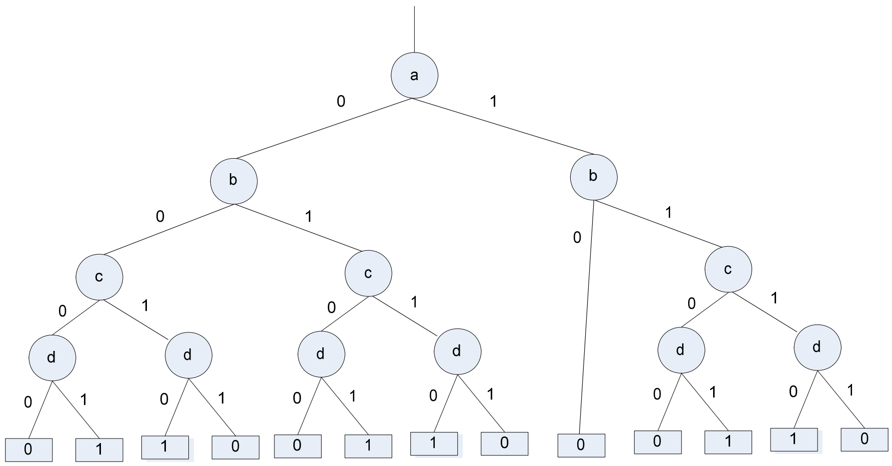
COE 561, Term 081

Digital System Design and Synthesis

HW# 1

Due date: Tuesday, Nov. 11

Q.1. Consider the following OBDD with the variable ordering {a, b, c, d}. Reduce it based on **Reduce** function to obtain the ROBDD. Show the details of your work.



Q.2. Consider the functions f1=ab+ac+bc, f2=a(b⊕c)+bc and f3=a(a⊕b)'+c(a⊕b):

(i) Draw the **ROBDD** for the functions **f1, f2** and **f3** using the variable order {a, b, c}.

(ii) What do you conclude from the results obtained in (i).

Q.3. Consider the two functions f=a⊕b⊕c and g=ab+ac+bc.

(i) Compute the function f⊕g.

(ii) Draw the **ITE DAG** for the function f⊕g. Show the details of the ITE algorithm step by step. Use the variable ordering {a, b, c}

Q.4. Consider the following given matrix representing a covering problem:

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Find a **minimum cover** using **EXACT_COVER** procedure. Show all the details of the algorithm. Assume the following order in branching selection when needed: C_1, C_2, C_3, C_4, C_5, C_6.

Q.5. Consider the function $F(A,B,C) = AB + \overline{A}C + \overline{B}\,\overline{C}$.

(i) Represent the function using **positional cube notation**.

(ii) Using positional cube notation, compute the **cofactor** F_A.

(iii) Using positional cube notation, compute the **consensus** between the two cubes $\overline{A}C$ and $\overline{B}\,\overline{C}$.

(iv) Using positional cube notation, based on the **sharp** operation, compute the complement of the function F.

(v) Using positional cube notation, determine if the cube BC is **covered** by the function $F = AB + \overline{A}C + \overline{B}\,\overline{C}$.

Q.6. Consider the function $F(A,B,C,D) = \overline{A}\,\overline{C} + A\overline{B} + \overline{A}\,\overline{B}C + \overline{A}C\overline{D}$:

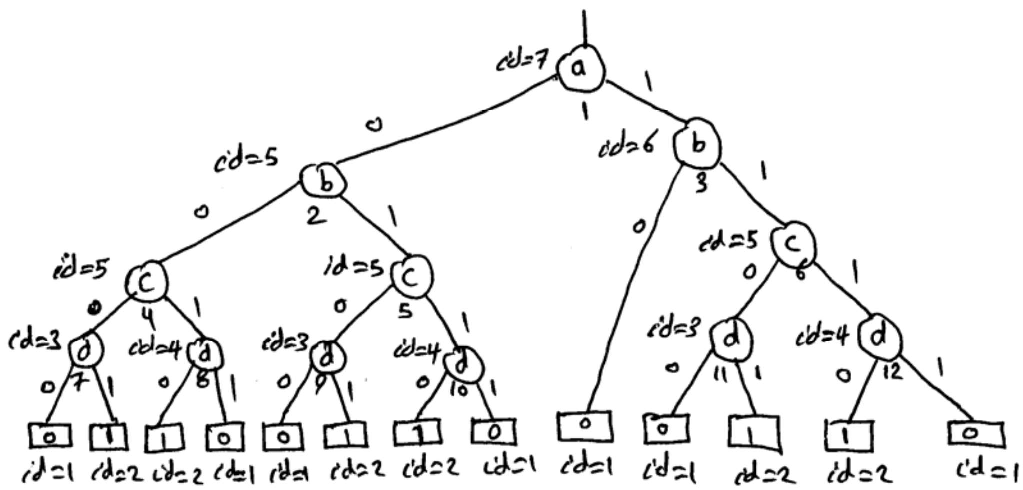
(i) Compute the **complement** of the function using the recursive complementation procedure outlined in section 7.3.4.

