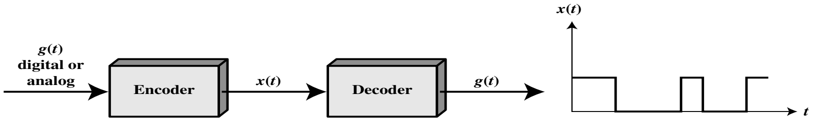
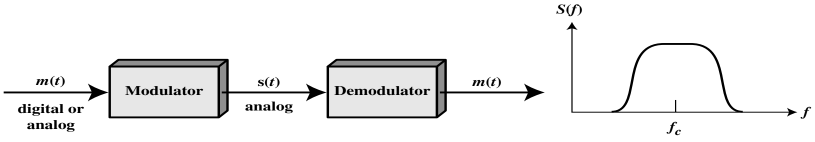
# William Stallings
# Data and Computer Communications

## Chapter 5
## Data Encoding

# Encoding and Modulation Techniques



(a) Encoding onto a digital signal



(b) Modulation onto an analog signal

Figure 5.1 Encoding and Modulation Techniques

# Digital & Analog Signaling

- Digital signaling
  - Data source g(t) encoded into digital signal x(t)
  - g(t) may be analog or digital
  - x(t) dependent on coding technique, chosen to optimize use of transmission medium
    - Conserve bandwidth or minimize errors
- Analog signaling
  - Based on continuous constant frequency signal, called carrier signal
  - Carrier signal frequency chosen to be compatible with transmission medium
  - Data transmitted by carrier signal modulation

# Analog Signaling

- Modulation
  - Process of encoding source data onto a carrier signal with frequency $f_c$
  - Operation on one or more of three fundamental frequency-domain parameters: amplitude, frequency, and phase
- Input signal m(t)
  - Can be analog or digital
  - Called *modulating signal* or *baseband signal*
  - Modulated signal s(t) is result of modulating carrier signal; called *bandlimited* or *bandpass* signal
  - Location of bandwidth on spectrum related to carrier frequency $f_c$

## Encoding Techniques

- Digital data, digital signal
  - Simple and inexpensive equipment
- Analog data, digital signal
  - Data needs to be converted to digital form
- Digital data, analog signal
  - Take advantage of existing analog transmission media
- Analog data, analog signal
  - Transmitted as baseband signal easily and cheaply
  - Modulation to shift bandwidth to another portion of spectrum
  - Multiple signals on different position on spectrum can share same transmission medium (frequency-division multiplexing)

## Digital Data, Digital Signal

- Digital signal
  - Sequence of discrete, discontinuous voltage pulses
  - Each pulse is a signal element
  - Binary data encoded into signal elements
- Unipolar signal
  - All signal elements have same sign
- Polar signal
  - One logic state represented by positive voltage the other by negative voltage

## Digital Data, Digital Signal

- Data rate
  - Rate of data transmission in bits per second
- Duration or length of a bit
  - Time taken for transmitter to emit the bit
  - For a data rate R bps, duration of each bit is 1/R
- Modulation rate
  - Rate at which the signal level changes
  - Measured in baud = signal elements per second
- Mark and Space
  - Binary 1 and Binary 0 respectively

## Interpreting Digital Signal at Receiver

- Receiver need to know
  - Timing of bits - when they start and end
  - Signal level
  - Sampling & comparison with a threshold value
- Factors affecting successful interpreting of signals: signal to noise ratio, data rate, bandwidth
  - Increase in data rate increases bit-error-rate (BER)
  - Increase in SNR decreases BER
  - Increase in bandwidth allows for increase in data rate

## Comparison of Encoding Schemes

- Encoding scheme
  - Mapping from data bits to signal elements
- Signal Spectrum
  - Lack of high frequencies reduces required bandwidth
  - Lack of dc component allows ac coupling via transformer, providing isolation & reducing interference
  - Transfer function of a channel is worse near the band edges
  - Concentrate power in the middle of the bandwidth
- Clocking
  - Synchronizing transmitter and receiver
  - Sync mechanism based on signal

## Comparison of Encoding Schemes

- Error detection
  - Can be built in to signal encoding
- Signal interference and noise immunity
  - Some codes are better than others
- Cost and complexity
  - Higher signal rate (& thus data rate) lead to higher costs
  - Some codes require signal rate greater than data rate

## Encoding Schemes

- Nonreturn to Zero-Level (NRZ-L)
- Nonreturn to Zero Inverted (NRZI)
- Bipolar –AMI (alternate mark inversion)
- Pseudoternary
- Manchester
- Differential Manchester
- B8ZS
- HDB3

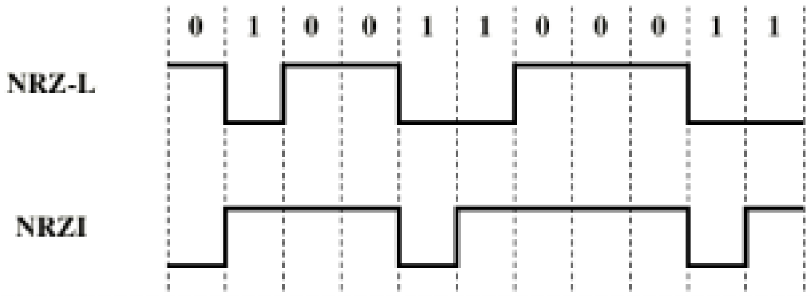
## Nonreturn to Zero-Level (NRZ-L)

- Two different voltages for 0 and 1 bits
- Voltage constant during bit interval
  - no transition,  no return to zero voltage
- e.g. Absence of voltage for zero, constant positive voltage for one
- More often, negative voltage for one value (1) and positive for the other (0)
- Used to generate or interpret digital data by terminals

## Nonreturn to Zero Inverted

- Nonreturn to zero inverted on ones
- Constant voltage pulse for duration of bit
- Data encoded as presence or absence of signal transition at beginning of bit time
- Transition (low to high or high to low) denotes a binary 1
- No transition denotes binary 0
- An example of differential encoding

## NRZ



## Differential Encoding

- Data represented by changes rather than levels
- More reliable detection of transition rather than level in presence of noise
- In complex transmission layouts it is easy to lose sense of polarity

## NRZ pros and cons

- Pros
  - Easy to engineer
  - Make good use of bandwidth
- Cons
  - dc component
  - Lack of synchronization capability
- Used for magnetic recording
- Not often used for signal transmission

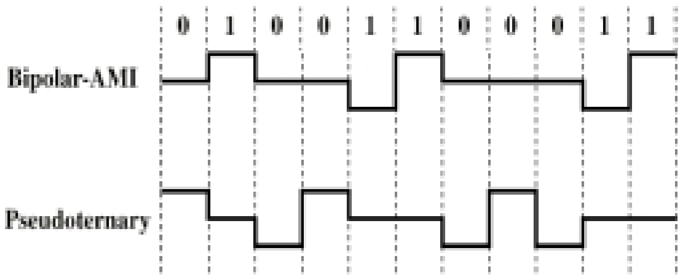
## Multilevel Binary

- Use more than two levels
- Bipolar-AMI
  - zero represented by no line signal
  - one represented by positive or negative pulse
  - one pulses alternate in polarity
  - No loss of sync if a long string of ones (zeros still a problem)
  - No net dc component
  - Lower bandwidth
  - Easy error detection

