Elmore Delay

The Elmore Delay

Consider a general RC tree network, as shown in Fig. 6.25. Note that (i) there are no resistor loops in this circuit, (ii) all of the capacitors in an RC tree are connected between a node and the ground, and (iii) there is one input node in the circuit. Also notice that there is a unique resistive path, from the input node to any other node in the circuit.

Inspecting the general topology of this RC tree network, we can make the following path definitions:

- Let P_i denote the unique path from the input node to node i, $i = 1, 2, 3, \ldots, N$.
- Let $P_{ij} = P_i \cap P_j$ denote the portion of the path between the input and the node i, which is *common* to the path between the input and node j.

Assuming that the input signal is a step pulse at time $t = 0$, the Elmore delay at node i of this RC tree is given by the following expression.

$$\tau_{Di} = \sum_{j=1}^{N} C_j \sum_{\substack{for\ all \\ k \in P_{ij}}} R_k \tag{6.60}$$

Calculation of the Elmore delay is equivalent to deriving the first-order time constant (first moment of the impulse response) of this circuit. Note that although this delay is still an *approximation* for the actual signal propagation delay from the input node to node i, it provides a fairly simple and accurate means of predicting the behavior of the RC line. The procedure to calculate the delay at any node in the circuit is very

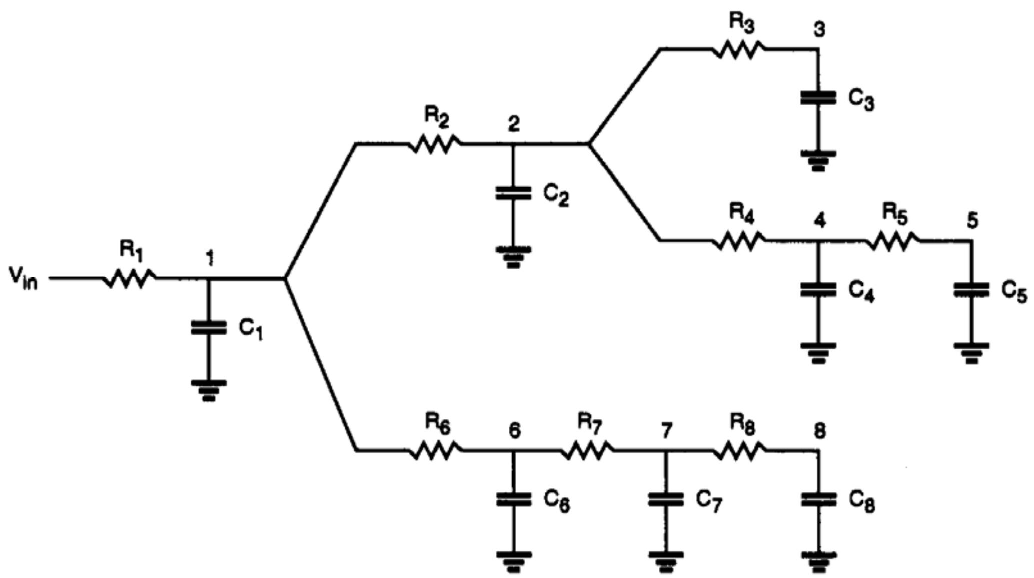

Figure 6.25 A general RC tree network consisting of several branches.

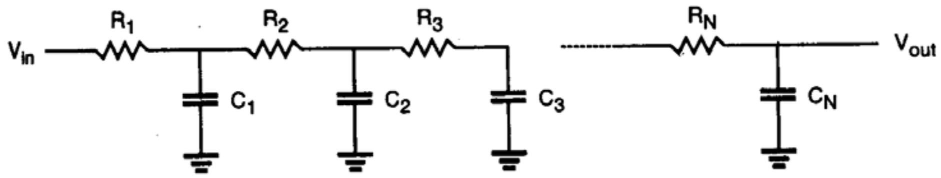

Figure 6.26 Simple *RC* ladder network consisting of one branch.

straightforward. For example, the Elmore delay at node 7 can be found according to (6.60) as

$$\tau_{D7} = R_1C_1 + R_1C_2 + R_1C_3 + R_1C_4 + R_1C_5 + (R_1 + R_6)C_6 + (R_1 + R_6 + R_7)C_7 + (R_1 + R_6 + R_7)C_8 \quad (6.61)$$

Similarly, the Elmore delay at node 5 can be calculated as

$$\tau_{D7} = R_1C_1 + (R_1 + R_2)C_2 + (R_1 + R_2)C_3 + (R_1 + R_2 + R_4)C_4 + (R_1 + R_2 + R_4 + R_5)C_5 + R_1C_6 + R_1C_7 + R_1C_8 \quad (6.62)$$

As a special case of the general *RC* tree network, consider now the simple *RC* ladder network shown in Fig. 6.26. Here, the entire network consists of one single branch, and the Elmore delay from the input to the output (node N) is found according to (6.60) as

$$\tau_{DN} = \sum_{j=1}^{N} C_j \sum_{k=1}^{j} R_k \quad (6.63)$$

If we further assume a *uniform RC* ladder network, consisting of identical elements (R/N) and (C/N) as shown in Fig. 6.24, then the Elmore delay from the input to the output node becomes

$$\tau_{DN} = \sum_{j=1}^{N} \left(\frac{C}{N}\right) \sum_{k=1}^{j} \left(\frac{R}{N}\right) = \left(\frac{C}{N}\right)\left(\frac{R}{N}\right)\left(\frac{N(N+1)}{2}\right) = RC\left(\frac{N+1}{2N}\right) \quad (6.64)$$

For very large N (distributed *RC* line behavior), this delay expression reduces to

$$\tau_{DN} = \frac{RC}{2} \quad \text{for} \quad N \to \infty \quad (6.65)$$