(ii) Compute all the **prime implicants** of the function using the method outlined in section 7.3.4.

Note that you do not need to use the positional cube notation in your solution of this question.

HW #1

Q1.



First, we set $id(v) = 1$ for all leaf vertices with value 0 and $id(v) = 2$ for leaf vertices with value 1.

We initialize ROBDD with two leaf vertices for 0 and 1.

Then, we process vertices at level 4, i.e nodes with index $= d$.

$V = \{7, 8, 9, 10, 11, 12\}$.

None of the vertices is removed since $id(low(v)) \neq id(high(v))$.

We assign keys to all vertices $\in V$.

$Key(7) = (1,2)$, $Key(8) = (2,1)$, $Key(9) = (1,2)$,
$Key(10) = (2,1)$, $Key(11) = (1,2)$, $Key(12) = (2,1)$.

$oldKey = (0,0)$.

We next sort the vertices in V according to their keys. Thus, $V = \{7, 9, 11, 8, 10, 12\}$.

$v = \{7\}$: Since $key(7) \neq oldKey$, $nextid = 3$, $id(7) = 3$
$oldKey = (1,2)$. We add $v = \{7\}$ to the ROBDD.

$v = \{9\}$: Since $Key(9) = oldKey$, $id(8) = 3$.

$v = \{11\}$: Since $Key(11) = oldKey$, $id(11) = 3$.

$v = \{8\}$: Since $Key(8) \neq oldKey$, $nextid = 4$. $Key(8) = 4$
$oldKey = (2,1)$. We add $v = \{8\}$ to the ROBDD.

$v = \{10\}$: Since $Key(10) = oldKey$, $id(10) = 4$.

$v = \{12\}$: Since $Key(12) = oldKey$, $id(12) = 4$.

Next, we process vertices at level 3 with index = c.

$V = \{4, 5, 6\}$

None of the vertices is removed since $id(low(v)) \neq id(high(v))$.

We assign Keys to all vertices $\in V$.

$Key(4) = (3,4)$, $Key(5) = (3,4)$, $Key(6) = (3,4)$.

$oldKey = (0,0)$

We sort the vertices according to their Keys.

$V = \{4, 5, 6\}$.

$v = \{4\}$: Since $Key(4) \neq oldKey$, $nextid = 5$, $id(4) = 5$,
$oldKey = (3,4)$ We add $v = \{4\}$ to the ROBDD.

$v = \{5\}$: Since $Key(5) = oldKey$, $id(5) = 5$.

$v = \{6\}$: Since $Key(6) = oldKey$, $id(6) = 5$.

Next, we process vertices at level 2 with index = b.

$V = \{2, 3\}$.

Since $id(low(2)) = id(high(2)) = 5$, $id(2) = 5$ and vertex 2 is removed from V.

$Key(3) = (1,5)$, $oldKey = (0,0)$

Since $Key(3) \neq oldKey$, $nextid = 6$, $id(3) = 6$,
$oldKey = (1,5)$. We add $v = \{3\}$ to the ROBDD.

Finally, we process vertices at level 1 with index = a.

$V = \{1\}$.

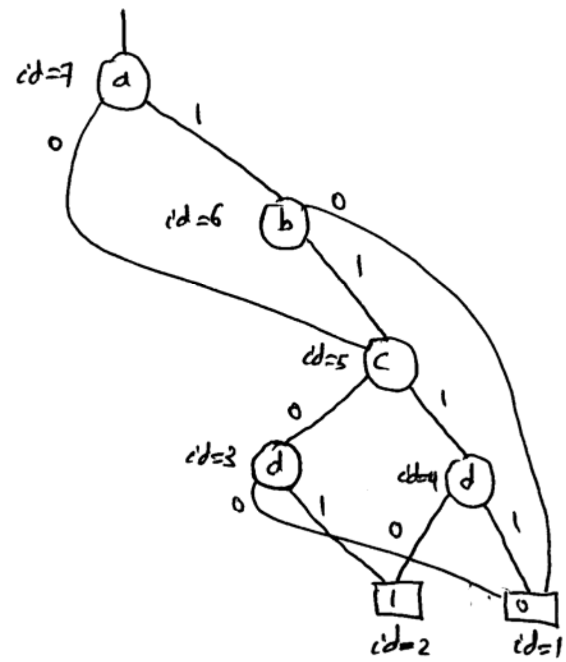
Since $id(low(1)) \neq id(high(1))$, the vertex is not removed.