## Pseudoternary

- One represented by absence of line signal
- Zero represented by alternating positive and negative
- No advantage or disadvantage over bipolar-AMI

## Bipolar-AMI and Pseudoternary



## Trade Off for Multilevel Binary

- Not as efficient as NRZ
  - Each signal element only represents one bit
  - In a 3 level system could represent $\log_2 3 = 1.58$ bits
  - Receiver must distinguish between three levels (+A, -A, 0)
  - Requires approximately 3dB more signal power for same probability of bit error

# Theoretical Bit Error Rate for Various Encoding Schemes

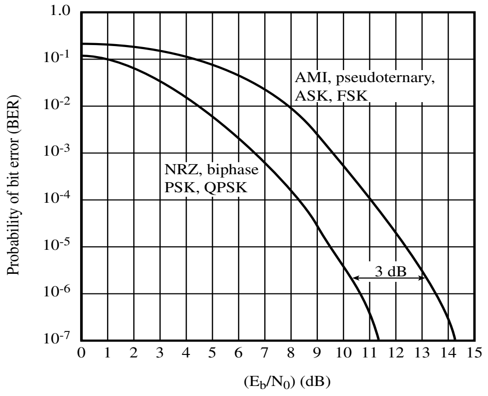


**Figure 5.4 Theoretical Bit Error Rate for Various Encoding Schemes**

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# Biphase

- Manchester
  - Transition in middle of each bit period
  - Transition serves as clock and data
  - Low to high represents one
  - High to low represents zero
  - Used by IEEE 802.3 (Standard for baseband coaxial cable & twisted pair CSMA/CD bus LANs).
- Differential Manchester
  - Mid bit transition is clocking only
  - Transition at start of a bit period represents zero
  - No transition at start of a bit period represents one
  - Note: this is a differential encoding scheme
  - Used by IEEE 802.5 (Token ring LAN)

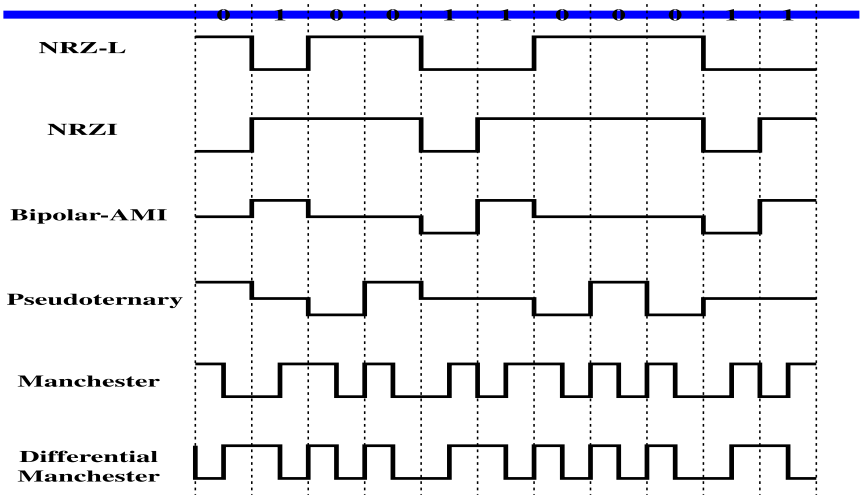


Figure 5.2 Digital Signal Encoding Formats

# Biphase Pros and Cons

- Con
  - At least one transition per bit time and possibly two
  - Maximum modulation rate is twice NRZ
  - Requires more bandwidth
- Pros
  - Synchronization on mid bit transition (self clocking)
  - No dc component
  - Error detection
    - Absence of expected transition

## Modulation Rate

- Data rate
  - Bits per second, or bit rate
  - $1/t_B$, where $t_B$ is bit duration
- Modulation rate
  - Rate at which signal elements generated
  - Measured in Baud
  - Modulation Rate D= R/b
    - R is data rate in bps
    - b is number of bits per signal element

## Manchester Code Modulation Rate

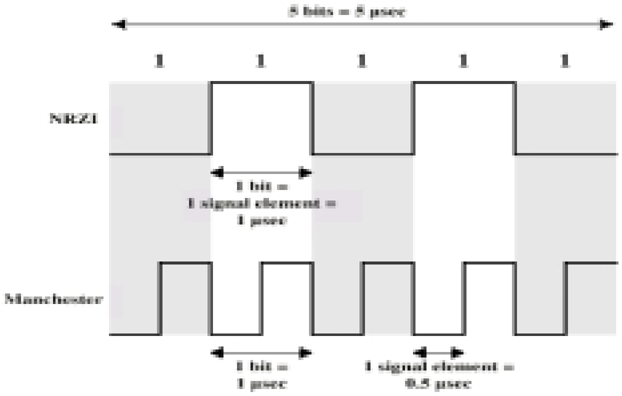


Figure 5.5 A Stream of Binary Ones at 1 Mbps

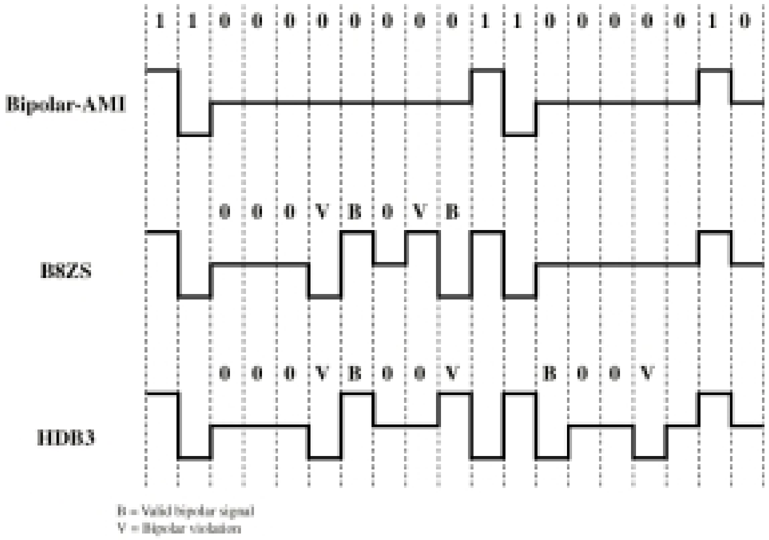
# Scrambling

- Use scrambling to replace sequences that would produce constant voltage
- Filling sequence
  - Must produce enough transitions to sync
  - Must be recognized by receiver and replaced with original
  - Same length as original
- No dc component
- No long sequences of zero level line signal
- No reduction in data rate
- Error detection capability

# B8ZS

- Bipolar With 8 Zeros Substitution
- Based on bipolar-AMI
- If octet of all zeros and last voltage pulse preceding was positive encode as 000+-0-+
- If octet of all zeros and last voltage pulse preceding was negative encode as 000-+0+-
- In general an octet of all zeros is replaced by 000VB0VB
- Causes two violations of AMI code
- Unlikely to occur as a result of noise
- Receiver detects and interprets as octet of all zeros

## B8ZS and HDB3



## HDB3

- High Density Bipolar 3 Zeros
  - Commonly used in Europe & Japan
  - Based on bipolar-AMI
- Allows at most 3 consecutive 0 voltage codes to be transmitted
- Four contiguous binary 0s are encoded by replacing the final encoded symbol with a non-zero voltage pulse with the same polarity as the last non-zero pulse, called the V (violation) pulse.

