover:

```
64 sunfire7>
64 sunfire7> espresso -Dirred two_expanded.pla > two_irred.pla
65 sunfire7> espresso -Dreduce two_irred.pla
.olb y
.dc
.i 4
.o 1
.ilb a b c d
.p 4
0000 1
--10 1
01-1 1
-0-1 1
.e
66 sunfire7>
```

So the Reduced cover is {a\`b\`c\`d\`, cd\`, a\`bd, b\`d}
Note that the reduced cover we obtained manually is {a\`b\`c\`d\`, cd\`. a\`d, ab\`d}

This because we had three implicants with same weight (i.e 18) so the order in which we start reducing matter. So if we started with a\`b\` then a\`d we would have a\`b\` reduced to a\`b\`c\`d\` and a\`d reduced to a\`bd. The implicates cd\` and b\`d will not be reduced also. So we would have obtained the same result as ESPRESSO. But it is ok since reduce is not unique.

Running EXPAND on the reduced cover:

```
67 sunfire7> espresso -Dreduce two_irred.pla > two_reduced.pla
68 sunfire7> espresso -Dexpand two_reduced.pla
.olb y
.dc
.i 4
.o 1
.ilb a b c d
.p 4
00-- 1
0--1 1
--10 1
-0-1 1
.e
```

So the expanded cover is {a\`b\`, a\`d, cd\`, b\`d} which is the same as we obtained.