$key(1) = (5,6)$, $oldkey = (0,0)$.

since $key(1) \neq oldkey$, $nextid = 7$, $id(1) = 7$.

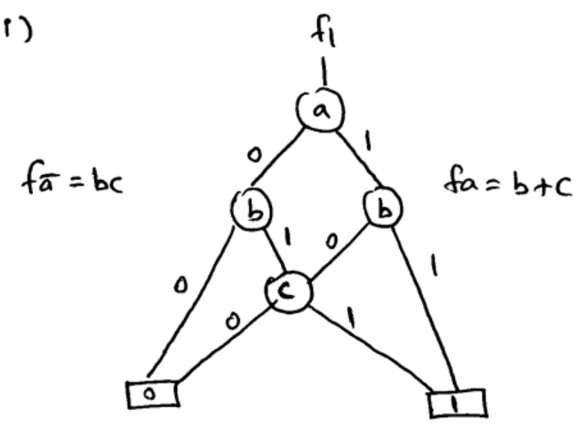
we add $v = \{1\}$ to the ROBDD.

Thus, the ROBDD formed is:



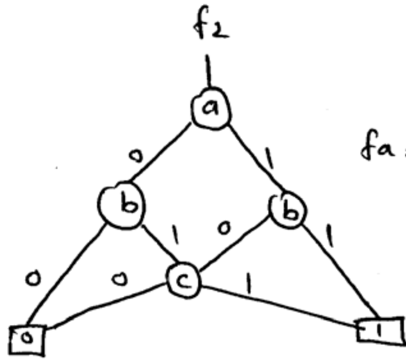
Q2.

(1)



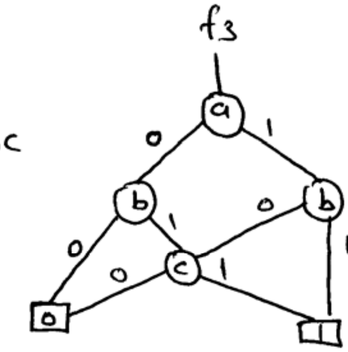
$f_{\bar{a}} = bc$

$f_a = (b \oplus c) + bc$



$f_{\bar{a}} = bc$

$f_a = b + \bar{b}c$



(ii) Since we have obtained equivalent ROBDDs for the three functions, we conclude that $f_1 = f_2 = f_3$.

Q3

(i) We first express the two functions f and g using the orthonormal basis; $\bar{a}\bar{b}$, $\bar{a}b$, $a\bar{b}$, ab

$$f = \bar{a}\bar{b}\,(c) + \bar{a}b\,(\bar{c}) + a\bar{b}\,(\bar{c}) + ab\,(c)$$

$$g = \bar{a}\bar{b}\,(0) + \bar{a}b\,(c) + a\bar{b}\,(c) + ab\,(1)$$

$$f \oplus g = \bar{a}\bar{b}\,(c \oplus 0) + \bar{a}b\,(\bar{c} \oplus c) + a\bar{b}\,(\bar{c} \oplus c) + ab\,(c \oplus 1)$$

$$= \bar{a}\bar{b}\,(c) + \bar{a}b\,(1) + a\bar{b}\,(1) + ab\,(\bar{c})$$

$$= \bar{a}\bar{b}c + \bar{a}b + a\bar{b} + ab\bar{c}$$

(ii) ITE diagram for the function $f \oplus g$

$f \oplus g = ITE(f, \bar{g}, g)$

$= ITE(a \oplus b \oplus c, \overline{ab+ac+bc}, ab+ac+bc)$

- $x = a$

$t = ITE(\overline{b \oplus c}, \overline{b+c}, b+c)$

- $x = b$

$t = ITE(c, 0, 1) = \bar{c}$ (trivial case)

we assign $id = 3 \Rightarrow t = 3$

$e = ITE(\bar{c}, \bar{c}, c) = 1$ (trivial case)

$\Rightarrow e = 2$

since $t \neq e$, an entry will be added in the table for $(b, 3, 2)$ with $id = 4$

$t = 4$.