## HDB3

- ⌘ This breaks the AMI rule and can lead to short term DC offsets.
- ⌘ DC offsets can be removed by adding a pulse in agreement with the AMI scheme, known as the B pulse, in the position of the first of the four 0s to encode.
- ⌘ B pulses are only inserted if an even number of pulses occur between the last V pulse and the next V pulse, but not in the case of the first V pulse emitted.

## HDB3

- ⌘ A rule is added to ensure that successive violations are of alternate polarity to avoid dc component

**Number of ones since last substitution**

| **Polarity of Preceding Pulse** | **Odd** | **Even** |
|---|---|---|
| **-** | 000- | +00+ |
| **+** | 000+ | -00- |

## Spectral Density of Various Encoding Techniques

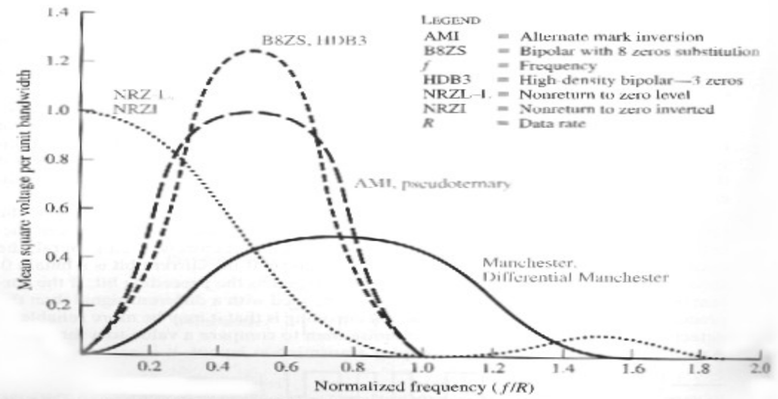


**Figure 5.3** Spectral Density of Various Signal Encoding Schemes

## Digital Data, Analog Signal

- Transmission of digital data through public telephone network
- Public telephone system
  - 300Hz to 3400Hz
  - Use modem (modulator-demodulator)
- Encoding techniques modify one of three characteristics of carrier signal
  - Amplitude => Amplitude shift keying (ASK)
  - Frequency => Frequency shift keying (FSK)
  - Phase => Phase shift keying (PK)
- Resulting signal has a bandwidth centered on carrier frequency

## Amplitude Shift Keying

- Binary values represented by different amplitudes of carrier
- Usually, one amplitude is zero
  - i.e. presence and absence of carrier is used
- For a carrier signal $s(t) = A\cos(2\pi f_c t)$ resulting signal is

$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ 0 & \text{binary } 0 \end{cases}$$

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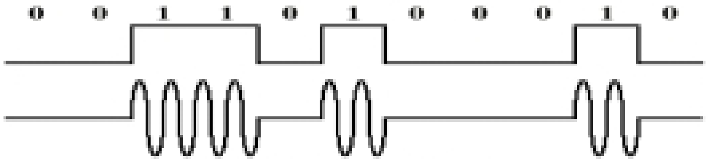
## Amplitude Shift Keying



(a) Amplitude-shift keying

- Inefficient: up to 1200bps on voice grade lines
- Used to transmit digital data over optical fiber

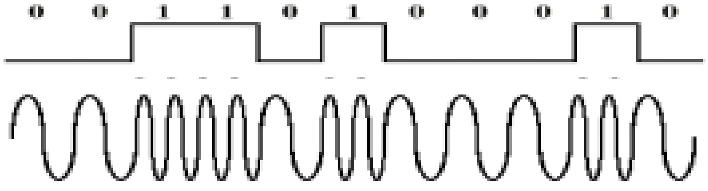
## Frequency Shift Keying (FSK)

- Binary values represented by two different frequencies near carrier frequency
- Resulting signal is

$$s(t) = \begin{cases} A\cos(2\pi f_1 t) & \text{binary } 1 \\ A\cos(2\pi f_2 t) & \text{binary } 0 \end{cases}$$

  - $f_1$ and $f_2$ are offset from carrier frequency $f_c$ by equal but opposite amounts

## Frequency Shift Keying (FSK)



(b) Frequency-shift keying

- Less susceptible to error than ASK
- Up to 1200bps on voice grade lines
- Used for high frequency (3 to 30 MHz) radio transmission
- Even higher frequencies on LANs using coaxial cables

## Frequency Shift Keying (FSK)

- Full-duplex transmission over voice grade line
  - In one direction $f_c$ is 1170 Hz with $f_1$ and $f_2$ given by 1170+100=1270 Hz and 1170–100=1070 Hz
  - In other direction $f_c$ is 2125 Hz with $f_1$ and $f_2$ given by 2125+100=2225 Hz and 2125–100=2025 Hz

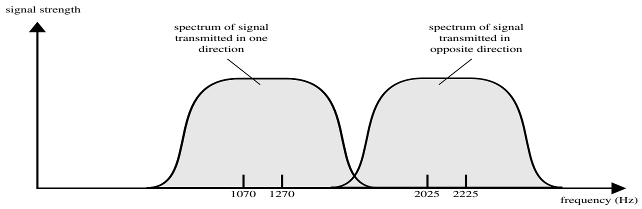
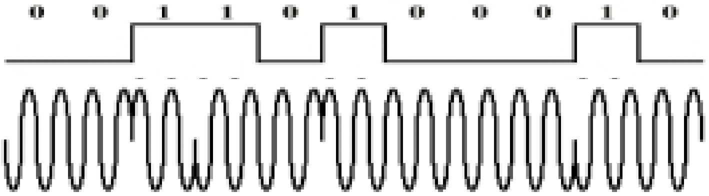


**Figure 5.8 Full-Duplex FSK Transmission on a Voice-Grade Line**

## Phase Shift Keying

- Phase of carrier signal is shifted to represent different values
- Two-phase system with differential PSK
  - Phase shift relative to previous bit transmitted rather than some constant reference signal
  - Binary 0 represented by sending a signal burst of same phase as previous signal burst
  - Binary 1 represented by sending a signal burst of opposite phase as previous signal burst
  - Phase shift of 180º

# Phase Shift Keying



(c) Phase-shift keying

- Resulting signal with phase measured relative to previous bit interval is

$$s(t)=\begin{cases} A\cos(2\pi f_c t+\pi) & \text{binary 1} \\ A\cos(2\pi f_c t) & \text{binary 0} \end{cases}$$

# Quadrature PSK (QPSK)

- More efficient Bandwidth use by each signal element representing more than one bit
  - Shifts of $\pi/2$ (90°)
  - Resulting signal is

$$s(t)=\begin{cases} A\cos(2\pi f_c t+\frac{\pi}{4}) & \text{binary } 11 \\ A\cos(2\pi f_c t+\frac{3\pi}{4}) & \text{binary } 10 \\ A\cos(2\pi f_c t+\frac{5\pi}{4}) & \text{binary } 00 \\ A\cos(2\pi f_c t+\frac{7\pi}{4}) & \text{binary } 01 \end{cases}$$

