COE 360 Principles of VLSI Design
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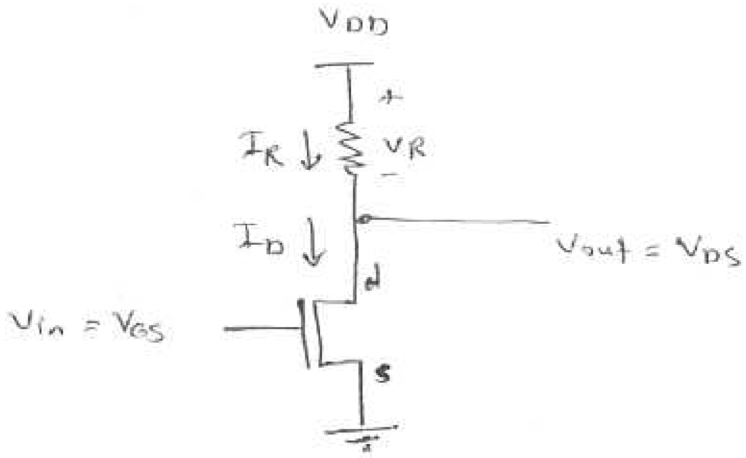
Lecture#12

Resistive-Load Inverter

Resistive - Load Inverter

Consider the structure of the resistive-load inverter shown below.



* When $V_{in} < V_t$, the transistor is in cutoff and does not conduct any drain current. Since the voltage drop across the load resistor is equal to zero, the output voltage $= V_{DD}$.

* As the input voltage is increased beyond V_t (i.e. $V_{in} \geq V_t$), the driver transistor starts conducting a nonzero drain current. The transistor will be initially in saturation since $V_{out} = V_{DS}$ will be slightly under V_{DD} and $V_{in} = V_{gs}$ close to V_t i.e. $V_{gs} - V_t < V_{ds}$ or

$$V_{gd} = V_{gs} - V_{ds} < V_t.$$

Thus, $I_R = \frac{\beta_n}{2} (V_{in} - V_t)^2$

* With increasing input voltage, the drain current of the driver also increases, and the output voltage V_{out} starts to drop. Eventually, for input voltages larger than $V_{out} + V_t$ (i.e. $V_{gs} > V_{ds} + V_t = V_{gs} - V_{ds} > V_t$) the transistor enters the linear region.

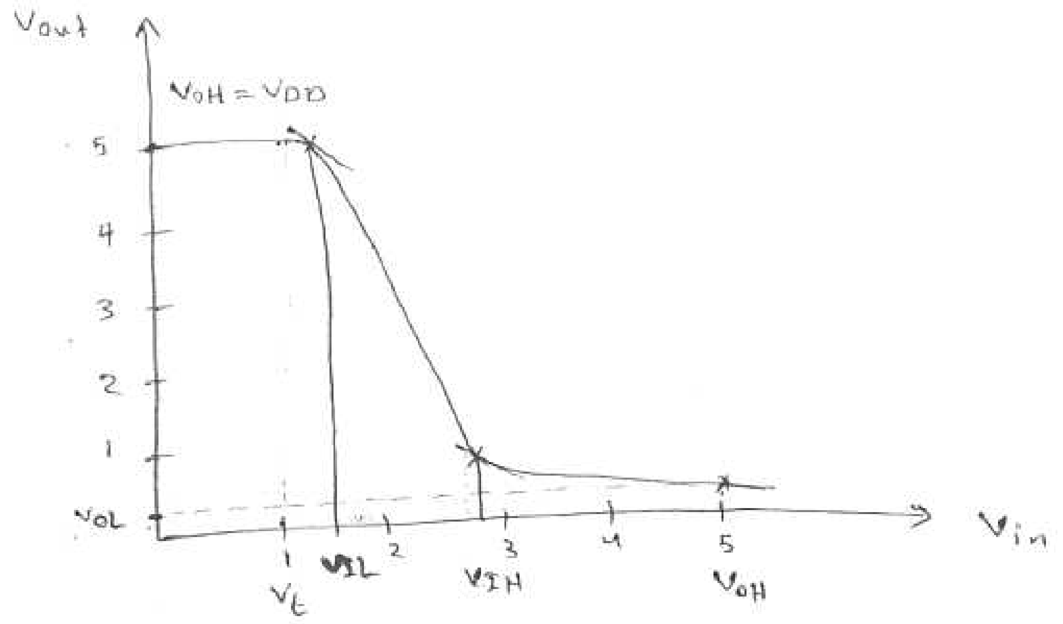
Thus, the current $I_R = I_D$ is given as:

$$I_R = \frac{\beta_n}{2} \left[2 (V_{in} - V_t) \cdot V_{out} - V_{out}^2 \right]$$