$e = ITE(b \oplus c, \overline{bc}, bc)$

- $x = b$

$t = ITE(\bar{c}, \bar{c}, c) = 1$ (trivial case)

$\Rightarrow t = 2$

$e = ITE(c, 1, 0) = c$ (trivial case)

we assign $id = 5 \Rightarrow e = 5$

since $t \neq e$, an entry will be added in the table for $(b, 2, 5)$ with $id = 6$

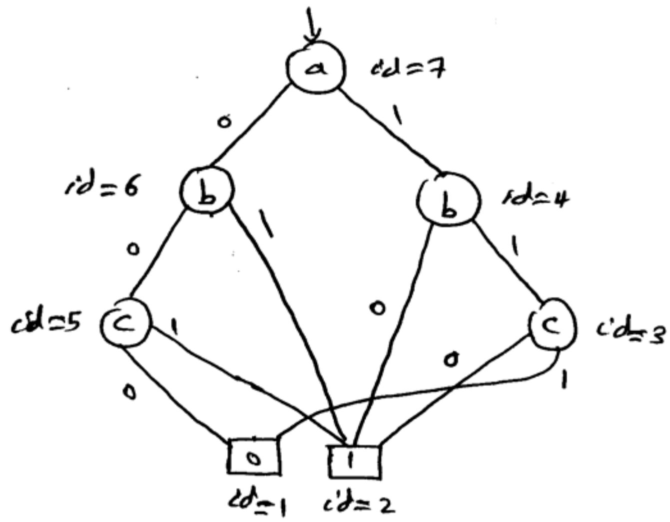
$e = 6$

Since $t \neq e$, an entry will be added in the table for $(a, 4, 6)$, with $id = 7$.

Thus, the unique table produced is:

id	var	right child	left child
3	c	1	2
4	b	3	2
5	c	2	1
6	b	2	5
7	a	4	6

The corresponding ITE DAG is ;



Q4. The matrix to be covered :

	c_1	c_2	c_3	c_4	c_5	c_6
r_1	1	0	1	1	1	0
r_2	1	0	0	0	0	1
r_3	0	1	0	1	1	0
r_4	0	1	0	1	0	0
r_5	1	1	0	1	0	1
r_6	0	1	1	0	1	0

Initially, there are no essential columns.

c_1 dominates c_6 and hence c_6 is removed

c_5 dominates c_3 and hence c_3 is removed

r_3 dominates r_4 and hence r_3 is removed

r_5 dominates r_2 and hence r_5 is removed

The reduced matrix is :

	c_1	c_2	c_4	c_5
r_1	1	0	1	1
r_2	1	0	0	0
r_4	0	1	1	0
r_6	0	1	0	1

we can see now that c_1 is essential and it has to be selected and r_1 and r_2 are removed since they are coverd.
The obtained reduced matrix will be :

	c_2	c_4	c_5
r_4	1	1	0
r_5	1	0	1

Now we see that c_2 dominates c_4 and it also dominates c_5. Thus, c_4 and c_5 are removed and
c_2 becomes essential and it is selected.
Thus, the selected columns are c_1 and c_2 and the returned solution is $x = (1, 1, 0, 0, 0, 0)$.
Note that in this problem, there was no need for branching as applying reduction techniques led to the minimum cover.

Q5. $F = AB + \bar{A}C + \bar{B}\bar{C}$

(i) Positional cube notation:

	A	B	C
AB	01	01	11
$\bar{A}C$	10	11	01
$\bar{B}\bar{C}$	11	10	10

(ii) Cofactor F_A

	A	B	C
B	11	01	11
$\bar{B}\bar{C}$	11	10	10

$F_A = B + \bar{B}\bar{C}$

(iii) Consensus between $\bar{A}C$ and $\bar{B}\bar{C}$

	A	B	C	
$\bar{A}C$	10	11	01	
$\bar{B}\bar{C}$	11	10	10	
	11	10	00	void
	10	11	00	void
$\bar{a}\bar{b}$	10	10	11	

(iv) Complement of F using sharp operation

First, we compute $U \# AB$

	A	B	C	
U	11	11	11	
AB	01	01	11	
$\bar{A}$	10	11	11	
$\bar{B}$	11	10	11	
	11	11	00	void

