

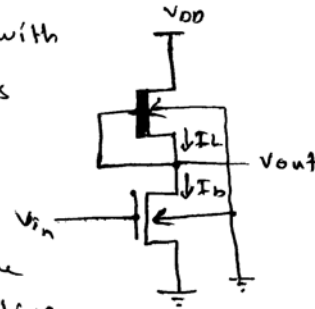
**COE 360 Principles of VLSI Design**  
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**Lecture#14**

**Inverter with Depletion-Type NMOS Load**

Inverter with Depletion-Type nMOS load

In this inverter, the driver is an enhancement-type nmos transistor with  $V_{T0, driver} > 0$ , whereas the load is a depletion-type nmos transistor with  $V_{T0, load} < 0$ .



$V_{GS, load} = 0 > V_{T, load}$ , thus the load device always has a conducting channel regardless of the input and output levels.

Also, note that  $V_{SB, load} = V_{out} \neq 0$ , thus the load device is subject to the substrate-bias effect and its threshold voltage is

$$V_{T, load} = V_{T0, load} + \gamma \left[ \sqrt{2\phi_b + V_{out}} - \sqrt{2\phi_b} \right]$$

The operating mode of the load transistor is determined by the output voltage level.

When  $V_{out} < V_{DD} + V_{T, load}$ , the load transistor is in saturation ( $V_{gd} = V_{out} - V_{DD} < V_{T, load}$ )

Then, the load current is given by the following equation

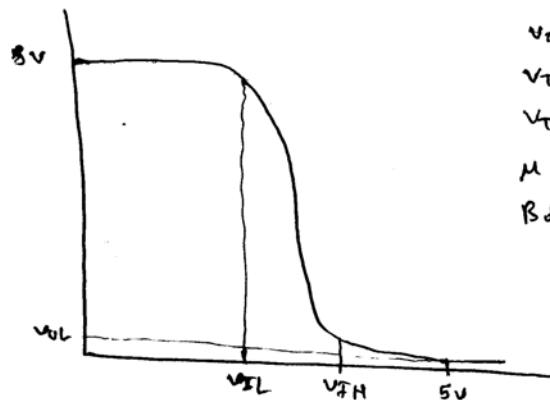
$$I_{D, load} = \frac{\beta_{load}}{2} \left[ -V_{T, load}(V_{out}) \right]^2$$

For larger output voltage levels, i.e., for  $V_{out} > V_{DD} + V_{T,load}$ , the depletion-type load transistor operates in the linear region.

The load current in this case is

$$I_{D,load} = \frac{\beta_{load}}{2} \left[ 2 |V_{T,load}(V_{out})| (V_{DD} - V_{out}) - (V_{DD} - V_{out})^2 \right]$$

The VTC of a typical depletion-load inverter is shown below, where  $(\mu C_{ox})_{load} = (\mu C_{ox})_{drive}$



$V_{DD} = 5V$   
 $V_{T,driver} = 1V$   
 $V_{T,load} = -3V$   
 $\mu C_{ox} = 40 \mu A/V^2$   
 $\beta_{drive}/\beta_{load} = 5$

Next, we will consider the critical voltage points  $V_{OH}$ ,  $V_{OL}$ ,  $V_{IL}$ , and  $V_{IH}$  for this inverter circuit.

\* Calculation of  $V_{OH}$

When  $V_{in} < V_{T0, driver}$ , the driver transistor will be cutoff and  $V_{OH} = V_{DD}$  since  $V_{GS, load} = 0 > V_{T0, load}$ .

\* Calculation of  $V_{OL}$

For  $V_{out} = V_{OL}$ , we assume  $V_{in} = V_{OH} = V_{DD}$   
 $V_{GD, driver} = V_{in} - V_{out} = V_{DD} - V_{OL} > V_{T0, driver}$   
 $\Rightarrow$  the driver transistor operates in the linear region.

$V_{GD, load} = V_{out} - V_{DD} = V_{OL} - V_{DD} < V_{T, load}$   
 $\Rightarrow$  the depletion-type load transistor operates in saturation

By KCL,  $I_L = I_D$

$$\frac{\beta_{load}}{2} [-V_{T, load} (V_{OL})]^2 = \frac{\beta_{driver}}{2} [2 (V_{OH} - V_{T0}) V_{OL} - V_{OL}^2]$$

$$\Rightarrow V_{OL}^2 - 2 (V_{OH} - V_{T0}) V_{OL} + \frac{\beta_{load}}{\beta_{driver}} |V_{T, load} (V_{OL})| = 0$$

$$\Rightarrow V_{OL} = (V_{OH} - V_{T0}) - \sqrt{(V_{OH} - V_{T0})^2 - \frac{\beta_{load}}{\beta_{driver}} |V_{T, load} (V_{OL})|}$$

This equation can be solved with the threshold equation  $V_{T,load}$  numerically as illustrated before.