The various operating regions of the driver transistor and the corresponding input-output conditions are summarized below:

Input Voltage Range	Operating Mode
$V_{in} < V_t$	Cutoff
$V_t \leq V_{in} \leq V_{out} + V_t$	Saturation
$V_{in} > V_{out} + V_t$	linear

The voltage transfer characteristic (VTC) of a typical resistive-load inverter circuit indicating the operating modes of the transistor and the critical voltage points is shown below:



Now, let us calculate the five critical voltage points which determine the steady state input-output behavior of the inverter.

* Calculation of V_{OH}

The output voltage V_{out} is given by

$$V_{out} = V_{DD} - R_L I_R$$

When the input voltage V_{in} is low i.e., smaller than V_t, the driver transistor is cutoff. Since the drain current of the driver transistor is equal to the load current, $I_R = I_D = 0$. Thus, the output voltage $\boxed{V_{OH} = V_{DD}}$.

* Calculation of V_{OL}

To calculate the output low voltage V_{OL}, we assume that the input voltage is equal to V_{OH}, i.e. $V_{in} = V_{OH} = V_{DD}$

Since $V_{in} - V_t > V_{out}$, the driver transistor operates in the linear region. Thus, the load current I_R is

$$I_R = I_D = \frac{\beta_n}{2}\left[2(V_{DD} - V_t)V_{OL} - V_{OL}^2\right]$$

Note also that $I_R = \dfrac{V_{DD} - V_{out}}{R_L}$

Thus, $$\frac{V_{DD} - V_{OL}}{R_L} = \frac{\beta_n}{2}\left[2(V_{DD} - V_t)V_{OL} - V_{OL}^2\right]$$

This equation yields a simple quadratic in V_{OL}, which is solved to find the value of the output low voltage

$$V_{OL}^2 - 2\left[V_{DD} - V_t + \frac{1}{\beta_n R_L}\right] V_{OL} + \frac{2}{\beta_n R_L} V_{DD} = 0$$

Note : $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$V_{OL} = V_{DD} - V_t + \frac{1}{\beta_n R_L} - \sqrt{\left(V_{DD} - V_t + \frac{1}{\beta_n R_L}\right)^2 - \frac{2V_{DD}}{\beta_n R_L}}$$

* Calculation of V_{IL}

By definition, V_{IL} is the smaller of the two input voltage values at which the slope of the VTC becomes equal to -1

i.e. $dV_{out} / dV_{in} = -1$

When $V_{in} = V_{IL}$, the output voltage V_{out} is slightly smaller than V_{OH}.

Consequently, $V_{out} > V_{in} - V_t$, and the transistor operates in saturation.

Thus,

$$\frac{V_{DD} - V_{out}}{R_L} = \frac{\beta_n}{2}\left(V_{in} - V_t\right)^2$$

To satisfy the derivative condition, we differentiate both sides of the equation with respect to V_{in}

$$-\frac{1}{R_L} \cdot \frac{dV_{out}}{dV_{in}} = \beta_n (V_{in} - V_t)$$

Then, we substitute $\frac{dV_{out}}{dV_{in}} = -1$

$$\Rightarrow \quad -\frac{1}{R_L} \cdot (-1) = \beta_n (V_{IL} - V_t)$$

$$\Rightarrow \quad \boxed{V_{IL} = V_t + \frac{1}{\beta_n R_L}}$$

The value of the output voltage when the input is equal to V_{IL} can also be found by substituting V_{IL} in the original equation

$$V_{out}(V_{in} = V_{IL}) = V_{DD} - \frac{\beta_n R_L}{2}\left[V_t + \frac{1}{\beta_n R_L} - V_t\right]^2$$

$$\boxed{V_{out} = V_{DD} - \frac{1}{2\beta_n R_L}}$$

* ## Calculation of V_{IH}

V_{IH} is the larger of the two voltage points at which the slope $= -1$. When the input voltage $V_{in} = V_{IH}$, V_{out} is slightly larger than the output low voltage V_{OL}. Hence, $V_{out} < V_{in} - V_t$, and the driver transistor operates in the linear region. Thus,

$$\frac{V_{DD} - V_{out}}{R_L} = \frac{\beta_n}{2}\left[2(V_{in} - V_t)V_{out} - V_{out}^2\right]$$