Then, we compute $\{\bar{A} + \bar{B}\} \# \bar{A}C$

	A	B	C	
$\bar{A}$	10	11	11	
$\bar{A}C$	10	11	01	
	00	11	11	void
	10	00	11	void
$\bar{A}\bar{C}$	10	11	10	

	A	B	C	
$\bar{B}$	11	10	11	
$\bar{A}C$	10	11	01	
$A\bar{B}$	01	10	11	
	11	00	11	void
$\bar{B}\bar{C}$	11	10	10	

Thus, we obtained the result $\{\bar{A}\bar{C}, A\bar{B}, \bar{B}\bar{C}\}$

Then, we perform $\{\bar{A}\bar{C}, A\bar{B}, \bar{B}\bar{C}\} \# \bar{B}\bar{C}$

	A	B	C	
$\bar{A}\bar{C}$	10	11	10	
$\bar{B}\bar{C}$	11	10	10	
	00	11	10	void
$\bar{A}B\bar{C}$	10	01	10	
	10	11	00	void

	A	B	C	
$A\bar{B}$	01	10	11	
$\bar{B}\bar{C}$	11	10	10	
	00	10	11	void
	01	00	11	void
$A\bar{B}C$	01	10	01	

	A	B	C	
$\bar{B}\bar{C}$	11	10	10	
$\bar{B}\bar{C}$	11	10	10	
	00	10	10	void
	11	00	10	void
	11	10	00	void

Thus, the obtained result is $\bar{A}B\bar{C} + A\bar{B}C$

(v) To determine if BC is covered by F, we need to compute F_{BC} and check if it is tautology.

F_{BC} is :

A	B	C
01	11	11
10	11	11

which is tautology. Thus, BC is covered by F.

Q6. $F = \bar{A}\bar{C} + A\bar{B} + \bar{A}\bar{B}C + \bar{A}C\bar{D}$

(i) Complement of the function

we will expand on unate variables

$$F = B\,F_B + F_{\bar{B}}$$

$$= B\,[\bar{A}\bar{C} + \bar{A}C\bar{D}] + [\bar{A}\bar{C} + A + \bar{A}C + \bar{A}C\bar{D}]$$

$$= B\,[A\,[0] + [\bar{C} + C\bar{D}]]$$

$$+ [D\,[\bar{A}\bar{C} + A + \bar{A}C] + \bar{D}\,[\bar{A}\bar{C} + A + \bar{A}C + \bar{A}C]]$$

$$= B\,[A\,[0] + [D\,[\bar{C}] + [\bar{C} + C]]]$$

$$+ [D\,[\bar{A}\,[\bar{C} + C] + A\,[1]]$$

$$+ \bar{D}\,[\bar{A}\,[\bar{C} + C] + A\,[1]]]$$

$$= B\,[A\,[0] + [D\,[\bar{C}] + 1]]$$

$$+ [D\,[\bar{A}\,[1] + A\,[1]]$$

$$+ \bar{D}\,[\bar{A}\,[1] + A\,[1]]]$$

Thus, $\bar{F} = B\,[A\,[1] + [D\,[C] + 0]]$

$$+ [D\,[\bar{A}\,[0] + A\,[0]]$$

$$+ \bar{D}\,[\bar{A}\,[0] + A\,[0]]]$$

$$= BA + BDC$$

(ii) Prime implicants

$$F = \bar{A}\ [\ \bar{C} + \bar{B}C + C\bar{D}\ ] + A\ [\bar{B}]$$

$$= \bar{A}\ [\ \bar{C}\,[1] + C\,[\bar{B} + \bar{D}]] + A\,[\bar{B}]$$

Prime implicants for $F_{\bar{A}\bar{C}} = \{1\}$

prime implicants for $F_{\bar{A}C} = \{\bar{B}, \bar{D}\}$

prime implicants for $F_{\bar{A}} \overset{scc}{=} \{\bar{C}, C\bar{B}, C\bar{D}, \bar{B}, \bar{D}\}$

$$= \{\bar{C}, \bar{B}, \bar{D}\}$$

prime implicants for $F_A = \{\bar{B}\}$

prime implicants for $F = scc\{A\bar{B}, \bar{A}\bar{C}, \bar{A}\bar{B}, \bar{A}\bar{D}, \bar{B}\bar{C}, \bar{B}, \bar{B}\bar{D}\}$

$$= \{\bar{A}\bar{C}, \bar{A}\bar{D}, \bar{B}\}.$$