  - Each signal element represents two bits
  - Can use 8 phase angles and have more than one amplitude
  - 9600 bps modem uses 12 angles, four of which have two amplitudes

## Data Rate & Modulation rate

- In general

$$D = \frac{R}{b} = \frac{R}{\log_2 L}$$

  - D: modulation rate (signals per second or bauds)
  - R: data rate (bits per second)
  - L: number of different signal elements
  - b: number of bits per signal element
- With line signaling speed of 2400 baud
  - For NRZ-L, data rate is 2400 bps
  - For PSK, using L=16 different combinations of amplitude and phase, data rate is 9600 bps

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## Performance of Digital to Analog Modulation Schemes

- Bandwidth of modulated signal depends on factors such as Filtering technique used to create bandpass signal
- ASK and PSK bandwidth directly related to bit rate
- Transmission bandwidth $B_T$ for **ASK** and **PSK** is

$$B_T = (1 + r)R$$

  - R is data rate
  - r is related to filtering technique; 0< r <1
- Transmission bandwidth $B_T$ for **FSK** is

$$B_T = 2\Delta F + (1 + r)R$$

  - where $\Delta F = f_2 - f_c = f_c - f_1$

# Performance of Digital to Analog Modulation Schemes

- FSK bandwidth related to data rate for lower frequencies, but to offset of modulated frequency from carrier at high frequencies
  - For high frequencies ΔF term dominates
  - FSK signaling on coaxial cable multipoint network uses ΔF=1.25 MHz, $f_c$=5 MHz and R=1 Mbps
    - 2ΔF=2.5 MHz; dominant factor
  - For low frequency (voice grade line modem), ΔF=100 Hz, $f_c$=1170 Hz in one direction, and R=300 bps
    - The term (1+r)R dominates

# Performance of Digital to Analog Modulation Schemes

- With multilevel signaling, bandwidth can improve significantly

$$B_T = \left(\frac{1+r}{b}\right)R = \left(\frac{1+r}{\log_2 L}\right)R$$

- In the presence of noise, bit error rate of PSK and QPSK are about 3dB superior to ASK and FSK

## Bandwidth Efficiency

- Bandwidth efficiency is the ratio of data rate to transmission bandwidth

| | r=0 | r=0.5 | r=1 |
|---|---|---|---|
| **ASK** | 1.0 | 0.67 | 0.5 |
| **FSK (wideband)** | 0 | 0 | 0 |
| **FSK (narrowband)** | 1.0 | 0.67 | 0.5 |
| **PSK** | 1.0 | 0.67 | 0.5 |
| **L=4, b=2** | 2.0 | 1.33 | 1.0 |
| **L=8, b=3** | 3.0 | 2.00 | 1.5 |
| **L=16, b=4** | 4.0 | 2.67 | 2.0 |
| **L=32, b=5** | 5.0 | 3.33 | 2.5 |

## Bandwidth Efficiency & Bit Error Rate

- The bit error rate (BER) can be reduced by increasing $E_b/N_0$
- Bit error rate can be reduced by decreasing bandwidth efficiency
  - Increasing bandwidth
  - Decreasing data rate

$$\frac{E_b}{N_0} = \frac{S}{N_0 R} = \frac{S}{N}\frac{B_T}{R}$$

## Example

- ⌘ What is the bandwidth efficiency for FSK, ASK, PSK, and QPSK for a bit error rate of $10^{-7}$ on a channel with a SNR of 12dB ?

$$\frac{E_b}{N_0} = 12\ dB - \left(\frac{R}{B_T}\right)_{dB}$$

- ⌘ For FSK and ASK, $E_b/N_0$= 14.2dB, $(R/B_T)_{dB}$=-2.2dB, $R/B_T$=0.6

## Example

- ⌘ For PSK, $E_b/N_0$= 11.2dB, $(R/B_T)_{dB}$=0.8dB, $R/B_T$=1.2
- ⌘ For QPSK, D=R/2, $R/B_T$=2.4
- ⌘ For digital signaling

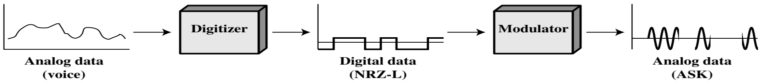
$$B_T = 0.5(1+r)D$$

- ⌘ For NRZ, D=R

$$\frac{R}{B_T} = \frac{2}{1+r}$$

# Analog Data, Digital Signal

- Digitization
  - Conversion of analog data into digital data
  - Digital data can be transmitted using NRZ-L
  - Digital data can be transmitted using code other than NRZ-L
  - Digital data can then be converted to analog signal
- Analog to digital conversion done using a codec
  - Pulse code modulation
  - Delta modulation



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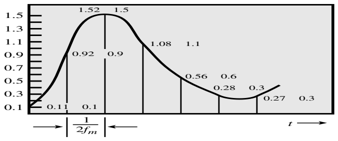
# Pulse Code Modulation (PCM)

- ***Sampling Theorem:*** If a signal is sampled at regular intervals at a rate higher than twice the highest signal frequency, the samples contain all the information of the original signal
- Signal maybe constructed from samples using a low-pass filter
- Voice data limited to below 4000Hz
- Require 8000 sample per second
- Analog samples (Pulse Amplitude Modulation, PAM)
- Each sample assigned digital value

# Pulse Code Modulation (PCM)



(a)

| Digit | Binary Equivalent | PCM waveform |
|---|---|---|
| 0 | 0000 | |
| 1 | 0001 | |
| 2 | 0010 | |
| 3 | 0011 | |
| 4 | 0100 | |
| 5 | 0101 | |
| 6 | 0110 | |
| 7 | 0111 | |
| 8 | 1000 | |
| 9 | 1001 | |
| 10 | 1010 | |
| 11 | 1011 | |
| 12 | 1100 | |
| 13 | 1101 | |
| 14 | 1110 | |
| 15 | 1111 | |

(b)

**Figure 5.10 Pulse-Code Modulation**

# Pulse Code Modulation (PCM)

- 4 bit system gives 16 levels
- Quantized
  - Quantizing error or noise
  - Approximations mean it is impossible to recover original exactly
  - SNR for quantizing error is

$$SNR = 20\log 2^n + 1.76\,dB = 6.02n + 1.76\,dB$$

  - For each additional bit used for quantizing, SNR increases by about 6 dB or a factor of 4
- 8 bit sample gives 256 levels
- Quality comparable with analog transmission
- 8000 samples per second of 8 bits each gives 64kbps

## PCM Example

- Suppose that we want to code an analog signal that has voltage levels 0-5v using 2-bit PCM.
- Then, we divide the the voltage level in four intervals such that the size of each interval is 5/4=1.25
  - 0-1.25, 1.25-2.5, 2.5-3.75, 3.75-5
- We choose the values to be in the middle of each interval.
  - Selected values are: 0.625, 1.875, 3.125, 4.375
  - This guarantees that the maximum quantization error is ½*5/4=0.625.