\* Calculation of  $V_{FL}$

When  $V_{in} = V_{FL}$ ,  $V_{out}$  is slightly less than  $V_{OH}$ .

$$V_{gd, driver} = V_{in} - V_{out} = V_{FL} - V_{OH} < V_{T, driver}$$

$\Rightarrow$  driver is in saturation

$$V_{gd, load} = V_{out} - V_{DD} \approx 0 > V_{T, load}$$

$\Rightarrow$  load is in linear region

By KCL,  $I_L = I_D$

$$\frac{\beta_{load}}{2} \left[ 2 | -V_{T, load}(V_{out}) | \cdot (V_{DD} - V_{out}) - (V_{DD} - V_{out})^2 \right]$$

$$= \frac{\beta_{driver}}{2} \cdot (V_{in} - V_{T_0})^2$$

when  $V_{in} = V_{FL}$ ,  $\frac{dV_{out}}{dV_{in}} = -1$

$$\frac{\beta_{load}}{2} \left[ 2 | V_{T, load}(V_{out}) | \cdot \left( \frac{-dV_{out}}{dV_{in}} \right) + 2 (V_{DD} - V_{out}) \cdot \left( - \frac{dV_{T, load}}{dV_{in}} \right) \right]$$

$$- 2 (V_{DD} - V_{out}) \cdot \left( \frac{-dV_{out}}{dV_{in}} \right) = \beta_{driver} (V_{in} - V_{T_0})$$

substituting  $\frac{dV_{out}}{dV_{in}} = -1$

$$\Rightarrow V_{IL} = V_{T_0} + \frac{\beta_{load}}{\beta_{driver}} \left[ |V_{T,load}(V_{out})| + V_{out} - V_{DD} + (V_{DD} - V_{out}) \left( -\frac{dV_{T,load}}{dV_{in}} \right) \right]$$

$$\frac{dV_{T,load}}{dV_{in}} = \frac{dV_{T,load}}{dV_{out}} \cdot \frac{dV_{out}}{dV_{in}}$$

$$\frac{dV_{T,load}}{dV_{out}} = \frac{\gamma}{2\sqrt{2\phi_b + V_{out}}}$$

The equation for  $V_{IL}$  and the current equation of the driver and the load can be solved with the threshold voltage equation  $V_{T,load}(V_{out})$  numerically.

### \* Calculation of $V_{IH}$

When  $V_{in} = V_{IH}$ , the output will be slightly higher than  $V_{OL}$ .

$$V_{gd,driver} = V_{in} - V_{out} = V_{IH} - V_{out} > V_{T_0,driver}$$

$\Rightarrow$  driver is in linear region

$$V_{gd, load} = V_{out} - V_{DD} < V_{T, load}$$

⇒ load is in saturation

By KCL,  $I_L = I_D$

$$\frac{\beta_{load}}{2} [-V_{T, load}(v_{out})]^2 = \frac{\beta_{driver}}{2} [2(v_{in} - V_{T_0})v_{out} - v_{out}^2]$$

when  $v_{in} = V_{IH}$ ,  $\frac{dv_{out}}{dv_{in}} = -1$

Differentiating both sides:

$$\beta_{load} [-V_{T, load}(v_{out})] \cdot \left(-\frac{dV_{T, load}}{dv_{out}}\right) \cdot \left(\frac{dv_{out}}{dv_{in}}\right) = \beta_{driver} \left[ v_{out} + (v_{in} - V_{T_0}) \left(\frac{dv_{out}}{dv_{in}}\right) - v_{out} \left(\frac{dv_{out}}{dv_{in}}\right) \right]$$

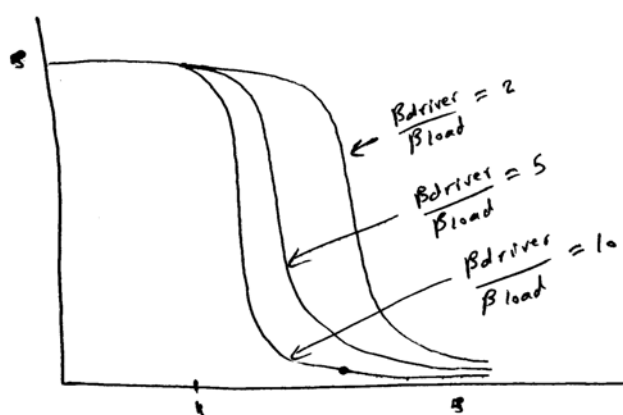
Now, substituting  $\frac{dv_{out}}{dv_{in}} = -1$

$$\Rightarrow V_{IH} = V_{T_0} + 2v_{out} + \frac{\beta_{load}}{\beta_{driver}} [V_{T, load}(v_{out})] \cdot \left(\frac{dV_{T, load}}{dv_{out}}\right)$$

This can be solved together with the current equation and the threshold equation numerically.