Differentiating both sides with respect to V_{in}, we obtain

$$-\frac{1}{R_L} \cdot \frac{dV_{out}}{dV_{in}} = \frac{\beta_n}{2}\left[2(V_{in} - V_t)\frac{dV_{out}}{dV_{in}} + 2V_{out} - 2V_{out} \cdot \frac{dV_{out}}{dV_{in}}\right]$$

Now substituting $\frac{dV_{out}}{dV_{in}} = -1$ at $V_{in} = V_{IH}$

$$-\frac{1}{R_L} \cdot (-1) = \beta_n\left[(V_{IH} - V_t) \cdot (-1) + 2V_{out}\right]$$

$$\Rightarrow \boxed{V_{IH} = V_t + 2V_{out} - \frac{1}{\beta_n R_L}}$$

Next, we can find V_{out} when $V_{in} = V_{IH}$ by substituting V_{IH} in the original equation

$$\frac{V_{DD} - V_{out}}{R_L} = \frac{\beta_n}{2}\left[2\left(V_t + 2V_{out} - \frac{1}{\beta_n R_L} - V_t\right)V_{out} - V_{out}^2\right]$$

The positive solution of this second-order equation gives

$$\boxed{V_{out}\,(V_{in} = V_{IH}) = \sqrt{\frac{2}{3}\,\frac{V_{DD}}{\beta_n R_L}}}$$

The four critical voltage points V_{OL}, V_{OH}, V_{IL}, V_{IH} can now be used to determine the noise margins, NM_L and NM_H of the resistive-load inverter.

* <u>Calculation of the inverter threshold voltage V_{th}:</u>

The inverter threshold voltage V_{th}, which is considered as the transition voltage is defined as the point on VTC where $V_{in} = V_{out}$. Note that under this condition, the transistor will be in saturation since $V_{in} - V_t < V_{out}$.

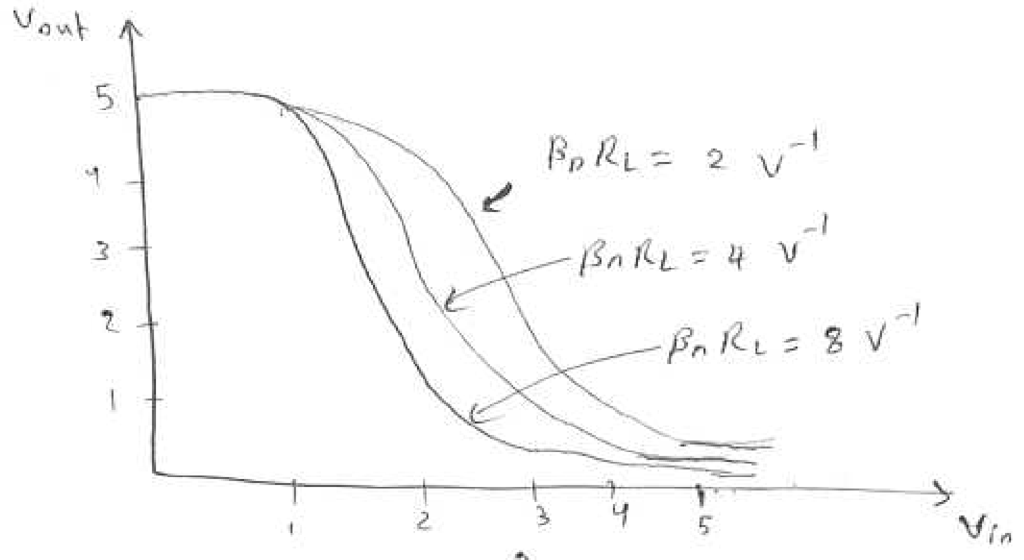
Thus, substituting $V_{out} = V_{in}$ in the following equation:

$$\frac{V_{DD} - V_{out}}{R_L} = \frac{\beta_n}{2}\left[V_{in} - V_t\right]^2$$

$$\Rightarrow \quad \frac{V_{DD} - V_{out}}{R_L} = \frac{\beta_n}{2}\left[V_{out} - V_t\right]^2$$

Solving this equation for V_{out} gives V_{th}.