## Nonlinear Encoding

- Aboslute error for each sample is the same regardless of signal level
  - Lower amplitude values are relatively more distorted
- Solution is to make quantization levels not evenly spaced
- Greater number of quantization steps for lower amplitudes and smaller number of steps for higher amplitudes
- Reduces overall signal distortion

## Effect of Nonlinear Coding

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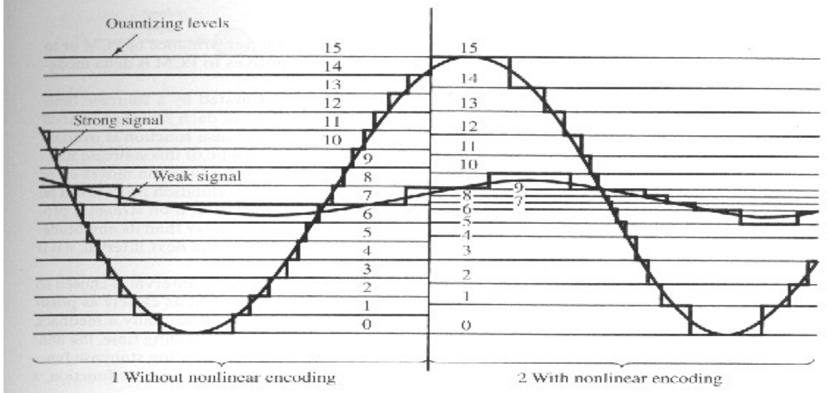


**Figure 5.11** Effect of Nonlinear Coding

## Companding

- ⌘ Effect of nonlinear coding can also be reduced by companding
  - Compressing-expanding
  - More gain to weak signals than to strong signals on input
  - Reverse operation at output

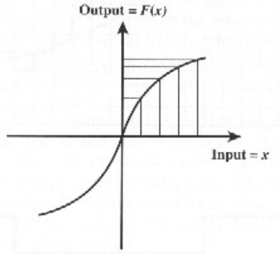
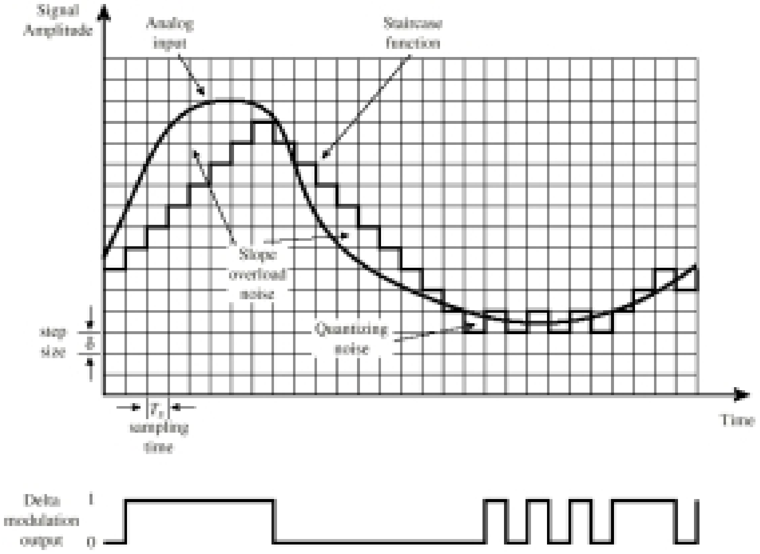


**Figure 5.12** Typical Companding Function

## Delta Modulation

- Analog input is approximated by a staircase function
- Move up or down one level (δ) at each sample interval
- Binary behavior
  - Function moves up or down at each sample interval
- A bit stream produced approximates derivative of analog signal rather than its amplitude
  - Produce a 1 if stair function is to go up
  - Produce a 0 if stair function is to go down

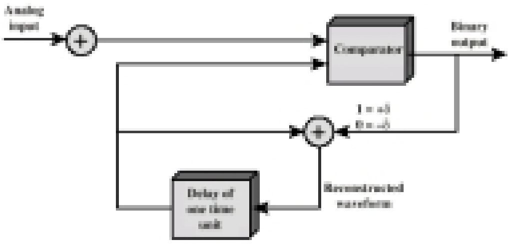
## Delta Modulation - example



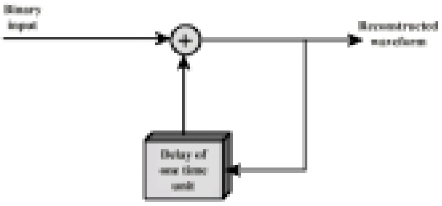
## Delta Modulation - Operation

- Analog input compared to most recent value of approximating staircase function
  - If value exceeds staircase function, generate a 1
  - Otherwise generate a 0
- Output of DM process is a binary sequence to be used for reconstructing staircase function
  - Reconstructed stair function is smoothed by a low pass filter to reconstruct approximated analog signal

## Delta Modulation - Operation



(a) Transmission



(b) Reception

## Delta Modulation

- Two important parameters in DM scheme
  - Size of step assigned to each binary digit δ
    - Must be chosen to produce a balance between two types of errors or noise
    - If waveform changes slowly, quantizing noise increases with increase in δ
    - If waveform changes rapidly, slope overload noise increases with decrease in δ
  - Increasing sampling rate
    - improves the accuracy of the scheme
    - Increases data rate
- Principal advantage of DM is implementation simplicity
- PCM has better SNR at same data rate

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## CODEC - Performance

- Good voice reproduction
  - PCM - 128 levels (7 bit)
  - Voice bandwidth 4 KHZ
  - Data rate should be 8000 x 7 = 56 kbps for PCM
- Bandwidth requirement
  - Digital transmission requires 56 kbps for 4 KHz analog signal
  - Using Nyquist theorem, this signal requires in the order of 28 KHz of Bandwidth

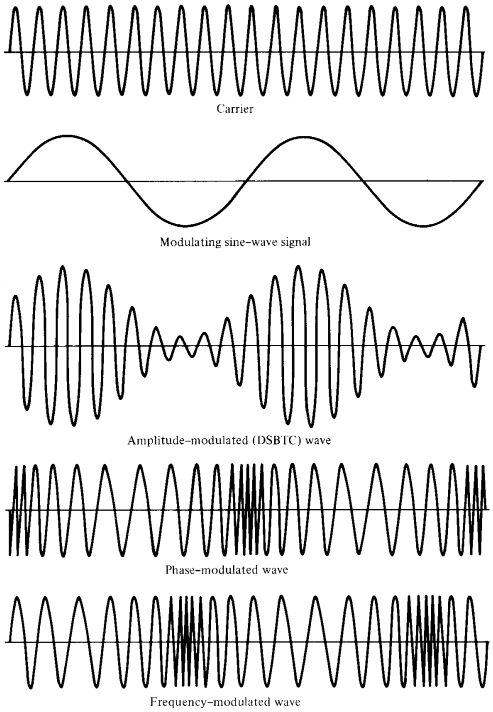
## CODEC - Performance

- A common PCM scheme for color TV uses 10-bit codes
  - For bandwidth=4.6 MHz => 92 Mbps
- Digital techniques continue to grow in popularity
  - Repeaters used with no additive noise
  - Time-division multiplexing (TDM) is sued for digital signals with no intermodulation noise
  - Use more efficient digital switching techniques
- More efficient codes are used to reduce required bit rate