It is seen from the analysis of the critical voltage points that the general shape of the inverter VTC is determined essentially by the threshold voltages of the driver and the load devices and by the driver-to-load ratio,  $\beta_{\text{driver}}/\beta_{\text{load}}$ .

Since the threshold voltages are usually set by the fabrication process, the driver-to-load ratio emerges as a primary design parameter which can be adjusted to achieve the desired VTC shape.



$$V_{DD} = 5V$$

$$V_{T0, \text{driver}} = 1V$$

$$V_{T0, \text{load}} = -3V$$

$$\mu C_{ox} = 40 \mu A/V^2$$

Unlike the enhancement-load inverter, a sharp VTC transition can be obtained with relatively small driver-to-load ratios.



## Power Dissipation

The steady-state DC power consumption of the depletion-load inverter circuit can be found by calculating the amount of current drawn from the power supply, during the input-low state and the input-high state.

When input is low, the driver is cutoff and  $V_{out} = V_{OH} = V_{DD}$  and there is no significant current flow. Consequently, the inverter does not dissipate power under this condition.

When the input is high, on the other hand, both the driver and the load conduct a significant current given by

$$\begin{aligned} I_{DC}(v_{in} = V_{DD}) &= \frac{\beta_{load}}{2} \left[ -V_{T,load}(V_{OL}) \right]^2 \\ &= \frac{\beta_{driver}}{2} \left[ 2(V_{OH} - V_{T0})V_{OL} - V_{OL}^2 \right] \end{aligned}$$

Assuming that the input is low 50% of the time and high during the other 50%,

$$P_{DC} = \frac{V_{DD}}{2} \cdot \frac{\beta_{load}}{2} \left[ -V_{T,load}(V_{OL}) \right]^2$$

### Example

Consider a depletion-load inverter with the following parameters:

$$V_{DD} = 5\text{ V}, \quad V_{T0, \text{driver}} = 1.0\text{ V}, \quad V_{T0, \text{load}} = -3.0\text{ V}$$

$$(W/L)_{\text{driver}} = 2, \quad (W/L)_{\text{load}} = 1/3, \quad \mu C_{ox} = 25\text{ }\mu\text{A/V}^2$$

$$\text{for driver and load, } \gamma = 0.4\text{ V}^{1/2}, \quad \phi_b = 0.3\text{ V}$$

Calculate the critical voltages ( $V_{OL}$ ,  $V_{OH}$ ,  $V_{FL}$ ,  $V_{FH}$ ) on the VTC and find the noise margins of the circuit.

### Solution

$$V_{OH} = V_{DD} = 5\text{ V}$$

For  $V_{OL}$ , we first assume  $V_{T, \text{load}} = V_{T0, \text{load}}$

$$\begin{aligned} V_{OL} &= V_{OH} - V_{T0} - \sqrt{(V_{OH} - V_{T0})^2 - \left(\frac{\beta_{\text{load}}}{\beta_{\text{driver}}}\right) |V_{T, \text{load}}|^2} \\ &= 5 - 1 - \sqrt{(5 - 1)^2 - \left(\frac{1}{6}\right) |3|^2} = 0.192\text{ V} \end{aligned}$$

Now, we can update the threshold voltage of the depletion load

$$\begin{aligned} V_{T, \text{load}} &= V_{T0, \text{load}} + \gamma \left[ \sqrt{2\phi_b + V_{OL}} - \sqrt{2\phi_b} \right] \\ &= -3 + 0.4 \left( \sqrt{0.6 + 0.192} - \sqrt{0.6} \right) \\ &= -2.95\text{ V} \end{aligned}$$

Using this new value for  $V_{T,load}$ , we now recalculate  $V_{OL}$

$$V_{OL} = 5 - 1 - \sqrt{(5-1)^2 - \left(\frac{1}{8}\right) |2.95|^2} = 0.186 \text{ V}$$

$$V_{T,load} = -3 + 0.4 \left[ \sqrt{0.6 + 0.186} - \sqrt{0.6} \right]$$

$$= -2.95 \text{ V}$$

At this point, we can stop the iteration process since  $V_{T,load}$  has not changed in the two significant digits after the decimal point. Thus,  $V_{OL} = \underline{0.186 \text{ V}}$