* As can be seen from the previous analysis that the term $\underline{\underline{\beta_n R_L}}$ plays an important role in determining the shape of the voltage transfer characteristic. V_{OH} is determined by the power supply voltage V_{DD}. Among the other three critical voltage points, V_{OL} receives primary attention while V_{IL} and V_{IH} are usually treated as secondary design variables



The larger the $\beta_n R_L$ values, the smaller the output low voltage and the shape of the VTC approaches that of the ideal inverter, with very large transition slope.

* Power Consumption

The average DC power consumption of the resistive load inverter is found by considering two cases $V_{in} = V_{OL}$ (low) and $V_{in} = V_{OH}$ (high)

$$P_{DC} = \frac{V_{DD}}{2}\left[I_{DC}(V_{in}=\text{low}) + I_{DC}(V_{in}=\text{high})\right]$$

When $V_{in} = V_{OL}$, the transistor is cutoff and $I_D = I_R = 0$, and power dissipation is zero.

When $V_{in} = V_{OH}$, the transistor is on and the current drawn from the power supply is given as

$$I_D = I_R = \frac{V_{DD} - V_{OL}}{R_L}$$

Thus, $$P_{DC}\,(\text{average}) = \frac{V_{DD}}{2} \cdot \frac{(V_{DD} - V_{OL})}{R_L}$$

Example

Consider the following inverter design problem:
Given $V_{DD} = 5\,V$, $\mu_n C_{ox} = 30\,\mu A/V^2$, and
$V_t = 1\,V$. Design a resistive-load inverter
with $V_{OL} = 0.2\,V$. Specifically, determine
the (W/L) ratio of the transistor and
the value of the load resistor R_L that
achieve the required V_{OL}.

Solution

Note that the transistor operates in the linear region when the output voltage is equal to V_{OL} and the input voltage is equal to $V_{OH} = V_{DD}$.

Thus,

$$\frac{V_{DD} - V_{OL}}{R_L} = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \left[2\,(V_{OH} - V_t) \cdot V_{OL} - V_{OL}^2 \right]$$

Assuming $V_{OL} = 0.2$,

$$\Rightarrow \quad \frac{5 - 0.2}{R_L} = \frac{30 \times 10^{-6}}{2} \cdot \frac{W}{L} \left((2)(4) \cdot (0.2) - (0.2)^2 \right)$$

$$\Rightarrow \quad \frac{W}{L} \cdot R_L = 2.05 \times 10^{5}\ \Omega$$

The selection of the pair of values to use for $\left(\frac{W}{L}\right)$ and R_L in the final design depends on tradeoff between the power consumption of the circuit and the silicon area.

Example

Consider a resistive load inverter with $V_{DD} = 5\,V$, $\mu_n C_{ox} = 20\,\mu A/V^2$, $V_t = 0.8\,V$, $R_L = 200\,K\Omega$, and $W/L = 2$. Calculate the critical voltages (V_{OL}, V_{OH}, V_{IL}, V_{IH}) and the noise margins of the circuit.

Solution

$$V_{OH} = V_{DD} = 5\,V$$

$$\beta_n = \mu_n C_{ox} \frac{W}{L} = 40\,\mu A/V^2$$

$$\beta_n R_L = 8\,V^{-1}$$

$$V_{OL} = V_{DD} - V_t + \frac{1}{\beta_n R_L} - \sqrt{\left(V_{DD} - V_t + \frac{1}{\beta_n R_L}\right)^2 - \frac{2V_{DD}}{\beta_n R_L}}$$

$$= 5 - 0.8 + \frac{1}{8} - \sqrt{\left(5 - 0.8 + \frac{1}{8}\right)^2 - \frac{2\times 5}{8}}$$

$$= 0.147\,V$$

$$V_{IL} = V_t + \frac{1}{\beta_n R_L} = 0.8 + \frac{1}{8} = 0.925\,V$$

$$V_{IH} = V_t + \sqrt{\frac{8}{3}\frac{V_{DD}}{\beta_n R_L}} - \frac{1}{\beta_n R_L}$$

$$= 0.8 + \sqrt{\frac{8}{3}\cdot\frac{5}{8}} - \frac{1}{8} = 1.97\,V$$

$$NM_L = V_{IL} - V_{OL} = 0.93 - 0.15 = 0.78\,V$$

$$NM_H = V_{OH} - V_{IH} = 5.0 - 1.97 = 3.03\,V$$

Note that NM_L is quite low here. A good NM_L should be at least $25\%\ V_{DD} = 1.25\,V$.