## Analog Data, Analog Signals

- Modulation
  - Combining an input signal *m(t)* and a carrier at frequency $f_c$ to produce signal *s(t)* with bandwidth centered on $f_c$
- Why modulate analog signals?
  - Higher frequency may be needed for effective transmission
    - For unguided transmission, impossible to send baseband signals as required antennas would be kilometers in diameter
  - Permits frequency division multiplexing
- Types of modulation
  - Amplitude
  - Frequency
  - Phase

# Analog Modulation



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# Amplitude Modulation

- Simplest form of modulation
- Signal is expressed as

$$s(t) = [1 + n_a x(t)] \cos 2\pi f_c t$$

  - $\cos 2\pi f_c t$ is carrier and x(t) is input signal both normalized to unity amplitude
  - $n_a$ is modulation index equal to ratio of amplitude of input signal to carrier
  - Additive 1 is a a DC component to prevent loss of information
  - Scheme is known as double sideband transmitted carrier (DSBTC)

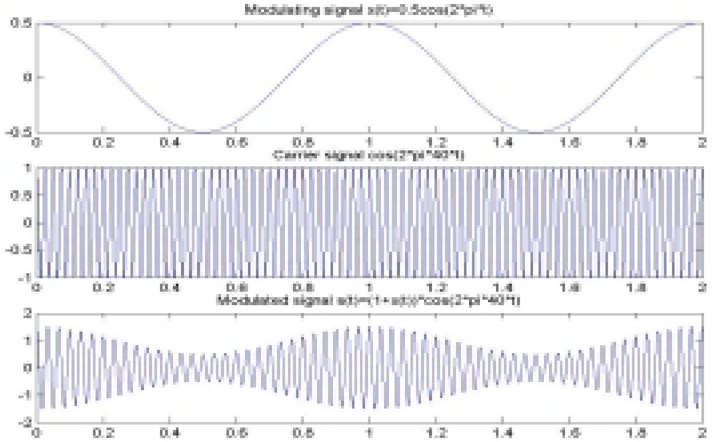
# Amplitude Modulation - Example

- Given the amplitude-modulating signal $x(t)=\cos 2\pi f_m t$ , find s(t)

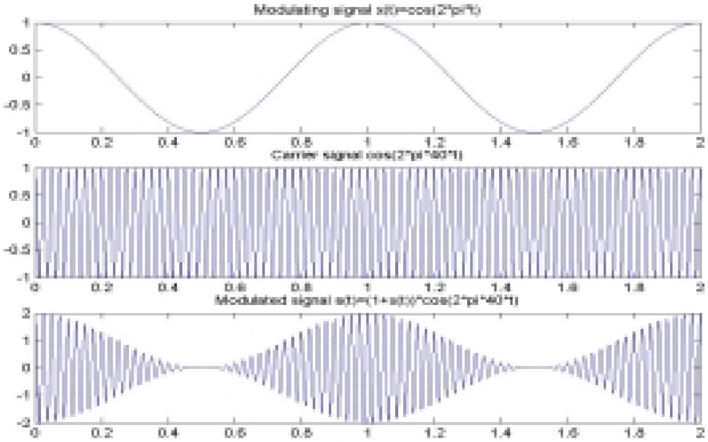
$$s(t) = [1 + n_a \cos 2\pi f_m t] \cos 2\pi f_c t$$
$$= \cos 2\pi f_c t + \frac{n_a}{2} \cos 2\pi (f_c \pm f_m) t$$

  - Resulting signal has a component at original carrier frequency as well as a pair of components each spaced $f_m$ Hz from the carrier
  - Envelope of resulting signal is $[1+n_a x(t)]$
    - ☒ With $n_a < 1$, envelope is exact reproduction of signal
    - ☒ With $n_a > 1$, envelope crosses the time axis and information is lost

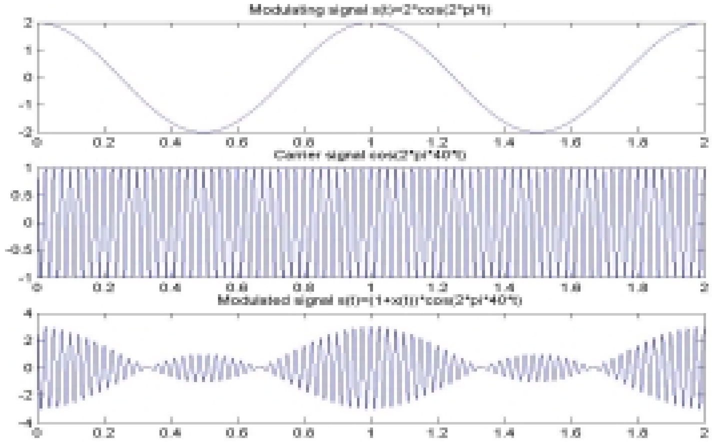
# Amplitude Modulation - Example



# Amplitude Modulation - Example



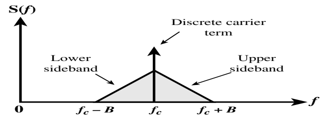
# Amplitude Modulation - Example



# Spectrum of an AM signal

- Spectrum of AM signal is original carrier plus spectrum of original signal translated to $f_c$
- Portion of spectrum $f > f_c$ is upper sideband
- Portion of spectrum $f < f_c$ is lower sideband
- Example: voice signal 300-3000Hz With $f_c$ 60 KHz
  - Upper sideband is 60.3-63 KHz
  - Lower sideband is 57-59.7 KHz

(a) Spectrum of modulating signal



(b) Spectrum of AM signal with carrier at $f_c$

**Figure 5.16 Spectrum of an AM Signal**

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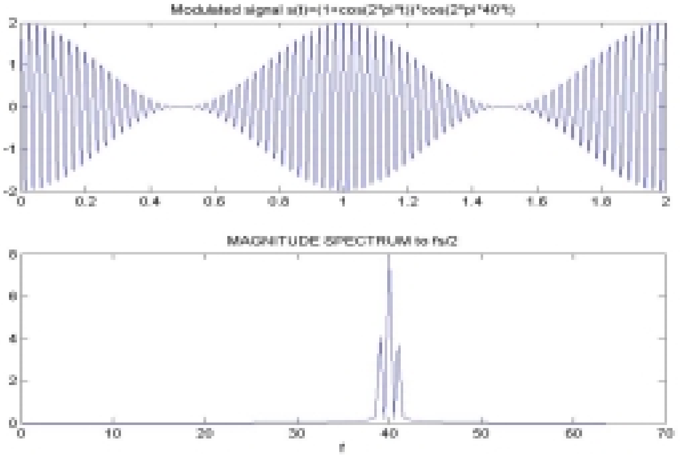
# Spectrum of an AM signal



# Amplitude Modulation

- Total transmitted power $P_t$ in *s(t)* is given by

$$P_t = P_c\left(1 + \frac{n_a^2}{2}\right)$$

  - $P_c$ is transmitted power in carrier
  - $n_a$ should be maximized (but <1) to allow most of signal power to carry information
- S(t) contains unnecessary information
  - Each of the sidebands contains complete spectrum of input
- Carrier used for synchronization purposes
- SSB: single sideband, eliminates one sideband and carrier
- DSBSC: double sideband suppressed carrier, carrier is not transmitted