To calculate  $V_{IL}$ , we know that the output is expected to be slightly lower than  $V_{OH}$ . As a first approximation, let us assume that  $V_{out} = V_{OH} = 5 \text{ V}$  for  $V_{in} = V_{IL}$ . Then, the threshold voltage of the load device can be estimated as

$$V_{T,load} (V_{out} = 5) = -3 + 0.4 \left( \sqrt{0.6 + 5} - \sqrt{0.6} \right)$$

$$= -2.36 \text{ V.}$$

$$V_{IL} (V_{out}) = V_{T0} + \frac{\beta_{load}}{\beta_{driver}} \left[ V_{out} - V_{OH} + |V_{T,load}(V_{out})| \right]$$

$$= 1 + \frac{1}{8} \left[ V_{out} - 5 + 2.36 \right]$$

$$= 0.167 V_{out} + 0.56$$

The output voltage level at this point can be found as  $V_{out} = (6)(1.36) - 3.35 = 4.81V$  which significantly improves our initial assumption of  $V_{out} = 5V$ . At this point, the threshold voltage must be recomputed to update its value.

$$V_{T,load} = -3 + 0.4(\sqrt{0.6 + 4.81} - \sqrt{0.6}) = -2.38V$$

Since this value is slightly higher than the previously calculated value, we can accept  $V_{FL} = 1.36V$  as a fairly accurate estimate.

To calculate  $V_{FH}$ , the output is expected to be slightly higher than  $V_{OL}$ . As a first order approximation, we can assume  $V_{out} = V_{OL}$  when  $V_{in} = V_{FH}$ . Thus, the threshold voltage can be estimated as

$$V_{T,load}(V_{out} = 0.186) = -3 + 0.4(\sqrt{0.6 + 0.186} - \sqrt{0.6}) = -2.95V.$$

$$\text{Thus, } \frac{dV_{T,load}}{dV_{out}} = \frac{\gamma}{2\sqrt{2Q_b + V_{out}}} = \frac{0.4}{2\sqrt{0.6 + 0.186}} = 0.22$$

THIS value can be used to find  $V_{IH}$  as a function of the output voltage

$$\begin{aligned} V_{IH}(V_{out}) &= V_{T_0} + 2V_{out} + \frac{\beta_{load}}{\beta_{driver}} \left[ V_{T,load}(V_{out}) \right] \\ &= 1 + 2V_{out} - \left(\frac{1}{6}\right) (2.95) \left(0.22 \frac{dV_{T,load}}{dV_{out}}\right) \\ &= 2V_{out} + 0.9 \end{aligned}$$

THIS expression can be rearranged as:

$$V_{out} = 0.5 V_{IH} - 0.45$$

Next, we substitute  $V_{out}$  in the KCL equation to obtain:

$$\begin{aligned} 2 \left[ 2(V_{IH}-1)(0.5 V_{IH} - 0.45) - (0.5 V_{IH} - 0.45)^2 \right] \\ = \frac{1}{3} (2.95)^2 \Rightarrow V_{IH}^2 - 8 V_{IH} - 13.31 = 0 \end{aligned}$$

THE solution of this simple quadratic equation yields two values for  $V_{IH}$ :

$$V_{IH} = \begin{cases} 2.36 \text{ v} \\ 5.64 \text{ v} \end{cases}$$

where  $V_{IH} = 2.36 \text{ v}$  is the physically correct solution.

The output voltage at this point is calculated as:

$$V_{out} = (0.5)(2.36) - (0.45) = 0.73 \text{ V}$$

With this updated output voltage value, we can now recalculate the load threshold

$$V_{T,load} = -3 + 0.4(\sqrt{0.6 + 0.73} - \sqrt{0.6}) \\ = -2.85 \text{ V}$$

$$\text{and } \frac{dV_{T,load}}{dV_{out}} = \frac{0.4}{2\sqrt{0.6 + 0.73}} = 0.173$$

Note that both of these values are fairly close to those used at the beginning of the iteration process. Thus, we may accept  $V_{IH} = 2.36 \text{ V}$  as a good estimate. The values can be more accurate by applying more iterations.

Thus, the noise margins for both high signal and low signal levels can be found as follows:

$$NMH = V_{OH} - V_{IH} = 5 - 2.36 = 2.64 \text{ V}$$

$$NML = V_{IL} - V_{OL} = 1.36 - 0.186 = 1.17 \text{ V}$$

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