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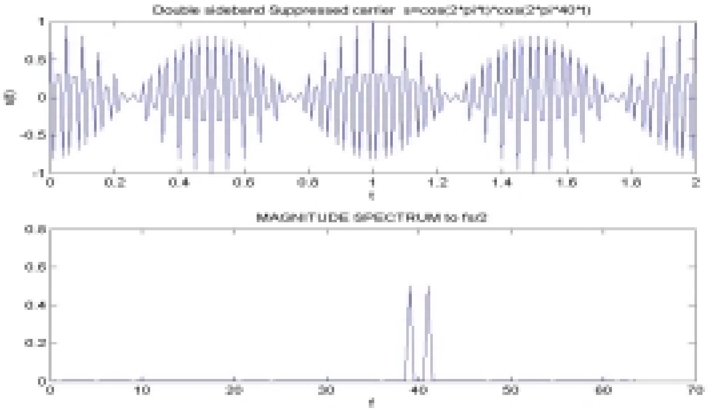
# Double Sideband Suppressed Carrier - Example

- Signal is expressed as $s(t) = A_c[m(t)]\cos 2\pi f_c t$



# Angle Modulation

- Encompasses frequency modulation (FM) and phase modulation (PM) as special cases
- Modulated signal is given by

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

- Phase modulation
  - Phase is proportional to modulating signal $\phi(t) = n_p m(t)$
  - $n_p$ is phase modulation index
- Frequency modulation
  - Derivative of phase proportional to modulating signal $\phi'(t) = n_f m(t)$
  - $n_f$ is frequency modulation index

---

# Angle Modulation

- The phase of s(t) at any instant is $2\pi f_c t + \phi(t)$
- Instantaneous phase deviation from carrier is $\phi(t)$
- In PM, $\phi(t) = n_p m(t)$, instantaneous phase deviation from carrier is proportional to m(t)
- Frequency can be defined as rate of change of phase of a signal
- Instantaneous frequency of s(t) is

$$2\pi f_i(t) = \frac{d}{dt}[2\pi f_c t + \phi(t)]$$

$$f_i(t) = f_c + \frac{1}{2\pi}\phi'(t)$$

  - Instantaneous frequency deviation from carrier frequency is $\phi'(t)$ which is in FM proportional to m(t)

## Angle Modulation

- Peak deviation $\Delta F$ is given as
  - $A_m$ is maximum value of m(t)

$$\Delta F = \frac{1}{2\pi} n_f A_m \ \text{HZ}$$

- An increase in magnitude of m(t) will increase $\Delta F$ which increases transmitted bandwidth $B_T$
- Average power level of FM signal is $A_C^2/2$, which does not increase with increasing $A_m$.
- In AM, $A_m$ affects the power in the AM signal but does not affect bandwidth

## Phase Modulation - Example

- Derive an expression for s(t) if phase-modulating signal $\phi(t) = n_p \cos 2\pi f_m t$; assume $A_c = 1$

$$s(t) = \cos[\,2\pi f_c t + n_p \cos 2\pi f_m t\,]$$

- Using Bessel's trigonometric identities where $J_n(n_p)$ is the nth order Bessel function:

$$s(t) = \sum_{n=-\infty}^{\infty} J_n(n_p) \cos[\,2\pi f_c t + 2\pi n f_m t + \frac{n\pi}{2}\,]$$

## Phase Modulation - Example

- Using the following properties:

$$J_{-n}(x) = (-1)^n J_n(x)$$

$$(-1)^n \cos(\theta - \frac{n\pi}{2}) = \cos(\theta + \frac{n\pi}{2})$$

- S(t) can be written as

$$s(t) = J_0(n_p)\cos 2\pi f_c t + \sum_{n=1}^{\infty} J_n(n_p)[\cos(2\pi(f_c + nf_m)t + \frac{n\pi}{2})$$
$$+\cos(2\pi(f_c - nf_m)t + \frac{n\pi}{2})]$$

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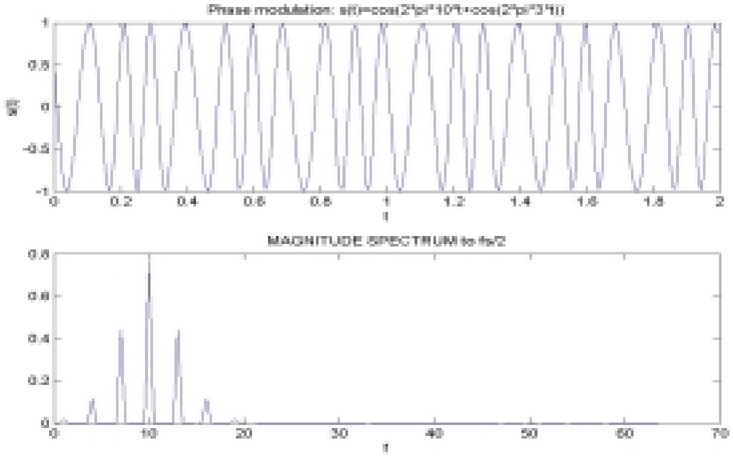
## Phase Modulation - Example



# Frequency Modulation - Example

- Derive an expression for s(t) if frequency-modulating signal $\phi'(t) = -n_f \sin 2\pi f_m t$

$$\phi(t) = \int -n_f \sin 2\pi f_m t \; dt = \frac{n_f}{2\pi f_m} \cos 2\pi f_m t$$

$$s(t) = \cos[2\pi f_c t + \frac{n_f}{2\pi f_m} \cos 2\pi f_m t]$$

$$s(t) = \cos[2\pi f_c t + \frac{\Delta F}{f_m} \cos 2\pi f_m t]$$

---

# Bandwidth Requirement

- AM, FM, and PM result in a signal whose bandwidth is centered at $f_c$
- For AM, $B_T = 2B$
- Angle modulation includes a term of the form $\cos(\phi(t))$ which is nonlinear producing a wide range of frequencies $f_c + f_m$, $f_c + 2f_m$, ...
- Infinite bandwidth is required to transmit an FM or PM signal

## Bandwidth Requirement

- Rule of thumb (Carson's rule)

$$B_T = 2(\beta + 1)B$$

$$\beta = \begin{cases} n_p A_m & \text{for PM} \\ \dfrac{\Delta \mathrm{F}}{\mathrm{B}} = \dfrac{n_f A_m}{2\pi B} & \text{for FM} \end{cases}$$

- For FM, $B_T = 2\Delta F + 2B$
- Both FM and PM require greater bandwidth than AM

## Quadrature Amplitude Modulation (QAM)

- Popular analog signaling technique used in asymmetric digital subscriber line (ADSL)
- Combination of amplitude and phase modulation
- Two signals transmitted simultaneously on same carrier frequency using two copies of carrier one shifted by 90°
- Each carrier is ASK modulated
- Input is a stream of binary digits arriving at a rate of R bps
  - Converted into two separate bits streams of R/2 bps

## Quadrature Amplitude Modulation (QAM)

- One stream is ASK modulated on a carrier of frequency $f_c$
- Other stream is ASK modulated on a carrier of frequency $f_c$ shifted by $90^o$
- The two modulated signals are combined together and transmitted
- Transmitted signal can be expressed as

$$s(t) = d_1(t)\cos 2\pi f_c t + d_2(t)\sin 2\pi f_c t$$

## QAM Modulator

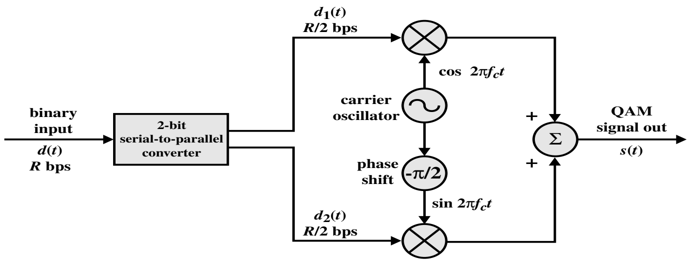


**Figure 5.18 QAM Modulator**

# Spread Spectrum

- Can be used to transmit analog or digital data using analog signal
- Spread data over wide bandwidth
- Makes jamming and interception harder

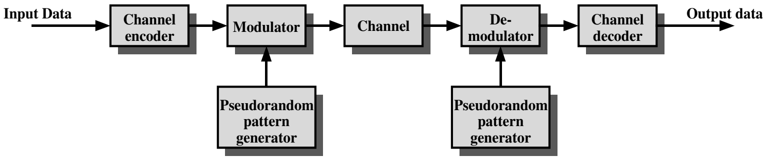


**Figure 5.19 General Model of Spread Spectrum Digital Communication System**

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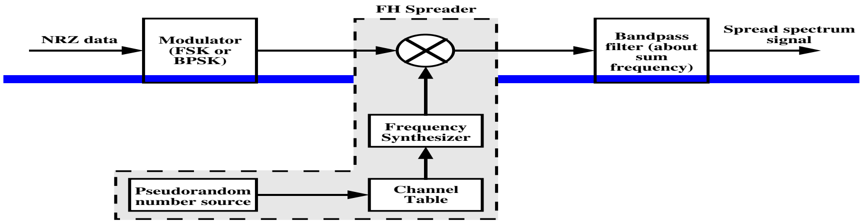
# Spread Spectrum

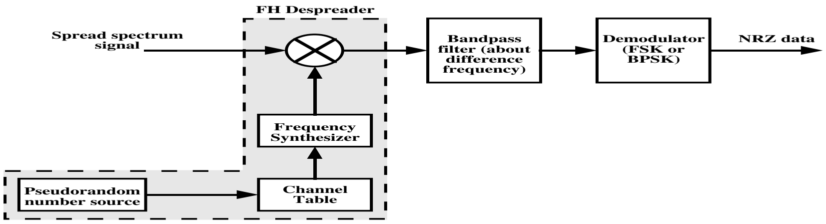
- Channel encoder receives input and converts it into analog signal with narrow bandwidth around center frequency
- Signal is further modulated using a pseudorandom sequence
- Modulation spreads the spectrum (increases bandwidth) of signal to be transmitted
- Same pseudorandom sequence used to demodulate the spread spectrum signal

# Frequency hoping Spread Spectrum

- Signal broadcast over seemingly random series of frequencies
- Hopping from one to another frequency in split-second intervals
- Receiver also hops on the same frequencies in synchronization with sender
- Difficult to catch and jam the signal without knowing the frequencies



(a) Transmitter



(b) Receiver

**Figure 5.20 Frequency Hopping Spread Spectrum System**

# Direct Sequence Spread Spectrum

- Each bit is represented by multiple bits in transmitted signal
- Multiple bits known as Chipping code
- Chipping code spreads signal across a wider frequency band in direct proportion to number of bits used
  - A 10-bit chipping code spreads signal across a frequency band 10 times larger than 1-bit code
- Combine digital information stream with pseudorandom bit stream using exclusive-or

# Direct Sequence Spread Spectrum

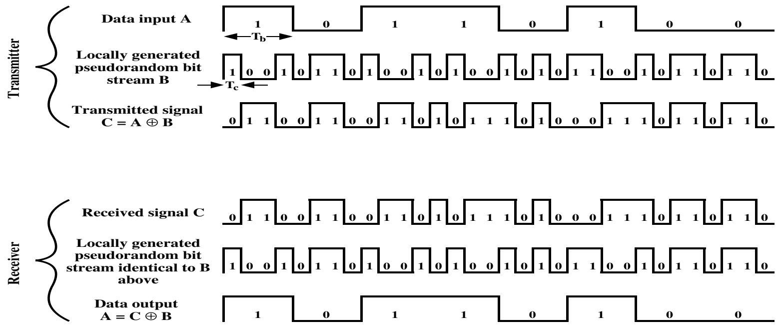
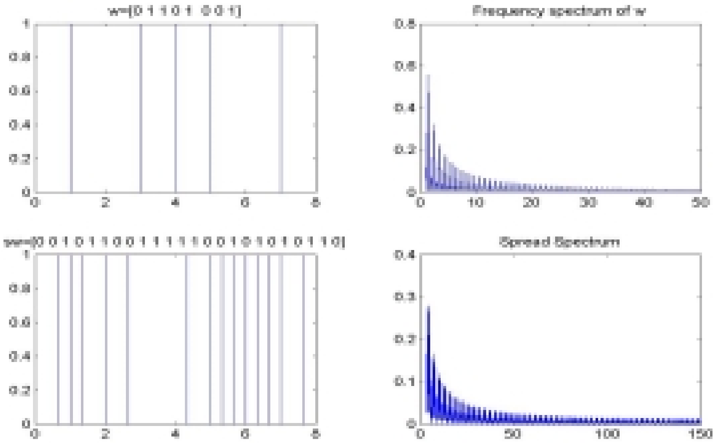
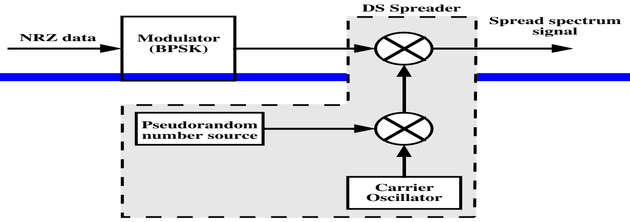


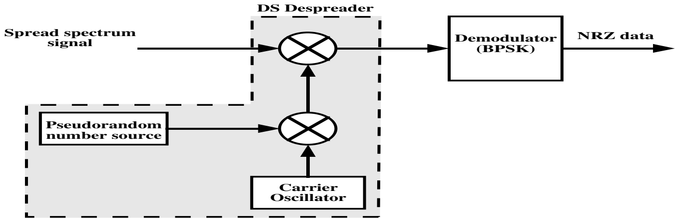
**Figure 5.21 Example of Direct Sequence Spread Spectrum**

# Direct Sequence Spread Spectrum - Example





(a) Transmitter



(b) Receiver

**Figure 5.22 Direct Sequence Spread Spectrum System**